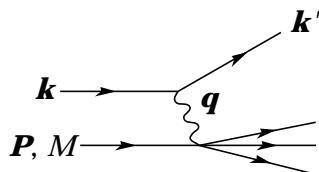


## 35. CROSS-SECTION FORMULAE FOR SPECIFIC PROCESSES

Revised April 1998 by R.N. Cahn (LBNL).

### 35.1. Leptonproduction



**Figure 35.1:** Kinematic quantities for description of lepton-nucleon scattering.  $k$  and  $k'$  are the four-momenta of incoming and outgoing leptons,  $P$  is the four-momentum of a nucleon with mass  $M$ . The exchanged particle is a  $\gamma$ ,  $W^\pm$ , or  $Z^0$ ; it transfers four-momentum  $q = k - k'$  to the target.

Invariant quantities:

$\nu = \frac{q \cdot P}{M} = E - E'$  is the lepton's energy loss in the lab (in earlier literature sometimes  $\nu = q \cdot P$ ). Here,  $E$  and  $E'$  are the initial and final lepton energies in the lab.

$Q^2 = -q^2 = 2(E E' - \vec{k} \cdot \vec{k}') - m_\ell^2 - m_{\ell'}^2$  where  $m_\ell(m_{\ell'})$  is the initial (final) lepton mass. If  $E E' \sin^2(\theta/2) \gg m_\ell^2, m_{\ell'}^2$ , then

$\approx 4E E' \sin^2(\theta/2)$ , where  $\theta$  is the lepton's scattering angle in the lab.

$x = \frac{Q^2}{2M\nu}$  In the parton model,  $x$  is the fraction of the target nucleon's momentum carried by the struck quark. [See section on Quantum Chromodynamics (Sec. 9 of this *Review*.)]

$y = \frac{q \cdot P}{k \cdot P} = \frac{\nu}{E}$  is the fraction of the lepton's energy lost in the lab.

$W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2$  is the mass squared of the system recoiling against the lepton.

$s = (k + P)^2 = \frac{Q^2}{xy} + M^2$

#### 35.1.1. Leptonproduction cross sections:

$$\begin{aligned} \frac{d^2\sigma}{dx dy} &= \nu (s - M^2) \frac{d^2\sigma}{d\nu dQ^2} = \frac{2\pi M\nu}{E'} \frac{d^2\sigma}{d\Omega_{\text{lab}} dE'} \\ &= x(s - M^2) \frac{d^2\sigma}{dx dQ^2}. \end{aligned} \quad (35.1)$$

## 2 35. Cross-section formulae for specific processes

**35.1.2. Leptoproduction structure functions:** The neutral-current process,  $eN \rightarrow eX$ , at low  $Q^2$  is just electromagnetic and parity conserving. It can be written in terms of two structure functions  $F_1^{\text{em}}(x, Q^2)$  and  $F_2^{\text{em}}(x, Q^2)$ :

$$\frac{d^2\sigma}{dx dy} = \frac{4\pi \alpha^2 (s - M^2)}{Q^4} \times \left[ (1 - y) F_2^{\text{em}} + y^2 x F_1^{\text{em}} - \frac{M^2}{(s - M^2)} xy F_2^{\text{em}} \right]. \quad (35.2)$$

The charged-current processes,  $e^-N \rightarrow \nu X$ ,  $\nu N \rightarrow e^-X$ , and  $\bar{\nu}N \rightarrow e^+X$ , are parity violating and can be written in terms of three structure functions  $F_1^{\text{CC}}(x, Q^2)$ ,  $F_2^{\text{CC}}(x, Q^2)$ , and  $F_3^{\text{CC}}(x, Q^2)$ :

$$\frac{d^2\sigma}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi} \frac{M_W^4}{(Q^2 + M_W^2)^2} \times \left\{ \left[ 1 - y - \frac{M^2 xy}{(s - M^2)} \right] F_2^{\text{CC}} + \frac{y^2}{2} 2x F_1^{\text{CC}} \pm \left( y - \frac{y^2}{2} \right) x F_3^{\text{CC}} \right\}, \quad (35.3)$$

where the last term is positive for the  $e^-$  and  $\nu$  reactions and negative for  $\bar{\nu}N \rightarrow e^+X$ . As explained below there are different structure functions for charge-raising and charge-lowering currents.

**35.1.3. Structure functions in the QCD parton model:** In the QCD parton model, the structure functions defined above can be expressed in terms of parton distribution functions. The quantity  $f_i(x, Q^2)dx$  is the probability that a parton of type  $i$  (quark, antiquark, or gluon), carries a momentum fraction between  $x$  and  $x + dx$  of the nucleon's momentum in a frame where the nucleon's momentum is large. For the cross section corresponding to the *neutral-current process*  $ep \rightarrow eX$ , we have for  $s \gg M^2$  (in the case where the incoming electron is either left- ( $L$ ) or right- ( $R$ ) handed):

$$\frac{d^2\sigma}{dx dy} = \frac{\pi\alpha^2}{sx^2 y^2} \left[ \sum_q \left( x f_q(x, Q^2) + x f_{\bar{q}}(x, Q^2) \right) \right] \times \left[ A_q + (1 - y)^2 B_q \right]. \quad (35.4)$$

Here the index  $q$  refers to a quark flavor (*i.e.*,  $u, d, s, c, b$ , or  $t$ ), and

$$A_q = \left( -q_q + g_{Lq} g_{Le} \frac{Q^2}{Q^2 + M_Z^2} \right)^2 + \left( -q_q + g_{Rq} g_{Re} \frac{Q^2}{Q^2 + M_Z^2} \right)^2, \quad (35.5)$$

$$B_q = \left( -q_q + g_{Rq} g_{Le} \frac{Q^2}{Q^2 + M_Z^2} \right)^2 + \left( -q_q + g_{Lq} g_{Re} \frac{Q^2}{Q^2 + M_Z^2} \right)^2. \quad (35.6)$$

### 35. Cross-section formulae for specific processes 3

Here  $q_q$  is the charge of flavor  $q$ . For a left-handed electron,  $g_{Re} = 0$  and  $g_{Le} = (-1/2 + \sin^2 \theta_W)/(\sin \theta_W \cos \theta_W)$ , while for a right-handed electron,  $g_{Le} = 0$  and  $g_{Re} = (\sin^2 \theta_W)/(\sin \theta_W \cos \theta_W)$ . For the quarks,  $g_{Lq} = (T_3 - q_q \sin^2 \theta_W)/(\sin \theta_W \cos \theta_W)$ , and  $g_{Rq} = (-q_q \sin^2 \theta_W) / (\sin \theta_W \cos \theta_W)$ .

For neutral-current *neutrino (antineutrino) scattering*, the same formula applies with  $g_{Le}$  replaced by  $g_{L\nu} = 1/(2 \sin \theta_W \cos \theta_W)$  ( $g_{L\bar{\nu}} = 0$ ) and  $g_{Re}$  replaced by  $g_{R\nu} = 0$  [ $g_{R\bar{\nu}} = -1/(2 \sin \theta_W \cos \theta_W)$ ].

In the case of the *charged-current processes*  $e_L^- p \rightarrow \nu X$  and  $\bar{\nu} p \rightarrow e^+ X$ , Eq. (35.3) applies with

$$F_2 = 2xF_1 = 2x \left[ f_u(x, Q^2) + f_c(x, Q^2) + f_t(x, Q^2) + f_{\bar{d}}(x, Q^2) + f_{\bar{s}}(x, Q^2) + f_{\bar{b}}(x, Q^2) \right], \quad (35.7)$$

$$F_3 = 2 \left[ f_u(x, Q^2) + f_c(x, Q^2) + f_t(x, Q^2) - f_{\bar{d}}(x, Q^2) - f_{\bar{s}}(x, Q^2) - f_{\bar{b}}(x, Q^2) \right]. \quad (35.8)$$

For the process  $\nu p \rightarrow e^- X$ :

$$F_2 = 2xF_1 = 2x \left[ f_d(x, Q^2) + f_s(x, Q^2) + f_b(x, Q^2) + f_{\bar{u}}(x, Q^2) + f_{\bar{c}}(x, Q^2) + f_{\bar{t}}(x, Q^2) \right], \quad (35.9)$$

$$F_3 = 2 \left[ f_d(x, Q^2) + f_s(x, Q^2) + f_b(x, Q^2) - f_{\bar{u}}(x, Q^2) - f_{\bar{c}}(x, Q^2) - f_{\bar{t}}(x, Q^2) \right]. \quad (35.10)$$

#### 35.2. $e^+e^-$ annihilation

For pointlike, spin-1/2 fermions, the differential cross section in the c.m. for  $e^+e^- \rightarrow f\bar{f}$  via single photon annihilation is ( $\theta$  is the angle between the incident electron and the produced fermion;  $N_c = 1$  if  $f$  is a lepton and  $N_c = 3$  if  $f$  is a quark).

$$\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4s} \beta [1 + \cos^2 \theta + (1 - \beta^2) \sin^2 \theta] Q_f^2, \quad (35.11)$$

where  $\beta$  is the velocity of the final state fermion in the c.m. and  $Q_f$  is the charge of the fermion in units of the proton charge. For  $\beta \rightarrow 1$ ,

$$\sigma = N_c \frac{4\pi\alpha^2}{3s} Q_f^2 = N_c \frac{86.8 Q_f^2 \text{ nb}}{s(\text{GeV}/c^2)^2}. \quad (35.12)$$

At higher energies, the  $Z^0$  (mass  $M_Z$  and width  $\Gamma_Z$ ) must be included. If the mass of a fermion  $f$  is much less than the mass of the  $Z^0$ , then the differential cross section for  $e^+e^- \rightarrow f\bar{f}$  is

## 4 35. Cross-section formulae for specific processes

$$\frac{d\sigma}{d\Omega} = N_c \frac{\alpha^2}{4s} \left\{ (1 + \cos^2 \theta) \left[ Q_f^2 - 2\chi_1 v_e v_f Q_f + \chi_2 (a_e^2 + v_e^2)(a_f^2 + v_f^2) \right] + 2 \cos \theta \left[ -2\chi_1 a_e a_f Q_f + 4\chi_2 a_e a_f v_e v_f \right] \right\} \quad (35.13)$$

where

$$\begin{aligned} \chi_1 &= \frac{1}{16 \sin^2 \theta_W \cos^2 \theta_W} \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}, \\ \chi_2 &= \frac{1}{256 \sin^4 \theta_W \cos^4 \theta_W} \frac{s^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}, \\ a_e &= -1, \\ v_e &= -1 + 4 \sin^2 \theta_W, \\ a_f &= 2T_{3f}, \\ v_f &= 2T_{3f} - 4Q_f \sin^2 \theta_W, \end{aligned} \quad (35.14)$$

where  $T_{3f} = 1/2$  for  $u, c$  and neutrinos, while  $T_{3f} = -1/2$  for  $d, s, b$ , and negatively charged leptons.

At LEP II it may be possible to produce the orthodox Higgs boson,  $H$ , (see the mini-review on Higgs bosons) in the reaction  $e^+e^- \rightarrow HZ^0$ , which proceeds dominantly through a virtual  $Z^0$ . The Standard Model prediction for the cross section [3] is

$$\sigma(e^+e^- \rightarrow HZ^0) = \frac{\pi\alpha^2}{24} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2 + 3M_Z^2}{(s - M_Z^2)^2} \cdot \frac{1 - 4 \sin^2 \theta_W + 8 \sin^4 \theta_W}{\sin^4 \theta_W \cos^4 \theta_W}. \quad (35.15)$$

where  $K$  is the c.m. momentum of the produced  $H$  or  $Z^0$ . Near the production threshold, this formula needs to be corrected for the finite width of the  $Z^0$ .

### 35.3. Two-photon process at $e^+e^-$ colliders

When an  $e^+$  and an  $e^-$  collide with energies  $E_1$  and  $E_2$ , they emit  $dn_1$  and  $dn_2$  virtual photons with energies  $\omega_1$  and  $\omega_2$  and 4-momenta  $q_1$  and  $q_2$ . In the equivalent photon approximation, the cross section for  $e^+e^- \rightarrow e^+e^-X$  is related to the cross section for  $\gamma\gamma \rightarrow X$  by (Ref. 1)

$$d\sigma_{e^+e^- \rightarrow e^+e^-X}(s) = dn_1 dn_2 d\sigma_{\gamma\gamma \rightarrow X}(W^2) \quad (35.16)$$

where  $s = 4E_1E_2$ ,  $W^2 = 4\omega_1\omega_2$  and

$$dn_i = \frac{\alpha}{\pi} \left[ 1 - \frac{\omega_i}{E_i} + \frac{\omega_i^2}{2E_i^2} - \frac{m_e^2 \omega_i^2}{(-q_i^2)E_i^2} \right] \frac{d\omega_i}{\omega_i} \frac{d(-q_i^2)}{(-q_i^2)}. \quad (35.17)$$

### 35. Cross-section formulae for specific processes 5

After integration (including that over  $q_i^2$  in the region  $m_e^2 \omega_i^2 / E_i(E_i - \omega_i) \leq -q_i^2 \leq (-q^2)_{\max}$ ), the cross section is

$$\begin{aligned} \sigma_{e^+e^- \rightarrow e^+e^-X}(s) &= \frac{\alpha^2}{\pi^2} \int_{z_{th}}^1 \frac{dz}{z} \left[ f(z) \left( \ln \frac{(-q^2)_{\max}}{m_e^2 z} - 1 \right)^2 \right. \\ &\quad \left. - \frac{1}{3} \left( \ln \frac{1}{z} \right)^3 \right] \sigma_{\gamma\gamma \rightarrow X}(zs) ; \\ f(z) &= \left( 1 + \frac{1}{2}z \right)^2 \ln \frac{1}{z} - \frac{1}{2}(1-z)(3+z) ; \\ z &= \frac{W^2}{s} . \end{aligned} \quad (35.18)$$

The quantity  $(-q^2)_{\max}$  depends on properties of the produced system  $X$ , in particular,  $(-q^2)_{\max} \sim m_\rho^2$  for hadron production ( $X = h$ ) and  $(-q^2)_{\max} \sim W^2$  for lepton pair production ( $X = \ell^+\ell^-$ ,  $\ell = e, \mu, \tau$ ).

For production of a resonance of mass  $m_R$  and spin  $J \neq 1$

$$\begin{aligned} \sigma_{e^+e^- \rightarrow e^+e^-R}(s) &= (2J+1) \frac{8\alpha^2 \Gamma_{R \rightarrow \gamma\gamma}}{m_R^3} \\ &\times \left[ f(m_R^2/s) \left( \ln \frac{sm_V^2}{m_e^2 m_R^2} - 1 \right)^2 - \frac{1}{3} \left( \ln \frac{s}{m_R^2} \right)^3 \right] \end{aligned} \quad (35.19)$$

where  $m_V$  is the mass that enters into the form factor of the  $\gamma\gamma \rightarrow R$  transition:  $m_V \sim m_\rho$  for  $R = \pi^0, \eta, f_2(1270), \dots$ ,  $m_V \sim m_R$  for  $R = c\bar{c}$  or  $b\bar{b}$  resonances.

#### 35.4. Inclusive hadronic reactions

One-particle inclusive cross sections  $E d^3\sigma/d^3p$  for the production of a particle of momentum  $p$  are conveniently expressed in terms of rapidity (see above) and the momentum  $p_T$  transverse to the beam direction (defined in the center-of-mass frame)

$$E \frac{d^3\sigma}{d^3p} = \frac{d^3\sigma}{d\phi dy p_T dp_T} . \quad (35.20)$$

In the case of processes where  $p_T$  is large or the mass of the produced particle is large (here large means greater than 10 GeV), the parton model can be used to calculate the rate. Symbolically

$$\sigma_{\text{hadronic}} = \sum_{ij} \int f_i(x_1, Q^2) f_j(x_2, Q^2) dx_1 dx_2 \hat{\sigma}_{\text{partonic}} , \quad (35.21)$$

where  $f_i(x, Q^2)$  is the parton distribution introduced above and  $Q$  is a typical momentum transfer in the partonic process and  $\hat{\sigma}$  is the partonic cross section. Some examples will

## 6 35. Cross-section formulae for specific processes

help to clarify. The production of a  $W^+$  in  $pp$  reactions at rapidity  $y$  in the center-of-mass frame is given by

$$\begin{aligned} \frac{d\sigma}{dy} = & \frac{G_F \pi \sqrt{2}}{3} \\ & \times \tau \left[ \cos^2 \theta_c \left( u(x_1, M_W^2) \bar{d}(x_2, M_W^2) \right. \right. \\ & \quad \left. \left. + u(x_2, M_W^2) \bar{d}(x_1, M_W^2) \right) \right. \\ & \quad \left. + \sin^2 \theta_c \left( u(x_1, M_W^2) \bar{s}(x_2, M_W^2) \right. \right. \\ & \quad \left. \left. + s(x_2, M_W^2) \bar{u}(x_1, M_W^2) \right) \right], \end{aligned} \quad (35.22)$$

where  $x_1 = \sqrt{\tau} e^y$ ,  $x_2 = \sqrt{\tau} e^{-y}$ , and  $\tau = M_W^2/s$ . Similarly the production of a jet in  $pp$  (or  $p\bar{p}$ ) collisions is given by

$$\begin{aligned} \frac{d^3\sigma}{d^2p_T dy} = & \sum_{ij} \int f_i(x_1, p_T^2) f_j(x_2, p_T^2) \\ & \times \left[ \hat{s} \frac{d\hat{\sigma}}{d\hat{t}} \right]_{ij} dx_1 dx_2 \delta(\hat{s} + \hat{t} + \hat{u}), \end{aligned} \quad (35.23)$$

where the summation is over quarks, gluons, and antiquarks. Here

$$s = (p_1 + p_2)^2, \quad (35.24)$$

$$t = (p_1 - p_{\text{jet}})^2, \quad (35.25)$$

$$u = (p_2 - p_{\text{jet}})^2, \quad (35.26)$$

$p_1$  and  $p_2$  are the momenta of the incoming  $p$  and  $p$  (or  $\bar{p}$ ) and  $\hat{s}$ ,  $\hat{t}$ , and  $\hat{u}$  are  $s$ ,  $t$ , and  $u$  with  $p_1 \rightarrow x_1 p_1$  and  $p_2 \rightarrow x_2 p_2$ . The partonic cross section  $\hat{s}[(d\hat{\sigma})/(d\hat{t})]$  can be found in Ref. 2. Example: for the process  $gg \rightarrow q\bar{q}$ ,

$$\hat{s} \frac{d\sigma}{dt} = 3\alpha_s^2 \frac{(\hat{t}^2 + \hat{u}^2)}{8\hat{s}} \left[ \frac{4}{9\hat{t}\hat{u}} - \frac{1}{\hat{s}^2} \right]. \quad (35.27)$$

The prediction of Eq. (35.23) is compared to data from the UA1 and UA2 collaborations in Fig. 37.8 in the Plots of Cross Sections and Related Quantities section of this *Review*.

The associated production of a Higgs boson and a gauge boson is analogous to the process  $e^+e^- \rightarrow HZ^0$  in Sec. 35.2. The required parton-level cross sections [4], averaged over initial quark colors, are

$$\begin{aligned} \sigma(q_i \bar{q}_j \rightarrow W^\pm H) &= \frac{\pi \alpha^2 |V_{ij}|^2}{36 \sin^4 \theta_W} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2 + 3M_W^2}{(s - M_W^2)^2} \\ \sigma(q\bar{q} \rightarrow Z^0 H) &= \frac{\pi \alpha^2 (a_q^2 + v_q^2)}{144 \sin^4 \theta_W \cos^4 \theta_W} \cdot \frac{2K}{\sqrt{s}} \cdot \frac{K^2 + 3M_Z^2}{(s - M_Z^2)^2}. \end{aligned}$$

Here  $V_{ij}$  is the appropriate element of the Kobayashi-Maskawa matrix and  $K$  is the c.m. momentum of the produced  $H$ . The axial and vector couplings are defined as in Sec. 35.2.

### 35.5. One-particle inclusive distributions

In order to describe one-particle inclusive production in  $e^+e^-$  annihilation or deep inelastic scattering, it is convenient to introduce a fragmentation function  $D_i^h(z, Q^2)$  where  $D_i^h(z, Q^2)$  is the number of hadrons of type  $h$  and momentum between  $zp$  and  $(z + dz)p$  produced in the fragmentation of a parton of type  $i$ . The  $Q^2$  evolution is predicted by QCD and is similar to that of the parton distribution functions [see section on Quantum Chromodynamics (Sec. 9 of this *Review*)]. The  $D_i^h(z, Q^2)$  are normalized so that

$$\sum_h \int z D_i^h(z, Q^2) dz = 1. \quad (35.28)$$

If the contributions of the  $Z$  boson and three-jet events are neglected, the cross section for producing a hadron  $h$  in  $e^+e^-$  annihilation is given by

$$\frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{dz} = \frac{\sum_i e_i^2 D_i^h(z, Q^2)}{\sum_i e_i^2}, \quad (35.29)$$

where  $e_i$  is the charge of quark-type  $i$ ,  $\sigma_{\text{had}}$  is the total hadronic cross section, and the momentum of the hadron is  $zE_{\text{cm}}/2$ .

In the case of deep inelastic muon scattering, the cross section for producing a hadron of energy  $E_h$  is given by

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dz} = \frac{\sum_i e_i^2 q_i(x, Q^2) D_i^h(z, Q^2)}{\sum_i e_i^2 q_i(x, Q^2)}, \quad (35.30)$$

where  $E_h = \nu z$ . (For the kinematics of deep inelastic scattering, see Sec. 34.4.2 of the Kinematics section of this *Review*.) The fragmentation functions for light and heavy quarks have a different  $z$  dependence; the former peak near  $z = 0$ . They are illustrated in Figs. 36.1 and 36.2 in the section on ‘‘Heavy Quark Fragmentation in  $e^+e^-$  Annihilation’’ (Sec. 36 of this *Review*).

#### References:

1. V.M. Budnev, I.F. Ginzburg, G.V. Meledin, and V.G. Serbo, Phys. Reports **15C**, 181 (1975);  
See also S. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. **D4**, 1532 (1971).
2. G.F. Owens, F. Reya, and M. Glück, Phys. Rev. **D18**, 1501 (1978).
3. B.W. Lee, C. Quigg, and B. Thacker, Phys. Rev. **D16**, 1519 (1977).
4. E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984).