10. ELECTROWEAK MODEL AND CONSTRAINTS ON NEW PHYSICS

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10.1. Introduction

The standard electroweak model is based on the gauge group [1] SU(2) × U(1), with gauge bosons W^i_μ, $i = 1, 2, 3$, and B_μ for the SU(2) and U(1) factors, respectively, and the corresponding gauge coupling constants g and g'. The left-handed fermion fields $\psi_i = \begin{pmatrix} \nu_i \\ \ell^-_i \end{pmatrix}$ and $\begin{pmatrix} u_i \\ d'_i \end{pmatrix}$ of the i^{th} fermion family transform as doublets under SU(2), where $d'_i \equiv \sum_j V_{ij}\, d_j$, and V is the Cabibbo-Kobayashi-Maskawa mixing matrix. (Constraints on V are discussed in the section on the Cabibbo-Kobayashi-Maskawa mixing matrix.) The right-handed fields are SU(2) singlets. In the minimal model there are three fermion families and a single complex Higgs doublet $\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.

After spontaneous symmetry breaking the Lagrangian for the fermion fields is

$$\begin{aligned}\mathscr{L}_F = &\sum_i \overline{\psi}_i \left(i \not\partial - m_i - \frac{g m_i H}{2M_W} \right) \psi_i \\ &- \frac{g}{2\sqrt{2}} \sum_i \overline{\psi}_i\, \gamma^\mu\, (1-\gamma^5)(T^+\, W^+_\mu + T^-\, W^-_\mu)\, \psi_i \\ &- e \sum_i q_i\, \overline{\psi}_i\, \gamma^\mu\, \psi_i\, A_\mu \\ &- \frac{g}{2\cos\theta_W} \sum_i \overline{\psi}_i\, \gamma^\mu (g^i_V - g^i_A \gamma^5)\, \psi_i\, Z_\mu\ . \end{aligned} \tag{10.1}$$

$\theta_W \equiv \tan^{-1}(g'/g)$ is the weak angle; $e = g \sin\theta_W$ is the positron electric charge; and $A \equiv B\cos\theta_W + W^3 \sin\theta_W$ is the (massless) photon field. $W^\pm \equiv (W^1 \mp i W^2)/\sqrt{2}$ and $Z \equiv -B \sin\theta_W + W^3 \cos\theta_W$ are the massive charged and neutral weak boson fields, respectively. T^+ and T^- are the weak isospin raising and lowering operators. The vector and axial couplings are

$$g^i_V \equiv t_{3L}(i) - 2q_i \sin^2\theta_W\ , \tag{10.2}$$

$$g^i_A \equiv t_{3L}(i)\ , \tag{10.3}$$

where $t_{3L}(i)$ is the weak isospin of fermion i ($+1/2$ for u_i and ν_i; $-1/2$ for d_i and e_i) and q_i is the charge of ψ_i in units of e.

The second term in $\mathscr{L}_F$ represents the charged-current weak interaction [2]. For example, the coupling of a W to an electron and a neutrino is

$$-\frac{e}{2\sqrt{2}\sin\theta_W} \left[W^-_\mu\ \overline{e}\ \gamma^\mu (1-\gamma^5)\nu + W^+_\mu\ \overline{\nu}\ \gamma^\mu\ (1-\gamma^5) e \right]\ . \tag{10.4}$$

For momenta small compared to M_W, this term gives rise to the effective four-fermion interaction with the Fermi constant given (at tree level, *i.e.*, lowest order in perturbation theory) by $G_F/\sqrt{2} = g^2/8M_W^2$. CP violation is incorporated in the Standard Model by a single observable phase in V_{ij}. The third term in $\mathscr{L}_F$ describes electromagnetic interactions (QED), and the last is the weak neutral-current interaction.

In Eq. (10.1), m_i is the mass of the i^{th} fermion ψ_i. For the quarks these are the current masses. For the light quarks, as described in the Particle Listings, $\overline{m}_u \approx 2-8$ MeV, $\overline{m}_d \approx 5-15$ MeV, and $\overline{m}_s \approx 100-300$ MeV (these are running $\overline{\rm MS}$ masses evaluated at $\mu = 1$ GeV). For the heavier quarks, the $\overline{\rm MS}$ masses are $\overline{m}_c(\mu = \overline{m}_c) \approx 1.0-1.6$ GeV and $\overline{m}_b(\mu = \overline{m}_b) \approx 4.1-4.5$ GeV. The average of the recent CDF [4] and DØ [5] values for the top quark "pole" mass is $m_t = 175 \pm 5$ GeV. See "The Note on Quark Masses" in the Particle Listings for more information.

H is the physical neutral Higgs scalar which is the only remaining part of ϕ after spontaneous symmetry breaking. The Yukawa coupling of H to ψ_i, which is flavor diagonal in the minimal model, is $g m_i/2M_W$. The H mass is not predicted by the model. Experimental limits are given in the Higgs section. In nonminimal models there are additional charged and neutral scalar Higgs particles [6].

10.2. Renormalization and radiative corrections

The Standard Model has three parameters (not counting M_H and the fermion masses and mixings). A particularly useful set is:

(*a*) The fine structure constant $\alpha = 1/137.0359895$ (61), determined from the quantum Hall effect. In most electroweak-renormalization schemes, it is convenient to define a running α dependent on the energy scale of the process, with $\alpha^{-1} \sim 137$ appropriate at low energy. (The running has recently been observed directly [7].) At energies of order M_Z, $\alpha^{-1} \sim 128$. For example, in the modified minimal subtraction ($\overline{\rm MS}$) scheme [8], one has $\widehat{\alpha}(M_Z)^{-1} = 127.88 \pm 0.09$, while the conventional (on-shell) QED renormalization yields [9] $\alpha(M_Z)^{-1} = 128.88 \pm 0.09$, which differs by finite constants from $\widehat{\alpha}(M_Z)^{-1}$. The uncertainty, due to the low-energy hadronic contribution to vacuum polarization, is the dominant theoretical uncertainty in the interpretation of precision data. Other recent evaluations [10–14] of this effect are in reasonable agreement. Further improvement will require better measurements of the cross section for $e^+e^- \to$ hadrons at low energy.

(*b*) The Fermi constant, $G_F = 1.16639(1) \times 10^{-5}$ GeV^{-2}, determined from the muon lifetime formula [15],

$$\begin{aligned}\tau_\mu^{-1} = &\frac{G_F^2 m_\mu^5}{192\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right) \\ &\times \left[1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]\ ,\end{aligned} \tag{10.5a}$$

where

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x\ , \tag{10.5b}$$

and

$$\alpha(m_\mu)^{-1} = \alpha^{-1} - \frac{2}{3\pi}\ln\left(\frac{m_\mu}{m_e}\right) + \frac{1}{6\pi} \approx 136\ , \tag{10.5c}$$

where the uncertainty in G_F is from the input quantities. There are additional uncertainties from higher order radiative corrections, which can be estimated from the magnitude of the known $\alpha^2 \ln(m_\mu/m_e)$ term of $\sim 1.8 \times 10^{-10}$ (alternatively, one can view Eq. (10.5) as the exact definition of G_F; then the theoretical uncertainty appears instead in the formulae for quantities derived from G_F).

(*c*) $\sin^2\theta_W$, determined from the Z mass and other Z pole observables, the W mass, and neutral-current processes [16]. The value of $\sin^2\theta_W$ depends on the renormalization prescription. There are a number of popular schemes [18–23] leading to $\sin^2\theta_W$ values which differ by small factors which depend on m_t and M_H. The notation for these schemes is shown in Table 10.1. Discussion of the schemes follows the table.

(*i*) The on-shell scheme promotes the tree-level formula $\sin^2\theta_W = 1 - M_W^2/M_Z^2$ to a definition of the renormalized $\sin^2\theta_W$ to all orders in perturbation theory, *i.e.*, $\sin^2\theta_W \to s_W^2 \equiv 1 - M_W^2/M_Z^2$. This scheme is simple conceptually. However, M_W is known much less precisely than M_Z and in practice one extracts s_W^2 from M_Z alone using

$$M_W = \frac{A_0}{s_W(1-\Delta r)^{1/2}}\ , \tag{10.6a}$$

$$M_Z = \frac{M_W}{c_W}\ , \tag{10.6b}$$

Table 10.1: Notations used to indicate the various schemes discussed in the text. Each definition of $\sin\theta_W$ leads to values that differ by small factors depending on m_t and M_H.

Scheme	Notation
On-shell	$s_W = \sin\theta_W$
NOV	$s_{M_Z} = \sin\theta_W$
$\overline{\text{MS}}$	$\widehat{s}_Z = \sin\theta_W$
$\overline{\text{MS}}$ *ND*	$\widehat{s}_{ND} = \sin\theta_W$
Effective angle	$\overline{s}_f = \sin\theta_W$

where $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, $A_0 = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.2802$ GeV, and Δr includes the radiative corrections relating α, $\alpha(M_Z)$, G_F, M_W, and M_Z. One finds $\Delta r \sim \Delta r_0 - \rho_t/\tan^2\theta_W$, where $\Delta r_0 \approx 1 - \alpha/\alpha(M_Z) \approx 0.06$ is due to the running of α and $\rho_t = 3G_F m_t^2/8\sqrt{2}\pi^2 \approx 0.0096(m_t/175 \text{ GeV})^2$ represents the dominant (quadratic) m_t dependence. There are additional contributions to Δr from bosonic loops, including those which depend logarithmically on the Higgs mass M_H. One has $\Delta r = 0.0349 \mp 0.0019 \pm 0.0007$ for $(m_t, M_H) = (175 \pm 5 \text{ GeV}, M_Z)$, where the second uncertainty is from $\alpha(M_Z)$. Thus the value of s_W^2 extracted from M_Z includes a large uncertainty (∓ 0.0006) from the currently allowed range of m_t.

(*ii*) A more precisely determined quantity $s_{M_Z}^2$ can be obtained from M_Z by removing the (m_t, M_H) dependent term from Δr [19], *i.e.*,

$$s_{M_Z}^2 c_{M_Z}^2 \equiv \frac{\pi\alpha(M_Z)}{\sqrt{2}\,G_F\,M_Z^2} \ . \tag{10.7}$$

This yields $s_{M_Z}^2 = 0.23116 \pm 0.00022$, with most of the uncertainty from α rather than M_Z. Scheme (*ii*) is equivalent to using M_Z rather than $\sin^2\theta_W$ as the third fundamental parameter. However, it recognizes that $s_{M_Z}^2$ is still a useful derived quantity. The small uncertainty in $s_{M_Z}^2$ compared to other schemes is because the m_t dependence has been removed by definition. However, the m_t uncertainty reemerges when other quantities (*e.g.*, M_W or other Z pole observables) are predicted in terms of M_Z.

Both s_W^2 and $s_{M_Z}^2$ depend not only on the gauge couplings but also on the spontaneous-symmetry breaking, and both definitions are awkward in the presence of any extension of the Standard Model which perturbs the value of M_Z (or M_W). Other definitions are motivated by the tree-level coupling constant definition $\theta_W = \tan^{-1}(g'/g)$.

(*iii*) In particular, the modified minimal subtraction ($\overline{\text{MS}}$) scheme introduces the quantity $\sin^2\widehat{\theta}_W(\mu) \equiv \widehat{g}'^2(\mu)/\left[\widehat{g}^2(\mu) + \widehat{g}'^2(\mu)\right]$, where the couplings $\widehat{g}$ and $\widehat{g}'$ are defined by modified minimal subtraction and the scale μ is conveniently chosen to be M_Z for electroweak processes. The value of $\widehat{s}_Z^2 = \sin^2\widehat{\theta}_W(M_Z)$ extracted from M_Z is less sensitive than s_W^2 to m_t (by a factor of $\tan^2\theta_W$), and is less sensitive to most types of new physics than s_W^2 or $s_{M_Z}^2$. It is also very useful for comparing with the predictions of grand unification. There are actually several variant definitions of $\sin^2\widehat{\theta}_W(M_Z)$, differing according to whether or how finite $\alpha\ln(m_t/M_Z)$ terms are decoupled (subtracted from the couplings). One cannot entirely decouple the $\alpha\ln(m_t/M_Z)$ terms from all electroweak quantities because $m_t \gg m_b$ breaks SU(2) symmetry. The scheme that will be adopted here decouples the $\alpha\ln(m_t/M_Z)$ terms from the $\gamma - Z$ mixing [8,20], essentially eliminating any $\ln(m_t/M_Z)$ dependence in the formulae for asymmetries at the Z pole when written in terms of $\widehat{s}_Z^2$. The various definitions are related by

$$\widehat{s}_Z^2 = c\,(m_t, M_H) s_W^2 = \overline{c}\,(m_t, M_H)\, s_{M_Z}^2 \ , \tag{10.8}$$

where $c = 1.0376 \pm 0.0021$ for $m_t = 175 \pm 5$ GeV and $M_H = M_Z$. Similarly, $\overline{c} = 1.0003 \mp 0.0007$. The quadratic m_t dependence is given by $c \sim 1 + \rho_t/\tan^2\theta_W$ and $\overline{c} \sim 1 - \rho_t/(1 - \tan^2\theta_W)$, respectively. The expressions for M_W and M_Z in the $\overline{\text{MS}}$ scheme are

$$M_W = \frac{A_0}{\widehat{s}_Z(1 - \Delta\widehat{r}_W)^{1/2}} \ , \tag{10.9a}$$

$$M_Z = \frac{M_W}{\widehat{\rho}^{1/2}\widehat{c}_Z} \ . \tag{10.9b}$$

One predicts $\Delta\widehat{r}_W = 0.0698 \pm 0.0001 \pm 0.0007$ for $m_t = 175 \pm 5$ GeV and $M_H = M_Z$. $\Delta\widehat{r}_W$ has no quadratic m_t dependence, because shifts in M_W are absorbed into the observed G_F, so that the error in $\Delta\widehat{r}_W$ is dominated by $\Delta r_0 = 1 - \alpha/\alpha(M_Z)$, which induces the second quoted uncertainty. Similarly, $\widehat{\rho} \sim 1 + \rho_t$. Including bosonic loops, $\widehat{\rho} = 1.0109 \pm 0.0006$ for $(m_t, M_H) = (175 \pm 5 \text{ GeV}, M_Z)$.

(*iv*) A variant $\overline{\text{MS}}$ quantity $\widehat{s}_{\text{ND}}^2$ (used in the 1992 edition of this *Review*) does not decouple the $\alpha\ln(m_t/M_Z)$ terms [21]. It is related to $\widehat{s}_Z^2$ by

$$\widehat{s}_Z^2 = \widehat{s}_{\text{ND}}^2 \Big/ \left(1 + \frac{\widehat{\alpha}}{\pi} d\right) \ , \tag{10.10a}$$

$$d = \frac{1}{3}\left(\frac{1}{\widehat{s}^2} - \frac{8}{3}\right)\left[(1 + \frac{\widehat{\alpha}_s}{\pi}) \ln\frac{m_t}{M_Z} - \frac{15\widehat{\alpha}_s}{8\pi}\right] \tag{10.10b}$$

where $\widehat{\alpha}_s$ is the QCD coupling at M_Z. Thus, $\widehat{s}_Z^2 - \widehat{s}_{\text{ND}}^2 \sim -0.0002$ for $m_t = 175$ GeV.

(*v*) Yet another definition, the effective angle [22,23] $\overline{s}_f^2$ for Z coupling to fermion f, is described at the end of Sec. 10.3.

Experiments are now at such a level of precision that complete $\mathcal{O}(\alpha)$ radiative corrections must be applied. For neutral-current and Z pole processes, these corrections are conveniently divided into two classes:

1. QED diagrams involving the emission of real photons or the exchange of virtual photons in loops, but not including vacuum polarization diagrams. These graphs often yield finite and gauge-invariant contributions to observable processes. However, they are dependent on energies, experimental cuts, *etc.*, and must be calculated individually for each experiment.
2. Electroweak corrections, including $\gamma\gamma$, γZ, ZZ, and WW vacuum polarization diagrams, as well as vertex corrections, box graphs, *etc.*, involving virtual W's and Z's. Many of these corrections are absorbed into the renormalized Fermi constant defined in Eq. (10.5). Others modify the tree-level expressions for Z pole observables and neutral-current amplitudes in several ways [16]. One-loop corrections are included for all processes. In addition, certain two-loop corrections are also important. In particular, two-loop corrections involving the top-quark modify ρ_t in $\widehat{\rho}$, Δr, and elsewhere by

$$\rho_t \to \rho_t[1 + R(M_H, m_t)\rho_t/3] \ . \tag{10.11}$$

$R(M_H, m_t)$ is best described as an expansion in M_Z^2/m_t^2. The unsuppressed terms were first obtained in Ref. 24, and are known analytically [25]. Contributions proportional to M_Z^2/m_t^2 were studied in Ref. 26 with the help of small and large Higgs mass expansions, which can be interpolated. These contributions are about as large as the leading ones in Refs. 24 and 25. Very recently, a subset of the relevant two-loop diagrams has been calculated numerically without any heavy mass expansion [27]. This serves as a valuable check on the M_H dependence of the leading terms obtained in Refs. 24–26. The difference turned out to be small. For M_H above its lower direct limit, $-17 < R < -11$. Mixed QCD-electroweak loops of order $\alpha\alpha_s m_t^2$ [28] and $\alpha\alpha_s^2 m_t^2$ [29]

increase the predicted value of m_t by 6%. This is, however, almost entirely an artifact of using the pole mass definition for m_t. The equivalent corrections when using the $\overline{\rm MS}$ definition $\overline{m}_t(\overline{m}_t)$ increase m_t by less than 0.5%. The leading electroweak [24,25] and mixed [30] two-loop terms are also known for the $Z \to b\bar{b}$ vertex, but not the respective subleading ones.

10.3. Cross-section and asymmetry formulas

It is convenient to write the four-fermion interactions relevant to ν-hadron, νe, and parity violating e-hadron neutral-current processes in a form that is valid in an arbitrary gauge theory (assuming massless left-handed neutrinos). One has

$$-\mathscr{L}^{\nu{\rm Hadron}} = \frac{G_F}{\sqrt{2}}\,\overline{\nu}\,\gamma^\mu\,(1-\gamma^5)\nu$$

$$\times \sum_i \left[\epsilon_L(i)\,\overline{q}_i\,\gamma_\mu(1-\gamma^5)q_i + \epsilon_R(i)\,\overline{q}_i\,\gamma_\mu(1+\gamma^5)q_i\right] \;, \qquad (10.12)$$

$$-\mathscr{L}^{\nu e} = \frac{G_F}{\sqrt{2}}\,\overline{\nu}_\mu\,\gamma^\mu(1-\gamma^5)\nu_\mu\,\overline{e}\,\gamma_\mu(g_V^{\nu e} - g_A^{\nu e}\gamma^5)e \qquad (10.13)$$

(for $\nu_e e$ or $\overline{\nu}_e e$, the charged-current contribution must be included), and

$$-\mathscr{L}^{e{\rm Hadron}} = -\frac{G_F}{\sqrt{2}}$$

$$\times \sum_i \left[C_{1i}\,\overline{e}\,\gamma_\mu\,\gamma^5\,e\,\overline{q}_i\,\gamma^\mu\,q_i + C_{2i}\,\overline{e}\,\gamma_\mu\,e\,\overline{q}_i\,\gamma^\mu\,\gamma^5\,q_i\right] \;. \qquad (10.14)$$

(One must add the parity-conserving QED contribution.)

The Standard Model expressions for $\epsilon_{L,R}(i)$, $g_{V,A}^{\nu e}$, and C_{ij} are given in Table 10.2. Note that $g_{V,A}^{\nu e}$ and the other quantities are coefficients of effective four-fermi operators, which differ from the quantities defined in Eq. (10.2) and Eq. (10.3) in the radiative corrections and in the presence of possible physics beyond the Standard Model.

A precise determination of the on-shell s_W^2, which depends only very weakly on m_t and M_H, is obtained from deep inelastic neutrino scattering from (approximately) isoscalar targets [31]. The ratio $R_\nu \equiv \sigma_{\nu N}^{NC}/\sigma_{\nu N}^{CC}$ of neutral- to charged-current cross sections has been measured to 1% accuracy by the CDHS [32] and CHARM [33] collaborations at CERN [34], and the CCFR collaboration at Fermilab [35,36] has obtained an even more precise result, so it is important to obtain theoretical expressions for R_ν and $R_{\overline{\nu}} \equiv \sigma_{\overline{\nu}N}^{NC}/\sigma_{\overline{\nu}N}^{CC}$ (as functions of $\sin^2\theta_W$) to comparable accuracy. Fortunately, most of the uncertainties from the strong interactions and neutrino spectra cancel in the ratio.

A simple zeroth-order approximation is

$$R_\nu = g_L^2 + g_R^2 r \;, \qquad (10.15a)$$

$$R_{\overline{\nu}} = g_L^2 + \frac{g_R^2}{r} \;, \qquad (10.15b)$$

where

$$g_L^2 \equiv \epsilon_L\,(u)^2 + \epsilon_L\,(d)^2 \approx \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W \;, \qquad (10.16a)$$

$$g_R^2 \equiv \epsilon_R\,(u)^2 + \epsilon_R\,(d)^2 \approx \frac{5}{9}\sin^4\theta_W \;, \qquad (10.16b)$$

and $r \equiv \sigma_{\overline{\nu}N}^{CC}/\sigma_{\nu N}^{CC}$ is the ratio of $\overline{\nu}$ and ν charged-current cross sections, which can be measured directly. (In the simple parton model, ignoring hadron energy cuts, $r \approx (\frac{1}{3}+\epsilon)/(1+\frac{1}{3}\epsilon)$, where $\epsilon \sim 0.125$ is the ratio of the fraction of the nucleon's momentum carried by antiquarks to that carried by quarks.) In practice, Eq. (10.15) must be corrected for quark mixing, quark sea effects, c-quark threshold effects, nonisoscalarity, $W-Z$ propagator differences, the finite muon mass, QED and electroweak radiative corrections. Details of the neutrino spectra, experimental cuts, x and Q^2 dependence of structure functions, and longitudinal structure functions enter only at the level of these corrections and therefore lead to very small uncertainties. The largest theoretical uncertainty is associated with the c-threshold, which mainly affects σ^{CC}. Using the slow rescaling prescription [37] the central value of $\sin^2\theta_W$ from CCFR varies as $0.0111(m_c\,[\text{GeV}]-1.31)$, where m_c is the effective mass. For $m_c = 1.31 \pm 0.24$ GeV (determined from ν-induced dimuon production [38]) this contributes ± 0.003 to the total uncertainty $\Delta\sin^2\theta_W \sim \pm 0.004$. This would require a high-energy neutrino beam for improvement. (The experimental uncertainty is also ± 0.003). The CCFR group quotes $s_W^2 = 0.2236 \pm 0.0041$ for $(m_t, M_H) = (175, 150)$ GeV with very little sensitivity to (m_t, M_H). Combining all of the precise deep-inelastic measurements, one obtains $s_W^2 = 0.2260 \pm 0.0039$.

Table 10.2: Standard Model expressions for the neutral-current parameters for ν-hadron, νe, and e-hadron processes. At tree level, $\rho = \kappa = 1$, $\lambda = 0$. If radiative corrections are included, $\rho_{\nu N}^{NC} = 1.0084$, $\widehat{\kappa}_{\nu N} = 0.9964$ (at $\langle Q^2\rangle = 35$ GeV^2), $\lambda_{uL} = -0.0031$, $\lambda_{dL} = -0.0025$, and $\lambda_{dR} = 2\,\lambda_{uR} = 7.5 \times 10^{-5}$ for $m_t = 175$ GeV and $M_H = M_Z = 91.1867$ GeV. For νe scattering, $\rho_{\nu e} = 1.0130$ and $\widehat{\kappa}_{\nu e} = 0.9970$ (at $\langle Q^2\rangle = 0.$). For atomic parity violation and the SLAC polarized electron experiment, $\rho'_{eq} = 0.9879$, $\rho_{eq} = 1.0009$, $\widehat{\kappa}'_{eq} = 1.0029$, $\widehat{\kappa}_{eq} = 1.0304$, $\lambda_{1d} = -2\,\lambda_{1u} = 3.7 \times 10^{-5}$, $\lambda_{2u} = -0.0121$ and $\lambda_{2d} = 0.0026$. The dominant m_t dependence is given by $\rho \sim 1 + \rho_t$, while $\widehat{\kappa} \sim 1$ ($\overline{\rm MS}$) or $\kappa \sim 1 + \rho_t/\tan^2\theta_W$ (on-shell).

Quantity	Standard Model Expression
$\epsilon_L(u)$	$\rho_{\nu N}^{NC}\left(\frac{1}{2} - \frac{2}{3}\widehat{\kappa}_{\nu N}\,\widehat{s}_Z^2\right) + \lambda_{uL}$
$\epsilon_L(d)$	$\rho_{\nu N}^{NC}\left(-\frac{1}{2} + \frac{1}{3}\widehat{\kappa}_{\nu N}\,\widehat{s}_Z^2\right) + \lambda_{dL}$
$\epsilon_R(u)$	$\rho_{\nu N}^{NC}\left(-\frac{2}{3}\widehat{\kappa}_{\nu N}\,\widehat{s}_Z^2\right) + \lambda_{uR}$
$\epsilon_R(d)$	$\rho_{\nu N}^{NC}\left(\frac{1}{3}\widehat{\kappa}_{\nu N}\,\widehat{s}_Z^2\right) + \lambda_{dR}$
$g_V^{\nu e}$	$\rho_{\nu e}\left(-\frac{1}{2} + 2\widehat{\kappa}_{\nu e}\,\widehat{s}_Z^2\right)$
$g_A^{\nu e}$	$\rho_{\nu e}\left(-\frac{1}{2}\right)$
C_{1u}	$\rho'_{eq}\left(-\frac{1}{2} + \frac{4}{3}\widehat{\kappa}'_{eq}\,\widehat{s}_Z^2\right) + \lambda_{1u}$
C_{1d}	$\rho'_{eq}\left(\frac{1}{2} - \frac{2}{3}\widehat{\kappa}'_{eq}\,\widehat{s}_Z^2\right) + \lambda_{1d}$
C_{2u}	$\rho_{eq}\left(-\frac{1}{2} + 2\widehat{\kappa}_{eq}\,\widehat{s}_Z^2\right) + \lambda_{2u}$
C_{2d}	$\rho_{eq}\left(\frac{1}{2} - 2\widehat{\kappa}_{eq}\,\widehat{s}_Z^2\right) + \lambda_{2d}$

The laboratory cross section for $\nu_\mu e \to \nu_\mu e$ or $\overline{\nu}_\mu e \to \overline{\nu}_\mu e$ elastic scattering is

$$\frac{d\sigma_{\nu_\mu,\overline{\nu}_\mu}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi}$$

$$\times\Big[(g_V^{\nu e} \pm g_A^{\nu e})^2 + (g_V^{\nu e} \mp g_A^{\nu e})^2(1-y)^2$$

$$-(g_V^{\nu e2} - g_A^{\nu e2})\frac{y\,m_e}{E_\nu}\Big] \;, \qquad (10.17)$$

where the upper (lower) sign refers to $\nu_\mu(\overline{\nu}_\mu)$, and $y \equiv E_e/E_\nu$ (which runs from 0 to $(1 + m_e/2E_\nu)^{-1}$) is the ratio of the kinetic energy of the recoil electron to the incident ν or $\overline{\nu}$ energy. For $E_\nu \gg m_e$ this yields a total cross section

$$\sigma = \frac{G_F^2\,m_e\,E_\nu}{2\pi}\left[(g_V^{\nu e} \pm g_A^{\nu e})^2 + \frac{1}{3}(g_V^{\nu e} \mp g_A^{\nu e})^2\right] \;. \qquad (10.18)$$

The most accurate leptonic measurements [39–41] of $\sin^2\theta_W$ are from the ratio $R \equiv \sigma_{\nu_\mu e}/\sigma_{\overline{\nu}_\mu e}$ in which many of the systematic uncertainties cancel. Radiative corrections (other than m_t effects) are small compared to the precision of present experiments and have negligible effect on the extracted $\sin^2\theta_W$. The most precise experiment (CHARM II) [41] determined not only $\sin^2\theta_W$ but $g_{V,A}^{\nu e}$ as well. The cross sections for $\nu_e e$ and $\overline{\nu}_e e$ may be obtained from

Eq. (10.17) by replacing $g_{V,A}^{\nu e}$ by $g_{V,A}^{\nu e}+1$, where the 1 is due to the charged-current contribution.

The SLAC polarized-electron experiment [42] measured the parity-violating asymmetry

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} , \qquad (10.19)$$

where $\sigma_{R,L}$ is the cross section for the deep-inelastic scattering of a right- or left-handed electron: $e_{R,L}N \to e\mathrm{X}$. In the quark parton model

$$\frac{A}{Q^2} = a_1 + a_2 \, \frac{1-(1-y)^2}{1+(1-y)^2} , \qquad (10.20)$$

where $Q^2 > 0$ is the momentum transfer and y is the fractional energy transfer from the electron to the hadrons. For the deuteron or other isoscalar targets, one has, neglecting the s-quark and antiquarks,

$$a_1 = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(C_{1u} - \frac{1}{2}C_{1d}\right) \approx \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(-\frac{3}{4} + \frac{5}{3}\sin^2\theta_W\right) , \qquad (10.21a)$$

$$a_2 = \frac{3G_F}{5\sqrt{2}\pi\alpha}\left(C_{2u} - \frac{1}{2}C_{2d}\right) \approx \frac{9G_F}{5\sqrt{2}\pi\alpha}\left(\sin^2\theta_W - \frac{1}{4}\right) . \qquad (10.21b)$$

There are now precise experiments measuring atomic parity violation [43] in cesium (at the 0.4% level) [44], thallium [45], lead [46], and bismuth [47]. The uncertainties associated with atomic wave functions are quite small for cesium, for which they are $\sim 1\%$ [48]. The theoretical uncertainties are 3% for thallium [49] but larger for the other atoms. For heavy atoms one determines the "weak charge"

$$\begin{aligned} Q_W &= -2\left[C_{1u}\,(2Z+N) + C_{1d}(Z+2N)\right] \\ &\approx Z(1-4\sin^2\theta_W) - N . \end{aligned} \qquad (10.22)$$

The recent Boulder experiment in cesium also observed the parity-violating weak corrections to the nuclear electromagnetic vertex (the anapole moment [50]).

In the future it should be possible to reduce the theoretical wave function uncertainties by taking the ratios of parity violation in different isotopes [43,51]. There would still be some residual uncertainties from differences in the neutron charge radii, however [52].

The forward-backward asymmetry for $e^+e^- \to \ell^+\ell^-$, $\ell = \mu$ or τ, is defined as

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} , \qquad (10.23)$$

where $\sigma_F(\sigma_B)$ is the cross section for ℓ^- to travel forward (backward) with respect to the e^- direction. A_{FB} and R, the total cross section relative to pure QED, are given by

$$R = F_1 , \qquad (10.24)$$

$$A_{FB} = 3F_2/4F_1 , \qquad (10.25)$$

where

$$F_1 = 1 - 2\chi_0\, g_V^e\, g_V^\ell\, \cos\delta_R + \chi_0^2\left(g_V^{e2} + g_A^{e2}\right)\left(g_V^{\ell 2} + g_A^{\ell 2}\right), \qquad (10.26a)$$

$$F_2 = -2\chi_0\, g_A^e\, g_A^\ell\, \cos\delta_R + 4\chi_0^2\, g_A^e\, g_A^\ell\, g_V^e\, g_V^\ell , \qquad (10.26b)$$

$$\tan\delta_R = \frac{M_Z\Gamma_Z}{M_Z^2 - s} , \qquad (10.27)$$

$$\chi_0 = \frac{G_F}{2\sqrt{2}\pi\alpha}\frac{sM_Z^2}{\left[(M_Z^2-s)^2 + M_Z^2\Gamma_Z^2\right]^{1/2}} , \qquad (10.28)$$

and $\sqrt{s}$ is the CM energy. Eq. (10.26) is valid at tree level. If the data is radiatively corrected for QED effects (as described above), then the remaining electroweak corrections can be incorporated [53,54] (in an approximation adequate for existing PEP, PETRA, and TRISTAN data, which are well below the Z pole) by replacing χ_0 by $\chi(s) \equiv (1+\rho_t)\chi_0(s)\alpha/\alpha(s)$, where $\alpha(s)$ is the running QED coupling, and evaluating g_V in the $\overline{\mathrm{MS}}$ scheme. Formulas for $e^+e^- \to$ hadrons may be found in Ref. 55.

At LEP and SLC, there are high-precision measurements of various Z pole observables [56–61]. These include the Z mass and total width, Γ_Z, and partial widths $\Gamma(f\overline{f})$ for $Z \to f\overline{f}$ where fermion $f = e$, μ, τ, hadrons, b, or c. The data is consistent with lepton-family universality, $\Gamma(e^+e^-) = \Gamma(\mu^+\mu^-) = \Gamma(\tau^+\tau^-)$, so one may work with an average width $\Gamma(\ell^+\ell^-)$. It is convenient to use the variables M_Z, Γ_Z, $R_\ell \equiv \Gamma(\mathrm{had})/\Gamma(\ell^+\ell^-)$, $\sigma_{\mathrm{had}} \equiv 12\pi\Gamma(e^+e^-)\Gamma(\mathrm{had})/M_Z^2\,\Gamma_Z^2$, $R_b \equiv \Gamma(b\overline{b})/\Gamma(\mathrm{had})$, and $R_c \equiv \Gamma(c\overline{c})/\Gamma(\mathrm{had})$, most of which are weakly correlated experimentally. ($\Gamma(\mathrm{had})$ is the partial width into hadrons.) The largest correlation coefficient of -0.20 occurs between R_b and R_c. R_ℓ is insensitive to m_t except for the $Z \to b\overline{b}$ vertex and final state corrections and the implicit dependence through $\sin^2\theta_W$. Thus it is especially useful for constraining α_s. The width for invisible decays [57], $\Gamma(\mathrm{inv}) = \Gamma_Z - 3\Gamma(\ell^+\ell^-) - \Gamma(\mathrm{had}) = 500.1 \pm 1.8$ MeV, can be used to determine the number of neutrino flavors much lighter than $M_Z/2$, $N_\nu = \Gamma(\mathrm{inv})/\Gamma^{\mathrm{theory}}(\nu\overline{\nu}) = 2.990 \pm 0.011$ for $(m_t, M_H) = (175 \pm 5~\mathrm{GeV}, M_Z)$.

There are also measurements of various Z pole asymmetries. These include the polarization or left-right asymmetry

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} , \qquad (10.29)$$

where $\sigma_L(\sigma_R)$ is the cross section for a left- (right)-handed incident electron. A_{LR} has been measured precisely by the SLD collaboration at the SLC [59], and has the advantages of being extremely sensitive to $\sin^2\theta_W$ and that systematic uncertainties largely cancel. In addition, the SLD collaboration has extracted the final-state couplings A_b, A_c, A_τ, and A_μ from left-right forward-backward asymmetries [57,60], using

$$A_{LR}^{FB}(f) = \frac{\sigma_{LF}^f - \sigma_{LB}^f - \sigma_{RF}^f + \sigma_{RB}^f}{\sigma_{LF}^f + \sigma_{LB}^f + \sigma_{RF}^f + \sigma_{RB}^f} = \frac{3}{4}A_f , \qquad (10.30)$$

where, for example, σ_{LF} is the cross section for a left-handed incident electron to produce a fermion f traveling in the forward hemisphere. Similarly, A_τ is measured at LEP [57] through the negative total τ polarization, $\mathcal{P}_\tau$, and A_e is extracted from the angular distribution of $\mathcal{P}_\tau$. An equation such as (10.30) assumes that initial state QED corrections, photon exchange, $\gamma - Z$ interference, the tiny electroweak boxes, and corrections for $\sqrt{s} \neq M_Z$ are removed from the data, leaving the pure electroweak asymmetries. This allows the use of effective tree-level expressions,

$$A_{LR} = A_e P_e , \qquad (10.31)$$

$$A_{FB} = \frac{3}{4}A_f\,\frac{A_e + P_e}{1 + P_e A_e} , \qquad (10.32)$$

where

$$A_f \equiv \frac{2\overline{g}_V^f\,\overline{g}_A^f}{\overline{g}_V^{f2} + \overline{g}_A^{f2}} , \qquad (10.33)$$

and

$$\overline{g}_V^f = \sqrt{\rho_f}\,(t_{3L}^{(f)} - 2q_f\kappa_f\sin^2\theta_W) , \qquad (10.33b)$$

$$\overline{g}_A^f = \sqrt{\rho_f}\,t_{3L}^{(f)} . \qquad (10.33c)$$

P_e is the initial e^- polarization, so that the second equality in Eq. (10.30) is reproduced for $P_e = 1$, and the Z pole forward-backward asymmetries at LEP ($P_e = 0$) are given by $A_{FB}^{(0,f)} = \frac{3}{4}A_e A_f$ where $f = e$, μ, τ, b, c, s, and q, and where $A_{FB}^{(0,q)}$ refers to the hadronic charge asymmetry. The initial state coupling, A_e, is also determined through the left-right charge asymmetry [61] and in polarized Bhabba scattering [60] at the SLC.

The electroweak-radiative corrections have been absorbed into corrections $\rho_f - 1$ and $\kappa_f - 1$, which depend on the fermion f and on the renormalization scheme. In the on-shell scheme, the quadratic m_t dependence is given by $\rho_f \sim 1 + \rho_t$, $\kappa_f \sim 1 + \rho_t/\tan^2\theta_W$, while in $\overline{\mathrm{MS}}$, $\widehat{\rho}_f \sim \widehat{\kappa}_f \sim 1$, for $f \neq b$ ($\widehat{\rho}_b \sim 1 - \frac{4}{3}\rho_t$, $\widehat{\kappa}_b \sim 1 + \frac{2}{3}\rho_t$). In the $\overline{\mathrm{MS}}$ scheme the normalization is changed according to $G_F M_Z^2/2\sqrt{2}\pi \to \widehat{\alpha}/4\widehat{s}_Z^2\widehat{c}_Z^2$.

(If one continues to normalize amplitudes by $G_F M_Z^2/2\sqrt{2}\pi$, as in the 1996 edition of this *Review*, then $\hat{\rho}_f$ contains an additional factor of $\hat{\rho}$.) In practice, additional bosonic and fermionic loops, vertex corrections, leading higher order contributions, *etc.*, must be included. For example, in the $\overline{\text{MS}}$ scheme one has, for $(m_t, M_H) = (175 \text{ GeV}, M_Z)$, $\widehat{\rho}_\ell = 0.9978$, $\widehat{\kappa}_\ell = 1.0013$, $\widehat{\rho}_b = 0.9868$ and $\widehat{\kappa}_b = 1.0067$. It is convenient to define an effective angle $\overline{s}_f^2 \equiv \sin^2\overline{\theta}_{Wf} \equiv \widehat{\kappa}_f \widehat{s}_Z^2 = \kappa_f s_W^2$, in terms of which $\overline{g}_V^f$ and $\overline{g}_A^f$ are given by $\sqrt{\rho_f}$ times their tree-level formulae. Because $\overline{g}_V^\ell$ is very small, not only $A_{LR}^0 = A_e$, $A_{FB}^{(0,\ell)}$, and $\mathcal{P}_\tau$, but also $A_{FB}^{(0,b)}$, $A_{FB}^{(0,c)}$, $A_{FB}^{(0,s)}$, and the hadronic asymmetries are mainly sensitive to $\overline{s}_\ell^2$. One finds that $\widehat{\kappa}_f$ ($f \neq b$) is almost independent of (m_t, M_H), so that one can write

$$\overline{s}_\ell^2 \sim \widehat{s}_Z^2 + 0.00029 \ . \tag{10.34}$$

Thus, the asymmetries determine values of $\overline{s}_\ell^2$ and $\widehat{s}_Z^2$ almost independent of m_t, while the κ's for the other schemes are m_t dependent.

10.4. *W* and *Z* decays

The partial decay width for gauge bosons to decay into massless fermions $f_1\overline{f}_2$ is

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226.5 \pm 0.3 \quad \text{MeV} \ , \tag{10.35a}$$

$$\Gamma(W^+ \to u_i\overline{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi}\,|V_{ij}|^2 \approx (707 \pm 1)\,|V_{ij}|^2 \quad \text{MeV} \ , \tag{10.35b}$$

$$\Gamma(Z \to \psi_i\overline{\psi}_i) = \frac{C G_F M_Z^3}{6\sqrt{2}\pi}\left[g_V^{i2} + g_A^{i2}\right] \tag{10.35c}$$

$$\approx \begin{cases} 167.25 \pm 0.08 \quad \text{MeV } (\nu\overline{\nu}), \quad 84.01 \pm 0.05 \quad \text{MeV } (e^+e^-), \\ 300.3 \pm 0.2 \quad \text{MeV } (u\overline{u}), \qquad 383.1 \pm 0.2 \quad \text{MeV } (d\overline{d}), \\ 376.0 \mp 0.1 \quad \text{MeV } (b\overline{b}), \end{cases}$$

where the numerical values are for $(m_t, M_H) = (175 \pm 5 \text{ GeV}, M_Z)$. For leptons $C = 1$, while for quarks $C = 3\Big(1 + \alpha_s(M_V)/\pi + 1.409\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3\Big)$, where the 3 is due to color and the factor in parentheses represents the universal part of the QCD corrections [62] for massless quarks [63]. The $Z \to f\overline{f}$ widths contain a number of additional corrections: universal (non-singlet) top-mass contributions [64]; fermion mass effects and further QCD corrections proportional to m_q^2 [65] (m_q is the running quark mass evaluated at the Z scale) which are different for vector and axial-vector partial widths; and singlet contributions starting from two-loop order which are large, strongly top-mass dependent, family universal, and flavor non-universal [66]. All QCD effects are known and included up to three loop order. The QED factor $1 + 3\alpha q_f^2/4\pi$, as well as two-loop $\alpha\alpha_s$ and α^2 corrections [67,68] are also included. Working in the on-shell scheme, *i.e.*, expressing the widths in terms of $G_F M_{W,Z}^3$, incorporates the largest radiative corrections from the running QED coupling [18,69]. Electroweak corrections to the Z widths are then incorporated by replacing $g_{V,A}^{i2}$ by $\overline{g}_{V,A}^{i2}$. Hence, in the on-shell scheme the Z widths are proportional to $\rho_i \sim 1 + \rho_t$. The $\overline{\text{MS}}$ normalization (see the end of the previous section) accounts also for the leading electroweak corrections [22]. There is additional (negative) quadratic m_t dependence in the $Z \to b\overline{b}$ vertex corrections [70] which causes $\Gamma(b\overline{b})$ to decrease with m_t. The dominant effect is to multiply $\Gamma(b\overline{b})$ by the vertex correction $1 + \delta\rho_{b\overline{b}}$, where $\delta\rho_{b\overline{b}} \sim 10^{-2}(-\frac{1}{2}\frac{m_t^2}{M_Z^2} + \frac{1}{5})$. In practice, the corrections are included in ρ_b and κ_b, as discussed before.

For 3 fermion families the total widths are predicted to be

$$\Gamma_Z \approx 2.496 \pm 0.001 \text{ GeV} \ , \tag{10.36}$$

$$\Gamma_W \approx 2.093 \pm 0.002 \text{ GeV} \ . \tag{10.37}$$

We have assumed $\alpha_s = 0.120$. An uncertainty in α_s of ± 0.003 introduces an additional uncertainty of 0.1% in the hadronic widths, corresponding to ± 1.6 MeV in Γ_Z. These predictions are to be compared with the experimental results $\Gamma_Z = 2.4948 \pm 0.0025$ GeV and $\Gamma_W = 2.062 \pm 0.059$ GeV.

10.5. Experimental results

Table 10.3: Principal LEP and other recent observables, compared with the Standard Model predictions for $M_Z = 91.1867 \pm 0.0020$ GeV, $M_H = M_Z$, and the global best fit values $m_t = 173 \pm 4$ GeV, $\alpha_s = 0.1214 \pm 0.0031$, and $\widehat{\alpha}(M_Z)^{-1} = 127.90 \pm 0.07$. The LEP averages of the ALEPH, DELPHI, L3, and OPAL results include common systematic errors and correlations [57]. $\overline{s}_\ell^2(A_{FB}^{(0,q)})$ is the effective angle extracted from the hadronic charge asymmetry. The values of $\Gamma(\ell^+\ell^-)$, $\Gamma(\text{had})$, and $\Gamma(\text{inv})$ are not independent of Γ_Z, R_ℓ, and σ_{had}. The first M_W value is from CDF, UA2, and DØ [71] while the second includes the measurements at LEP [57]. M_W and M_Z are correlated, but the effect is negligible due to the tiny M_Z error. The four values of A_ℓ are (i) from A_{LR} for hadronic final states [59]; (ii) the combined value from SLD including leptonic asymmetries; (iii) from the total τ polarization; and (iv) from the angular distribution of the τ polarization. The two values of s_W^2 from deep-inelastic scattering are from CCFR [36] and the global average, respectively. Similarly, the $g_{V,A}^{\nu e}$ are from CHARM II [41] and the world average. The second errors in Q_W are theoretical [48,49]. Older low-energy results are not listed but are included in the fits. In the Standard Model predictions, the uncertainty is from M_Z, m_t, $\Delta\alpha(M_Z)$ and α_s. In parentheses we show the shift in the predictions when M_H is changed to 300 GeV which is its 90% CL upper limit. The errors in Γ_Z, $\Gamma(\text{had})$, R_ℓ, and σ_{had} are completely dominated by the uncertainty in α_s.

Quantity	Value	Standard Model
m_t [GeV]	175 ± 5	$173 \pm 4\ (+5)$
M_W [GeV]	80.405 ± 0.089	$80.377 \pm 0.023\ (-0.036)$
	80.427 ± 0.075	
M_Z [GeV]	91.1867 ± 0.0020	$91.1867 \pm 0.0020\ (+0.0001)$
Γ_Z [GeV]	2.4948 ± 0.0025	$2.4968 \pm 0.0017\ (-0.0007)$
$\Gamma(\text{had})$ [GeV]	1.7432 ± 0.0023	$1.7433 \pm 0.0016\ (-0.0005)$
$\Gamma(\text{inv})$ [MeV]	500.1 ± 1.8	$501.7 \pm 0.2\ (-0.1)$
$\Gamma(\ell^+\ell^-)$ [MeV]	83.91 ± 0.10	$84.00 \pm 0.03\ (-0.04)$
σ_{had} [nb]	41.486 ± 0.053	$41.469 \pm 0.016\ (-0.005)$
R_ℓ	20.775 ± 0.027	$20.754 \pm 0.020\ (+0.003)$
R_b	0.2170 ± 0.0009	$0.2158 \pm 0.0001\ (-0.0002)$
R_c	0.1734 ± 0.0048	$0.1723 \pm 0.0001\ (+0.0001)$
$A_{FB}^{(0,\ell)}$	0.0171 ± 0.0010	$0.0162 \pm 0.0003\ (-0.0004)$
$A_{FB}^{(0,b)}$	0.0984 ± 0.0024	$0.1030 \pm 0.0009\ (-0.0013)$
$A_{FB}^{(0,c)}$	0.0741 ± 0.0048	$0.0736 \pm 0.0007\ (-0.0010)$
$A_{FB}^{(0,s)}$	0.118 ± 0.018	$0.1031 \pm 0.0009\ (-0.0013)$
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2322 ± 0.0010	$0.2315 \pm 0.0002\ (+0.0002)$
A_ℓ	0.1550 ± 0.0034	$0.1469 \pm 0.0013\ (-0.0018)$
	0.1547 ± 0.0032	
	0.1411 ± 0.0064	
	0.1399 ± 0.0073	
A_b	0.900 ± 0.050	$0.9347 \pm 0.0001\ (-0.0002)$
A_c	0.650 ± 0.058	$0.6678 \pm 0.0006\ (-0.0008)$
$s_W^2(\nu\text{N})$	0.2236 ± 0.0041	$0.2230 \pm 0.0004\ (+0.0007)$
	0.2260 ± 0.0039	
$g_V^{\nu e}$	-0.035 ± 0.017	$-0.0395 \pm 0.0005\ (+0.0002)$
	-0.041 ± 0.015	
$g_A^{\nu e}$	-0.503 ± 0.017	$-0.5064 \pm 0.0002\ (+0.0002)$
	-0.507 ± 0.014	
$Q_W(\text{Cs})$	$-72.41 \pm 0.25 \pm 0.80$	$-73.12 \pm 0.06\ (+0.01)$
$Q_W(\text{Tl})$	$-114.8 \pm 1.2 \pm 3.4$	-116.7 ± 0.1

The values of the principal Z pole observables are listed in Table 10.3, along with the Standard Model predictions for $M_Z = 91.1867 \pm 0.0020$, $m_t = 173 \pm 4$ GeV, $M_H = M_Z$ and $\alpha_s = 0.1214 \pm 0.0031$. Note, that the values of the Z pole observables (as well as M_W) differ from those in the Particle Listings because they include recent preliminary results [57,58,59,71]. The values and predictions of M_W [57,71], the Q_W for cesium [44] and thallium [45], and recent results from deep inelastic [32–36] and $\nu_\mu e$ scattering [39–41] are also listed. The agreement is excellent. Even the largest discrepancies, A^0_{LR}, $A^{(0,b)}_{FB}$, and $A^{(0,\tau)}_{FB}$, deviate by only 2.4 σ, 1.9 σ and 1.7 σ, respectively.

Other observables like $R_b = \Gamma(b\overline{b})/\Gamma(\mathrm{had})$ and $R_c = \Gamma(c\overline{c})/\Gamma(\mathrm{had})$ which showed significant deviations in the past, are now in perfect (R_c) or at least better agreement. In particular, R_b whose measured value deviated as much as 3.7 σ from the Standard Model prediction is now only 1.3 σ high. Many types of new physics could contribute to R_b (the implications of this possibility for the value of $\alpha_s(M_Z)$ extracted from the fits are discussed below) and A_b and as a consequence to $A^{(0,b)}_{FB} = \frac{3}{4} A_e A_b$. Indeed, A_b can be extracted from $A^{(0,b)}_{FB}$ when A_e is taken from leptonic asymmetries (using lepton universality), and combined with the measurement at the SLC. The result, $A_b = 0.877 \pm 0.023$, is 2.5 σ below the Standard Model prediction. (Alternatively, one can use $A_\ell = 0.1469 \pm 0.0013$ from the global fit and obtain $A_b = 0.894 \pm 0.021$ which is 1.9 σ low.) However, this deviation of about 6% cannot arise from new physics radiative corrections since a 30% correction to $\widehat{\kappa}_b$ would be necessary to account for the central value of A_b. Only a new type of physics which couples at the tree level preferentially to the third generation, and which does not contradict R_b (including the off-peak R_b measurements by DELPHI [72]), can conceivably account for a low A_b [73].

The left-right asymmetry, $A^0_{LR} = 0.1550 \pm 0.0034$ [59], based on all hadronic data from 1992–1996 has moved closer to the Standard Model expectation of 0.1469 ± 0.0013 than previous values. However, because of the smaller error A^0_{LR} is still 2.4 σ above the Standard Model prediction. There is also an experimental difference of $\sim 1.9\ \sigma$ between the SLD value of $A_\ell(\mathrm{SLD}) = 0.1547 \pm 00032$ from all A_{LR} and $A^{FB}_{LR}(\ell)$ data on one hand, and the LEP value $A_\ell(\mathrm{LEP}) = 0.1461 \pm 0.0033$ obtained from $A^{(0,\ell)}_{FB}$, $A_e(\mathcal{P}_\tau)$, $A_\tau(\mathcal{P}_\tau)$ on the other hand, in both cases assuming lepton-family universality.

Despite these discrepancies the χ^2 value of the fit for the Standard Model is excellent. It is 25 for 30 d.o.f. when fitting to the independent observables in Table 10.3, and 181 for 209 d.o.f. when the older neutral current observables are included. The probability of a larger χ^2 is 0.73 and 0.92 for the two cases, respectively. (The low χ^2 for the older data is likely due to overly conservative estimates of systematic errors.)

With the latest value of $A^{(0,\tau)}_{FB}$ the data is now in reasonable agreement with lepton-family universality, which will be assumed. The observables in Table 10.3 (including correlations on the LEP lineshape and LEP/SLD heavy flavor observables), as well as all low-energy neutral-current data [16,17], are used in the global fits described below. The parameter $\sin^2\theta_W$ can be determined from Z pole observables, M_W, and from a variety of neutral-current processes spanning a very wide Q^2 range. The results [16], shown in Table 10.4, are in impressive agreement with each other, indicating the quantitative success of the Standard Model. The one discrepancy is the value $\widehat{s}^2_Z = 0.23023 \pm 0.00043$ from $A_\ell(\mathrm{SLD})$ which is 2.3 σ below the value 0.23124 ± 0.00017 from the global fit to all data and 2.6 σ below the value 0.23144 ± 0.00019 obtained from all data other than $A_\ell(\mathrm{SLD})$.

The data allow a simultaneous determination of $\sin^2\theta_W$, m_t, and the strong coupling $\alpha_s(M_Z)$. The latter is determined mainly from R_ℓ, Γ_Z, and σ_{had}, and is only weakly correlated with the other variables. The global fit to all data, including the CDF/DØ value, $m_t = 175 \pm 5$ GeV, yields

$$\begin{aligned} \widehat{s}^2_Z &= 0.23124 \pm 0.00017\ (+0.00024)\ , \\ m_t &= 173 \pm 4\ (+5)\ \text{GeV}\ , \\ \alpha_s(M_Z) &= 0.1214 \pm 0.0031\ (+0.0018)\ , \\ M_H &= M_Z\ . \end{aligned} \tag{10.38}$$

In parentheses we show the effect of changing M_H to 300 GeV which is the conservative 90% CL upper limit (see below). In all fits, the errors include full statistical, systematic, and theoretical uncertainties. The $\widehat{s}^2_Z$ error reflects the error on $\overline{s}^2_f \sim \pm 0.00023$ from the Z pole asymmetries. In the on-shell scheme one has $s^2_W = 0.22304 \pm 0.00044$, the larger error due to the stronger sensitivity to m_t. The extracted value of α_s is based on a formula with negligible theoretical uncertainty (± 0.0005 in α_s) if one assumes the exact validity of the Standard Model. It is in excellent agreement with other precise values [74], such as 0.122 ± 0.005 from τ decays, 0.121 ± 0.005 from jet-event shapes in e^+e^- annihilation, and the very recent result [75], $0.119 \pm 0.002\ (\mathrm{exp}) \pm 0.004\ (\mathrm{scale})$, from deep-inelastic scattering. It is slightly higher than the values from lattice calculations of the $b\overline{b}$ (0.1174 ± 0.0024 [76]) and $c\overline{c}$ (0.116 ± 0.003 [77]) spectra, and from decays of heavy quarkonia (0.112 ± 0.006 [74]). For more details, see our Section 9 on "Quantum Chromodynamics" in this *Review*. The average $\alpha_s(M_Z)$ obtained from Section 9 when ignoring the precision measurements discussed in this Section is 0.1178 ± 0.0023. We use this value as an external constraint for the second fit in Table 10.5. The resulting value,

$$\alpha_s = 0.1191 \pm 0.0018\ (+0.0006)\ , \tag{10.39}$$

can be regarded as the present world average.

Table 10.4: Values obtained for s^2_W (on-shell) and $\widehat{s}^2_Z(\overline{\mathrm{MS}})$ from various reactions assuming the global best fit values (for $M_H = M_Z$) $m_t = 173 \pm 4$ GeV and $\alpha_s = 0.1214 \pm 0.0031$.

Reaction	s^2_W	$\widehat{s}^2_Z$
M_Z	0.2231 ± 0.0005	0.2313 ± 0.0002
M_W	0.2228 ± 0.0006	0.2310 ± 0.0005
Γ_Z/M^3_Z, R, $\sigma_{\mathrm{had}} M^2_Z$	0.2235 ± 0.0011	0.2316 ± 0.0011
$A^{(0,\ell)}_{FB}$	0.2225 ± 0.0007	0.2307 ± 0.0006
LEP asymmetries	0.2235 ± 0.0004	0.2317 ± 0.0003
A^0_{LR}	0.2220 ± 0.0005	0.2302 ± 0.0004
$\overline{A}_b, \overline{A}_c$	0.230 ± 0.016	0.239 ± 0.016
Deep inelastic (isocalar)	0.226 ± 0.004	0.234 ± 0.004
$\nu_\mu(\overline{\nu}_\mu)p \to \nu_\mu(\overline{\nu}_\mu)p$	0.203 ± 0.032	0.211 ± 0.032
$\nu_\mu(\overline{\nu}_\mu)e \to \nu_\mu(\overline{\nu}_\mu)e$	0.221 ± 0.008	0.229 ± 0.008
atomic parity violation	0.220 ± 0.003	0.228 ± 0.003
SLAC eD	0.213 ± 0.019	0.222 ± 0.018
All data	0.2230 ± 0.0004	0.23124 ± 0.00017

The value of R_b is 1.3 σ above the Standard Model expectation. If this is not just a fluctuation but is due to a new physics contribution to the $Z \to b\overline{b}$ vertex (many types would couple preferentially to the third family), the value of $\alpha_s(M_Z)$ extracted from the hadronic Z width would be reduced [17]. Allowing for this possibility one obtains $\alpha_s(M_Z) = 0.1166 \pm 0.0048\ (+0.0007)$. Similar remarks apply in principle for R_c and the other quark and lepton flavors, and one should keep in mind that the Z lineshape value of α_s is very sensitive to many types of new physics.

The data indicate a preference for a small Higgs mass. There is a strong correlation between the quadratic m_t and logarithmic M_H terms in $\widehat{\rho}$ in all of the indirect data except for the $Z \to b\overline{b}$ vertex. Therefore, observables (other than R_b) which favor m_t values higher than the Tevatron range favor lower values of M_H. This effect is enhanced by R_b, which has little direct M_H dependence but favors the lower end of the Tevatron m_t range. M_W has additional M_H dependence through $\Delta\widehat{r}_W$ which is not coupled to m^2_t effects. The strongest individual pulls towards smaller M_H are from M_W, A^0_{LR}, and $A^{(0,\ell)}_{FB}$ (when combined with M_Z), as well as R_b. The difference in χ^2 for the global fit is $\Delta\chi^2 = \chi^2(M_H = 1000 \text{ GeV}) - \chi^2(M_H = 77 \text{ GeV}) = 16.6$. Hence, the data favor a small value of M_H, as in supersymmetric extensions of the Standard Model, and m_t on the lower side of the Tevatron range. If one allows M_H as a free fit parameter and does not include any constraints from direct Higgs searches, one obtains $M_H = 69^{+85}_{-43}$ GeV, *i.e.*, the central value below the direct lower bound, $M_H \geq 77$ GeV (95% CL) [78]. Including the results of the direct searches as an extra contribution to the likelihood function drives the best fit value to the present kinematic reach ($M_H \sim 83$ GeV), and we obtain the upper limit $M_H < 236$ (287) GeV at 90 (95)% CL. The extraction of M_H from the precision data depends strongly on the value used for $\alpha(M_Z)$. The value derived by Martin and Zeppenfeld [11] relying on the predictions of perturbative QCD down to smaller values of $\sqrt{s}$ is higher and has a smaller stated error. Using this value would give a best fit at $M_H = 140$ GeV, and an upper limit $M_H < 300$ (361) GeV at 90 (95)% CL. Clearly, a consensus on the applicability of perturbative QCD in e^+e^- annihilation is highly desirable.

The most deviating observable, A_{LR}, has a strong impact on the Higgs mass limits as well [17,79]. The Introduction to this *Review* suggests an unbiased treatment of deviating observables r through the introduction of scale factors S_r. It is instructive to study the impact of this more conservative procedure on M_H. For the case of a fit to the Standard Model, we define

$$S_r = \max(\sqrt{\chi_r^2}, 1) \ , \tag{10.40}$$

where χ_r^2 is the χ^2 contribution of observable r to a global fit in which M_H is allowed as a free fit parameter (with no direct constraints included). We then repeat the fit with all errors multiplied by S_r, and proceed iteratively until the procedure has converged. This way we obtain

$$S_{A^0_{LR}} = 2.76, \qquad S_{A^{(0,b)}_{FB}} = 2.05, \qquad S_{A^{(0,\tau)}_{FB}} = 1.83,$$

$$S_{A^{FB}_{LR}(\tau)} = 1.45, \qquad S_{A^{FB}_{LR}(\mu)} = 1.34, \qquad S_{R_b} = 1.33,$$

as well as $S_{A_e(\mathcal{P}_\tau)} = 1.02$, and $S_r = 1$ for all other observables. The result of the global fit is

$$\begin{aligned} \hat{s}^2_Z &= 0.23141 \pm 0.00031 \ , \\ m_t &= 174 \pm 5 \text{ GeV} \ , \\ \alpha_s(M_Z) &= 0.1222 \pm 0.0034 \ , \\ M_H &= 122^{+134}_{-77} \text{ GeV} \ , \end{aligned} \tag{10.41}$$

where the larger errors compared to Eq. (10.38) are from M_H rather than the S_r. Since the central value of M_H is much larger than the present direct lower bound, and $\log(M_H)$ is approximately normal distributed, it is justified to include the error due to M_H (with all correlations properly taken into account) in a Gaussian way in the uncertainties of the other parameters. For comparison with other fits we also list the results for fixed M_H in Table 10.5. Including the direct constraint we obtain an upper limit $M_H < 329$ (408) GeV at 90 (95)% CL, which is higher by $\mathcal{O}(100 \text{ GeV})$ than the one without scale factors. It is in good agreement with the bound we obtained above by switching to the higher $\alpha(M_Z)$. Indeed, both analyses decrease the impact of A_{LR} on the Higgs mass limit.

A few comments are in order: (i) The procedure used here is not unambiguous. It depends on whether results from different experiments (*e.g.*, the various experimental groups at LEP or the Tevatron) are combined or used as individual pieces of input. We use combined result, primarily in order to avoid insurmountable complications with cross correlations between different experimental groups on top of the correlations between the observables. Even the result on a single observable quoted by an individual group, is in general a combination of various channels, with different types of systematic errors (which are the prime reason for the introduction of scale factors in the first place). Thus, ideally, one would prefer to define the S_r at this level. In practice, however, this is virtually impossible to achieve. In the case of M_W we use the individual determinations, since they are uncorrelated and are based on entirely different processes. (ii) None of the definitions of scale factors in the Introduction to this *Review* is directly applicable to our case. However, we have tried to work as closely as possible in spirit to the definitions given there. One major difference is that central values of fit parameters (in particular of M_H) change upon introducing S_r; on the other hand, central values of measurements remain unchanged. (iii) The procedure used here relies on the validity of the Standard Model, since in the presence of new physics, some discrepancies will be shifted into new physics parameters. When fits to new types of physics are to be compared to Standard Model fits as is done in Section 10.5 one has to refrain from using scale factors.

One can also carry out a fit to the indirect data alone, *i.e.*, without including the value $m_t = 175 \pm 5$ GeV observed directly by CDF and DØ. (The indirect prediction is for the $\overline{\text{MS}}$ mass which is in the end converted to the pole mass using an BLM optimized [80] version of the two-loop perturbative QCD formula [81]; this should correspond approximately to the kinematic mass extracted from the collider events.) One obtains $m_t = 170 \pm 7$ (+14) GeV, with little change in the $\sin^2\theta_W$ and α_s values, in remarkable agreement with the direct CDF/DØ value. The results of fits to various combinations of the data are shown in Table 10.5 and the relation between $\hat{s}^2_Z$ and m_t for various observables in Fig. 10.1.

Table 10.5: Values of $\hat{s}^2_Z$ and s^2_W (in parentheses), α_s, and m_t for various combinations of observables. The central values and uncertainties are for $M_H = M_Z$ while the third numbers show the shift (positive unless specified) from changing M_H to 300 GeV.

Data	$\hat{s}^2_Z$ (s^2_W)	α_s (M_Z)	m_t [GeV]
All indirect + m_t	0.23124(17)(24)	0.1214(31)(18)	173(4)(5)
	(0.2230±0.0004 (+0.0007))		
All indirect + m_t + α_s	0.23121(17)(22)	0.1191(18)(6)	173(4)(5)
	(0.2230±0.0004 (+0.0007))		
All indirect + m_t + S_r	0.23133(20)(32)	0.1218(31)(21)	173(4)(5)
	(0.2232±0.0005 (+0.0008))		
All indirect	0.23129(19)(11)	0.1216(31)(14)	170(7)(14)
	(0.2234±0.0007 (−0.0002))		
Z pole	0.23135(21)(10)	0.1218(31)(13)	168(8)(14)
	(0.2236±0.0008 (−0.0003))		
LEP 1	0.23170(24)(13)	0.1232(31)(14)	160(8)(14)
	(0.2247±0.0009 (−0.0002))		
SLD + M_Z	0.23023(43)	0.1200 (fixed)	203(13)(17)
	(0.2192±0.0017 (−0.0008))		
$A^{(0,b)}_{FB}$ + M_Z	0.23209(45)	0.1200 (fixed)	147(17)(21)
	(0.2261±0.0018 (−0.0009))		
M_W + M_Z	0.23101(43)(22)	0.1200 (fixed)	181(12)(12)
	(0.2221±0.0015)		

Using $\alpha(M_Z)$ and $\hat{s}^2_Z$ as inputs, one can predict $\alpha_s(M_Z)$ assuming grand unification. One predicts [82] $\alpha_s(M_Z) = 0.130 \pm 0.001 \pm 0.01$ for the simplest theories based on the minimal supersymmetric extension of the Standard Model, where the first (second) uncertainty is from the inputs (thresholds). This is consistent with the experimental $\alpha_s(M_Z) = 0.1216 \pm 0.0031 \pm 0.0003$ from the Z lineshape (with the second error corresponding to $M_H < 150$ GeV, as is appropriate to the lower M_H range appropriate for supersymmetry) and with the world average 0.119 ± 0.002. Nonsupersymmetric unified theories predict the low value $\alpha_s(M_Z) = 0.073 \pm 0.001 \pm 0.001$. See also the note on "Low-Energy Supersymmetry" in the Particle Listings.

One can also determine the radiative correction parameters Δr: including the CDF and DØ data, one obtains $\Delta r = 0.0355 \pm$

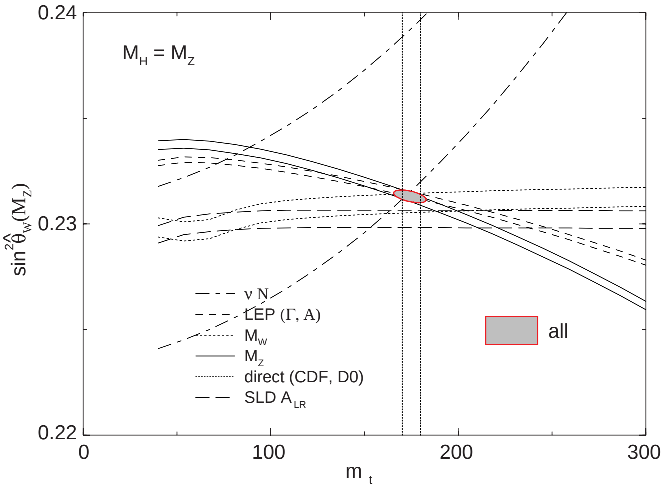

Figure 10.1: One-standard-deviation uncertainties in $\sin^2\widehat{\theta}_W$ as a function of m_t, the direct CDF and DØ range 175 ± 5 GeV, and the 90% CL region in $\sin^2\widehat{\theta}_W - m_t$ allowed by all data, assuming $M_H = M_Z$.

0.0014 (+0.0021) and $\Delta\widehat{r}_W = 0.0697 \pm 0.0005$ (+0.0001), in excellent agreement with the predictions 0.0349 ± 0.0020 and 0.0698 ± 0.0007. M_W measurements [57,71] (when combined with M_Z) are equivalent to measurements of $\Delta r = 0.0325 \pm 0.0045$.

Table 10.6: Values of the model-independent neutral-current parameters, compared with the Standard Model predictions for $M_Z = 91.1867 \pm 0.0020$ GeV, $M_H = M_Z$, and the global best fit values $m_t = 173 \pm 4$ GeV, $\alpha_s = 0.1214 \pm 0.0031$, and $\widehat{\alpha}(M_Z)^{-1} = 127.90 \pm 0.07$. There is a second $g_{V,A}^{\nu e}$ solution, given approximately by $g_V^{\nu e} \leftrightarrow g_A^{\nu e}$, which is eliminated by e^+e^- data under the assumption that the neutral current is dominated by the exchange of a single Z. θ_i, $i = L$ or R, is defined as $\tan^{-1}[\epsilon_i(u)/\epsilon_i(d)]$.

Quantity	Experimental Value	Standard Model Prediction	Correlation	
$\epsilon_L(u)$	0.328 ± 0.016	0.3461 ± 0.0002		
$\epsilon_L(d)$	-0.440 ± 0.011	-0.4292 ± 0.0002	non-	
$\epsilon_R(u)$	-0.179 ± 0.013	-0.1548 ± 0.0001	Gaussian	
$\epsilon_R(d)$	$-0.027\ ^{+0.077}_{-0.048}$	0.0775 ± 0.0001		
g_L^2	0.3009 ± 0.0028	0.3040 ± 0.0003		
g_R^2	0.0328 ± 0.0030	0.0300	small	
θ_L	2.50 ± 0.035	2.4629 ± 0.0001		
θ_R	$4.56\ ^{+0.42}_{-0.27}$	5.1765		
$g_V^{\nu e}$	-0.041 ± 0.015	-0.0395 ± 0.0005		-0.04
$g_A^{\nu e}$	-0.507 ± 0.014	-0.5064 ± 0.0002		
C_{1u}	-0.216 ± 0.046	-0.1885 ± 0.0003	-0.997	-0.78
C_{1d}	0.361 ± 0.041	0.3412 ± 0.0002		0.78
$C_{2u} - \frac{1}{2}C_{2d}$	-0.03 ± 0.12	-0.0488 ± 0.0008		

Most of the parameters relevant to ν-hadron, νe, e-hadron, and e^+e^- processes are determined uniquely and precisely from the data in "model independent" fits (*i.e.*, fits which allow for an arbitrary electroweak gauge theory). The values for the parameters defined in Eqs. (10.12)–(10.14) are given in Table 10.6 along with the predictions of the Standard Model. The agreement is excellent. The low-energy e^+e^- results are difficult to present in a model-independent way because Z propagator effects are non-negligible at TRISTAN, PETRA, and PEP energies. However, assuming e-μ-τ universality, the lepton asymmetries imply [55] $4(g_A^e)^2 = 0.99 \pm 0.05$, in good agreement with the Standard Model prediction $\simeq 1$.

The results presented here are generally in reasonable agreement with the ones obtained by the LEP Electroweak Working Group [57]. We obtain slightly higher values for α_s and significantly lower best fit values for M_H. We could trace the differences to be due to (i) the inclusion of recent higher order radiative corrections, in particular, $\mathcal{O}(\alpha^2 m_t^2)$ [26] and $\mathcal{O}(\alpha\alpha_s)$ vertex [68] corrections, as well as the leading $\mathcal{O}(\alpha_s^4)$ contribution to hadronic Z decays; (ii) the use of a slightly higher value of $\alpha(M_Z)$ [9]; (iii) a more complete set of low energy data (which is not very important for Standard Model fits, but is for physics beyond the Standard Model); and (iv) scheme dependences. Taking into account these differences, the agreement is excellent.

10.6. Constraints on new physics

The Z pole, W mass, and neutral-current data can be used to search for and set limits on deviations from the Standard Model. In particular, the combination of these indirect data with the direct CDF and DØ value for m_t allows stringent limits on new physics. We will mainly discuss the effects of exotic particles (with heavy masses $M_{\rm new} \gg M_Z$ in an expansion in $M_Z/M_{\rm new}$) on the gauge boson self-energies. (Brief remarks are made on new physics which is not of this type.) Most of the effects on precision measurements can be described by three gauge self-energy parameters S, T, and U. We will define these, as well as related parameters, such as ρ_0, ϵ_i, and $\widehat{\epsilon}_i$, to arise from new physics only. *I.e.*, they are equal to zero ($\rho_0 = 1$) exactly in the Standard Model, and do not include any contributions from m_t or M_H, which are treated seperately. Our treatment differs from most of the original papers. We also allow a $Zb\overline{b}$ vertex correction parameter γ_b.

Many extensions of the Standard Model are described by the ρ_0 parameter:

$$\rho_0 \equiv M_W^2/(M_Z^2\,\widehat{c}_Z^{\,2}\,\widehat{\rho})\ , \tag{10.42}$$

which describes new sources of SU(2) breaking that cannot be accounted for by Higgs doublets or m_t effects. In the presence of $\rho_0 \neq 1$, Eq. (10.42) generalizes Eq. (10.9*b*), while Eq. (10.9*a*) remains unchanged. Provided that the new physics which yields $\rho_0 \neq 1$ is a small perturbation which does not significantly affect the radiative corrections, ρ_0 can be regarded as a phenomenological parameter which multiplies G_F in Eqs. (10.12)–(10.14), (10.28), and Γ_Z in Eq. (10.35). There is now enough data to determine ρ_0, $\sin^2\theta_W$, m_t, and α_s simultaneously. In particular, the direct CDF and DØ events and R_b yield m_t independent of ρ_0, the asymmetries yield $\widehat{s}_Z^{\,2}$, R_ℓ gives α_s, and M_Z and the widths constrain ρ_0. From the global fit,

$$\rho_0 = 0.9998 \pm 0.0008\ (+0.0014)\ , \tag{10.43}$$
$$\widehat{s}_Z^{\,2} = 0.23126 \pm 0.00019\ (+0.00010)\ , \tag{10.44}$$
$$\alpha_s = 0.1219 \pm 0.0034\ (-0.0009)\ , \tag{10.45}$$
$$m_t = 174 \pm 5\ \text{ GeV}\ , \tag{10.46}$$

where the central values are for $M_H = M_Z$ and in parentheses we show the effect of changing M_H to 300 GeV. (As in the case $\rho_0 = 1$, the best fit value for M_H is below its direct lower limit.) The allowed regions in the $\rho_0 - \widehat{s}_Z^{\,2}$ plane are shown in Fig. 10.2.

The result in Eq. (10.43) is in remarkable agreement with the Standard Model expectation, $\rho_0 = 1$. It can be used to constrain higher-dimensional Higgs representations to have vacuum expectation values of less than a few percent of those of the doublets. Indeed, the relation between M_W and M_Z is modified if there are Higgs multiplets with weak isospin $> 1/2$ with significant vacuum expectation values. In order to calculate to higher orders in such theories one must define a set of four fundamental renormalized parameters which one may conveniently choose to be α, G_F, M_Z, and M_W, since M_W and M_Z are directly measurable. Then $\widehat{s}_Z^{\,2}$ and ρ_0 can be considered dependent parameters.

Eq. (10.43) can also be used to constrain other types of new physics. For example, nondegenerate multiplets of heavy fermions or scalars break the vector part of weak SU(2) and lead to a decrease in the value of M_Z/M_W. A nondegenerate SU(2) doublet $\binom{f_1}{f_2}$ yields a

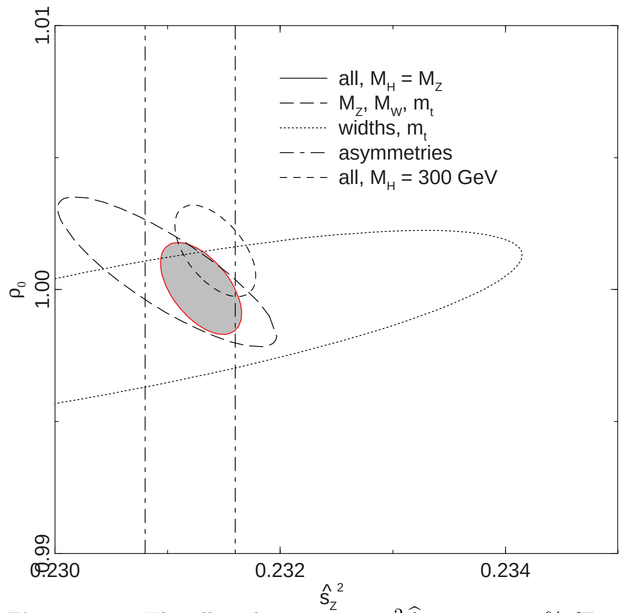

Figure 10.2: The allowed regions in $\sin^2\widehat{\theta}_W - \rho_0$ at 90% CL. m_t is a free parameter and $M_H = M_Z$ is assumed except for the dashed contour for all data which is for $M_H = 300$ GeV. The horizontal (width) band uses the experimental value of M_Z in Eq. (10.35).

positive contribution to ρ_t of [83]

$$\frac{CG_F}{8\sqrt{2}\pi^2}\Delta m^2 \ , \tag{10.47}$$

where

$$\Delta m^2 \equiv m_1^2 + m_2^2 - \frac{4m_1^2 m_2^2}{m_1^2 - m_2^2}\ln\frac{m_1}{m_2} \geq (m_1 - m_2)^2 \ , \tag{10.48}$$

and $C = 1$ (3) for color singlets (triplets). Thus, in the presence of such multiplets, one has

$$\frac{3G_F}{8\sqrt{2}\pi^2}\sum_i \frac{C_i}{3}\,\Delta m_i^2 = \rho_0 - 1 \ , \tag{10.49}$$

where the sum includes fourth-family quark or lepton doublets, $\binom{t'}{b'}$ or $\binom{E^0}{E^-}$, and scalar doublets such as $\binom{\tilde{t}}{\tilde{b}}$ in supersymmetry (in the absence of $L - R$ mixing). This implies

$$\sum_i \frac{C_i}{3}\,\Delta m_i^2 < (49 \text{ GeV})^2 \quad \text{and} \quad (83 \text{ GeV})^2 \tag{10.50}$$

for $M_H = M_Z$ and 300 GeV, respectively, at 90% CL.

Nondegenerate multiplets usually imply $\rho_0 > 1$. Similarly, heavy Z' bosons decrease the prediction for M_Z due to mixing and generally lead to $\rho_0 > 1$ [84]. On the other hand, additional Higgs doublets which participate in spontaneous symmetry breaking [85], heavy lepton doublets involving Majorana neutrinos [86], and the vacuum expectation values of Higgs triplets or higher-dimensional representations can contribute to ρ_0 with either sign. Allowing for the presence of heavy degenerate chiral multiplets (the S parameter, to be discussed below) affects the determination of ρ_0 from the data, at present leading to a smaller value.

A number of authors [87–92] have considered the general effects on neutral current and Z and W pole observables of various types of heavy (*i.e.*, $M_{\rm new} \gg M_Z$) physics which contribute to the W and Z self-energies but which do not have any direct coupling to the ordinary fermions. In addition to nondegenerate multiplets, which break the vector part of weak SU(2), these include heavy degenerate multiplets of chiral fermions which break the axial generators. The effects of one degenerate chiral doublet are small, but in technicolor theories there may be many chiral doublets and therefore significant effects [87].

Such effects can be described by just three parameters, S, T, and U at the (electroweak) one loop level. (Three additional parameters are needed if the new physics scale is comparable to M_Z [93].) T is proportional to the difference between the W and Z self-energies at $Q^2 = 0$ (*i.e.*, vector SU(2)-breaking), while S ($S + U$) is associated with the difference between the Z (W) self-energy at $Q^2 = M^2_{Z,W}$ and $Q^2 = 0$ (axial SU(2)-breaking). In the $\overline{\rm MS}$ scheme [20]

$$\begin{aligned}
\alpha(M_Z)T &\equiv \frac{\Pi^{\rm new}_{WW}(0)}{M_W^2} - \frac{\Pi^{\rm new}_{ZZ}(0)}{M_Z^2} \ , \\
\frac{\alpha(M_Z)}{4\widehat{s}^2_Z\widehat{c}^2_Z}S &\equiv \frac{\Pi^{\rm new}_{ZZ}(M_Z^2) - \Pi^{\rm new}_{ZZ}(0)}{M_Z^2} \ , \\
\frac{\alpha(M_Z)}{4\widehat{s}^2_Z}(S+U) &\equiv \frac{\Pi^{\rm new}_{WW}(M_W^2) - \Pi^{\rm new}_{WW}(0)}{M_W^2} \ ,
\end{aligned} \tag{10.51}$$

where $\Pi^{\rm new}_{WW}$ and $\Pi^{\rm new}_{ZZ}$ are, respectively, the contributions of the new physics to the W and Z self-energies. S, T, and U are defined with a factor of α removed, so that they are expected to be of order unity in the presence of new physics. They are related to other parameters ($\widehat{\epsilon}_i$, h_i, S_i) defined in [20,88,89] by

$$\begin{aligned}
T &= h_V = \widehat{\epsilon}_1/\alpha \ , \\
S &= h_{AZ} = S_Z = 4\widehat{s}^2_Z\widehat{\epsilon}_3/\alpha \ , \\
U &= h_{AW} - h_{AZ} = S_W - S_Z = -4\widehat{s}^2_Z\widehat{\epsilon}_2/\alpha \ .
\end{aligned} \tag{10.52}$$

A heavy nondegenerate multiplet of fermions or scalars contributes positively to T as

$$\rho_0 = \frac{1}{1-\alpha T} \simeq 1 + \alpha T \ , \tag{10.53}$$

where ρ_0 is given in Eq. (10.49). The effects of nonstandard Higgs representations cannot be separated from heavy nondegenerate multiplets unless the new physics has other consequences, such as vertex corrections. Most of the original papers defined T to include the effects of loops only. However, we will redefine T to include all new sources of SU(2) breaking, including nonstandard Higgs, so that T and ρ_0 are equivalent by Eq. (10.53).

A multiplet of heavy degenerate chiral fermions yields

$$S = C\sum_i \Big(t_{3L}(i) - t_{3R}(i)\Big)^2/3\pi \ , \tag{10.54}$$

where $t_{3L,R}(i)$ is the third component of weak isospin of the left- (right-) handed component of fermion i and C is the number of colors. For example, a heavy degenerate ordinary or mirror family would contribute $2/3\pi$ to S. In technicolor models with QCD-like dynamics, one expects [87] $S \sim 0.45$ for an isodoublet of technifermions, assuming $N_{TC} = 4$ technicolors, while $S \sim 1.62$ for a full technigeneration with $N_{TC} = 4$; T is harder to estimate because it is model dependent. In these examples one has $S \geq 0$. However, the QCD-like models are excluded on other grounds (flavor-changing neutral currents, and too-light quarks and pseudo-Goldstone bosons [94]). In particular, these estimates do not apply to models of walking technicolor [94], for which S can be smaller or even negative [95]. Other situations in which $S < 0$, such as loops involving scalars or Majorana particles, are also possible [96]. Supersymmetric extensions of the Standard Model generally give very small effects [97]. Most simple types of new physics yield $U = 0$, although there are counter-examples, such as the effects of anomalous triple-gauge vertices [89].

The Standard Model expressions for observables are replaced by

$$\begin{aligned}
M_Z^2 &= M_{Z0}^2\frac{1-\alpha T}{1 - G_F M_{Z0}^2 S/2\sqrt{2}\pi} \ , \\
M_W^2 &= M_{W0}^2\frac{1}{1 - G_F M_{W0}^2(S+U)/2\sqrt{2}\pi} \ ,
\end{aligned} \tag{10.55}$$

where M_{Z0} and M_{W0} are the Standard Model expressions (as functions of m_t and M_H) in the $\overline{\rm MS}$ scheme. Furthermore,

$$\begin{aligned} \Gamma_Z &= \frac{1}{1-\alpha T} M_Z^3 \beta_Z \ , \\ \Gamma_W &= M_W^3 \beta_W \ , \\ A_i &= \frac{1}{1-\alpha T} A_{i0} \ , \end{aligned} \qquad (10.56)$$

where β_Z and β_W are the Standard Model expressions for the reduced widths Γ_{Z0}/M_{Z0}^3 and Γ_{W0}/M_{W0}^3, M_Z and M_W are the physical masses, and A_i (A_{i0}) is a neutral current amplitude (in the Standard Model).

The $Z \to b\overline{b}$ vertex is sensitive to certain types of new physics which primarily couple to heavy families. It is useful to introduce an additional parameter γ_b by [98]

$$\Gamma(Z \to b\overline{b}) = \Gamma^0(Z \to b\overline{b})(1+\gamma_b) \ , \qquad (10.57)$$

where Γ^0 is the Standard Model expression (or the expression modified by S, T, and U). Experimentally, R_b is 1.3 σ above the Standard Model expectations, favoring a positive γ_b. Extended technicolor interactions generally yield negative values of γ_b of a few percent [99], although it is possible to obtain a positive γ_b in models for which the extended technicolor group does not commute with the electroweak gauge group [100] or for which diagonal interactions related to the extended technicolor dominate [101]. Topcolor and topcolor-assisted technicolor models do not generally give a significant contribution to γ_b because the extended technicolor contribution to m_t is small [102]. Supersymmetry can yield (typically small) contributions of either sign [103,104].

The data allow a simultaneous determination of $\widehat{s}^2_Z$ (*e.g.*, from the Z pole asymmetries), S (from M_Z), U (from M_W), T (*e.g.*, from the Z decay widths), α_s (from R_ℓ), m_t (from CDF and DØ), and γ_b (from R_b) with little correlation among the Standard Model parameters:

$$\begin{aligned} S &= -0.16 \pm 0.14 \ (-0.10) \ , \\ T &= -0.21 \pm 0.16 \ (+0.10) \ , \\ U &= \ \ \ 0.25 \pm 0.24 \ (+0.01) \ , \\ \gamma_b &= \ \ \ 0.007 \pm 0.005 \ , \end{aligned} \qquad (10.58)$$

and $\widehat{s}^2_Z = 0.23118 \pm 0.00023$, $\alpha_s = 0.1191 \pm 0.0051$, $m_t = 175 \pm 5$ GeV, where the uncertainties are from the inputs. The central values assume $M_H = M_Z$, and in parentheses we show the change for $M_H = 300$ GeV. The parameters in Eq. (10.58) which by definition are due to new physics only, are all consistent with the Standard Model values of zero near the 1σ level, although at present there is a slight tendency for negative S and T, and positive U and γ_b. With the latest value of R_b, the extracted $\alpha_s = 0.1191 \pm 0.0051$ is now in perfect agreement with other determinations, even in the presence of the large class of new physics allowed in this fit. Its error is slightly higher than in Eq. (10.38) for the Standard Model, but the central value is independent of M_H. Using Eq. (10.53) the value of ρ_0 corresponding to T is 0.9984 ± 0.0012 ($+0.0008$). The values of the $\widehat{\epsilon}$ parameters defined in Eq. (10.52) are

$$\begin{aligned} \widehat{\epsilon}_3 &= -0.0013 \pm 0.0012 \ (-0.0009) \ , \\ \widehat{\epsilon}_1 &= -0.0016 \pm 0.0012 \ (+0.0008) \ , \\ \widehat{\epsilon}_2 &= -0.0022 \pm 0.0021 \ (-0.0001) \ . \end{aligned} \qquad (10.59)$$

There is a strong correlation between γ_b and the predicted α_s (the correlation coefficient is -0.69), just as in the model with $S = T = U = 0$ [17]. For $\gamma_b = 0$ one obtains $\alpha_s = 0.1239 \pm 0.0037$, with little change in the other parameters. The largest correlation coefficient ($+0.73$) is between S and T. The allowed region in $S-T$ is shown in Fig. 10.3. From Eq. (10.58) one obtains $S < 0.03\,(0.08)$ and $T < 0.09\,(0.15)$ at 90 (95)% CL for $M_H = M_Z$ (S) and 300 GeV (T). If one fixes $M_H = 600$ GeV and requires the constraint $S \geq 0$ (as is appropriate in QCD-like technicolor models) then $S < 0.12\,(0.15)$. Allowing arbitrary S, an extra generation of ordinary fermions is now excluded at the 99.2% CL. This is in agreement with a fit to the number of light neutrinos, $N_\nu = 2.993 \pm 0.011$. The favored value of S is problematic for simple technicolor models with many techni-doublets and QCD-like dynamics, as is the value of γ_b. Although S is consistent with zero, the electroweak asymmetries, especially the SLD left-right asymmetry, favor $S < 0$. The simplest origin of $S < 0$ would probably be an additional heavy Z' boson [84], which could mimic $S < 0$. Similarly, there is a slight indication of negative T, while, as discussed above, nondegenerate scalar or fermion multiplets generally predict $T > 0$.

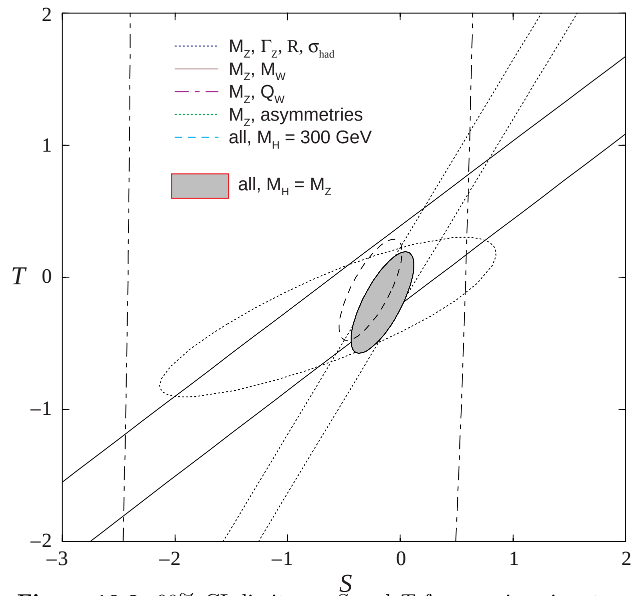

Figure 10.3: 90% CL limits on S and T from various inputs. S and T represent the contributions of new physics only. (Uncertainties from m_t are included in the errors.) The contours assume $M_H = M_Z$ except for the dashed contour for all data which is for $M_H = 300$ GeV. The fit to M_W and M_Z assumes $U = 0$, while U is arbitrary in the other fits.

There is no simple parametrization that is powerful enough to describe the effects of every type of new physics on every possible observable. The S, T, and U formalism describes many types of heavy physics which affect only the gauge self-energies, and it can be applied to all precision observables. However, new physics which couples directly to ordinary fermions, such as heavy Z' bosons [84] or mixing with exotic fermions [105] cannot be fully parametrized in the S, T, and U framework. It is convenient to treat these types of new physics by parametrizations that are specialized to that particular class of theories (*e.g.*, extra Z' bosons), or to consider specific models (which might contain, *e.g.*, Z' bosons and exotic fermions with correlated parameters). Constraints on various types of new physics are reviewed in [17,106,107]. Fits to models with technicolor, extended technicolor, and supersymmetry are described, respectively, in [100], [108], and [109]. An alternate formalism [110] defines parameters, ϵ_1, ϵ_2, ϵ_3, ϵ_b in terms of the specific observables M_W/M_Z, $\Gamma_{\ell\ell}$, $A_{FB}^{(0,\ell)}$, and R_b. The definitions coincide with those for $\widehat{\epsilon}_i$ in Eqs. (10.51) and (10.52) for physics which affects gauge self-energies only, but the ϵ's now parametrize arbitrary types of new physics. However, the ϵ's are not related to other observables unless additional model-dependent assumptions are made. Another approach [111–113] parametrizes new physics in terms of gauge-invariant sets of operators. It is especially powerful in studying the effects of new physics on nonabelian gauge vertices. The most general approach introduces deviation vectors [106]. Each type of new physics defines a deviation vector, the components of which are the deviations of each observable from its Standard Model prediction, normalized to the experimental uncertainty. The length (direction) of the vector represents the strength (type) of new physics.

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