NOTE ON THE $\Lambda(1405)$

Revised March 1998 by R.H. Dalitz (Oxford University).

It is generally accepted that the $\Lambda(1405)$ is a well-established $J^P = 1/2^-$ resonance. It is assigned to the lowest $L = 1$ supermultiplet of the 3-quark system and paired with the $J^P = 3/2^-$ $\Lambda(1520)$. Lying about 30 MeV below the $N\overline{K}$ threshold, the $\Lambda(1405)$ can be observed directly only as a resonance bump in the $(\Sigma\pi)^0$ subsystem in final states of production experiments. It was first reported by ALSTON 61B in the reaction $K^-p \to \Sigma\pi\pi\pi$ at 1.15 GeV/c and has since been seen in at least eight other experiments. However, only two of them had enough events for a detailed analysis: THOMAS 73, with about 400 $\Sigma^\pm\pi^\mp$ events from $\pi^-p \to K^0(\Sigma\pi)^0$ at 1.69 GeV/c; and HEMINGWAY 85, with 766 $\Sigma^+\pi^-$ and 1106 $\Sigma^-\pi^+$ events from $K^-p \to (\Sigma\pi\pi)^+\pi^-$ at 4.2 GeV/c, after the selections $1600 \leq M(\Sigma\pi\pi)^+ \leq 1720$ MeV and momentum transfer ≤ 1.0 (GeV/c)^2 to purify the $\Lambda(1405) \to (\Sigma\pi)^0$ sample. These experiments agree on a mass of about 1395–1400 MeV and a width of about 60 MeV. (Hemingway's mass of 1391 ± 1 MeV is from his best, but unacceptably poor, Breit-Wigner fit.)

The Byers-Fenster tests on these data allow any spin and either parity: neither J nor P has yet been determined *directly*. The early indications for $J^P = 1/2^-$ came from finding Re $A_{I=0}$ to be large and negative in a constant-scattering-length analysis of low-energy $N\overline{K}$ reaction data (see KIM 65, SAKITT 65, and earlier references cited therein). The first multichannel energy-dependent K-matrix analysis (KIM 67) strengthened the case for a resonance around 1400–1420 MeV strongly coupled to the $I = 0$ S-wave $N\overline{K}$ system.

THOMAS 73 and HEMINGWAY 85 both found the $\Lambda(1405)$ bump to be asymmetric and not well fitted by a Breit-Wigner resonance function with constant parameters. The asymmetry involves a rapid fall in intensity as the $N\overline{K}$ threshold energy is approached from below. This is readily understood as due to a strong coupling of the $\Lambda(1405)$ to the S-wave $N\overline{K}$ channel (see DALITZ 81). This striking S-shaped cusp behavior at a new threshold is characteristic of S-wave coupling; the other below-threshold hyperon, the $\Sigma(1385)$, has no such threshold distortion because its $N\overline{K}$ coupling is P-wave. For the $\Lambda(1405)$, this asymmetry is the *sole direct evidence* that $J^P = 1/2^-$.

Following the early work cited above, a considerable literature has developed on proper procedures for phenomenological extrapolation below the $N\overline{K}$ threshold, partly in order to strengthen the evidence for the spin-parity of the $\Lambda(1405)$, and partly to provide an estimate for the amplitude $f(N\overline{K})$ in the unphysical domain below the $N\overline{K}$ threshold; the latter is needed for the evaluation of the dispersion relation for $N\overline{K}$ and NK forward scattering amplitudes. For recent reviews, see MILLER 84 and BARRETT 89. In most recent work, the $(\Sigma\pi)^0$ production spectrum is included in the data fitted (see, *e.g.*, CHAO 73, MARTIN 81).

It is now accepted that the data can be fitted only with an S-wave pole in the reaction amplitudes below $N\overline{K}$ threshold (see, however, FINK 90), but there is still controversy about the physical origin of this pole (for a review, see DALITZ 81 and DALITZ 82). Two extreme possibilities are: (a) an $L = 1$ SU(3)-singlet *uds* state coupled with the S-wave meson-baryon systems; or (b) an unstable $N\overline{K}$ bound state, analogous to the (stable) deuteron in the NN system. The problem with (a) is that the $\Lambda(1405)$ mass is so much lower than that of its partner, the $\Lambda(1520)$. This requires, in the QCD-inspired quark model, rather large spin-orbit couplings, whether or not one uses relativistic kinetic energies. CAPSTICK 86 and CAPSTICK 89 conclude that a proper QCD calculation leads only to small energy splittings, whereas LEINWEBER 90, using QCD sum rules, obtains a good fit to this splitting.

On the other hand, the problem with (b) is that then another $J^P = 1/2^- \Lambda$ is needed to replace the $\Lambda(1405)$ in the $L = 1$ supermultiplet, and it would have to lie close to the $\Lambda(1520)$, a region already well explored by $N\overline{K}$ experiments without result. Intermediate structures are possible; for example, the cloudy bag model allows the configurations (a) and (b) to mix and finds the intensity of (a) in the $\Lambda(1405)$ to be only 14% (VEIT 84, VEIT 85, JENNINGS 86). Such models naturally predict a second $1/2^-$ Λ close to the $\Lambda(1520)$.

The determination of the mass and width of the resonance from $(\Sigma\pi)^0$ data is usually based on the "Watson approximation," which states that the production rate $R(\Sigma\pi)$ of the $(\Sigma\pi)^0$ state has a mass dependence proportional to $(\sin^2\delta_{\Sigma\pi})/q$, q being the $\Sigma\pi$ c.m. momentum, in a $\Sigma\pi$ mass range where $\delta_{\Sigma\pi}$ is not far from $\pi/2$ and only the $\Sigma\pi$ channel is open, *i.e.*, between the $\Sigma\pi$ and the $N\overline{K}$ thresholds. Then $q\,R(\Sigma\pi)$ is proportional to $\sin^2\delta_{\Sigma\pi}$, and the mass M may be defined as the energy at which $\sin^2\delta_{\Sigma\pi} = 1$. The width Γ may be determined from the rate at which $\delta_{\Sigma\pi}$ goes through $\pi/2$, or from the FWHM; this is a matter of convention.

This determination of M and Γ from the data suffers from the following defects:

(i) The determination of $\sin^2\delta_{\Sigma\pi}$ requires that $R(\Sigma\pi)$ be scaled to give $\sin^2\delta_{\Sigma\pi} = 1$ at the peak for the best fit to the data; *i.e.*, the bump must be *assumed* to arise from a resonance. However, this assumption is supported by the analysis of the low-energy $N\overline{K}$ data and its extrapolation below threshold.

(ii) Owing to the nearby $N\overline{K}$ threshold, the shape of the best fit to the $M(\Sigma\pi)$ bump is uncertain. For energies below this threshold at $E_{N\overline{K}}$, the general form for $\delta_{\Sigma\pi}$ is

$$q \cot \delta_{\Sigma\pi} = \frac{1+\kappa\alpha}{\gamma + \kappa(\alpha\gamma - \beta^2)} \ . \tag{1}$$

Here α, β, and γ are the (generally energy-dependent) NN, $N\Sigma$, and $\Sigma\Sigma$ elements of the $I = 0$ S-wave K-matrix for the $(\Sigma\pi, N\overline{K})$ system, and κ is the magnitude of the (imaginary) c.m. momentum k_K for the $N\overline{K}$ system below threshold. The elements α, β, γ are real functions of E; they have no branch cuts at the $\Sigma\pi$ and $N\overline{K}$ thresholds, but they are permitted to have poles in E along the real E axis. The resonance asymmetry arises from the effect of κ on $\delta_{\Sigma\pi}$. We note that $\delta_{\Sigma\pi} = \pi/2$ when $\kappa = -1/\alpha$.

Accepting this close connection of $\delta_{\Sigma\pi}$ with the low-energy $N\overline{K}$ data, it is natural to analyze the two sets of data together (*e.g.*, MARTIN 81), and there is now a large body of accurate $N\overline{K}$ data for laboratory momenta between 100 and 300 MeV/c (see MILLER 84). The two sets of data span c.m. energies from 1370 MeV to 1490 MeV, and the K-matrix elements will not be energy independent over such a broad range. For the $I = 0$ channels, a linear energy dependence for K^{-1} has been adopted routinely ever since the work of KIM 67, and it is essential when fitting the $q\,R(\Sigma\pi)$ and $N\overline{K}$ data together. However, $q\,R(\Sigma\pi)$ is not always well fitted in this procedure; the value obtained for the $\Lambda(1405)$ mass M varies a good deal with the type of fit, not a surprising result when the $\Sigma\pi$ mass spectrum below the pK^- threshold contributes only nine data points in a total of about 200. The value of M obtained from an overall fit is not necessarily much better than from one using only the $q\,R(\Sigma\pi)$ data; and M may be a function of the representation—K-matrix, K^{-1}-matrix, relativistic-separable or nonseparable potentials, *etc.*— used in fitting over the full energy range. DALITZ 91 fitted the $q\,R(\Sigma^+\pi^-)$ Hemingway data with each of the first three representations just mentioned, constrained to the $I = 0$ $N\overline{K}$ threshold scattering length from low-energy $N\overline{K}$ data. The (nonseparable) meson-exchange potentials of MÜLLER-GROELING 90, fitted to the low-energy $N\overline{K}$ (and NK) data, predicted an unstable $N\overline{K}$ bound state with mass and width compatible with the $\Lambda(1405)$.

From the measurement of $2p \rightarrow 1s$ x rays from kaonic-hydrogen, the energy-level shift ΔE and width Γ of its 1s state can give us two further constraints on the $(\Sigma\pi, N\overline{K})$ system, at an energy roughly midway between those from the low-energy hydrogen bubble chamber studies and those from $q\,R(\Sigma\pi)$ observations below the pK^- threshold. IWASAKI 97 have reported the first convincing observation of this x ray, with a good initial estimate:

$$\Delta E - i\Gamma/2 = (-323 \pm 63 \pm 11) - i(204 \pm 104 \pm 50)\ \text{eV}\ . \quad (2)$$

The errors here encompass about half of the predictions made following the various analyses and/or models for the in-flight K^-p and sub-threshold $q\,R(\Sigma\pi)$ data. Better measurements will be needed to discriminate between the analyses and predictions. Now that ΔE is known with some certainty, we can anticipate much-improved data on kaonic-hydrogen, perhaps from the DAΦNE storage ring at Frascati, information vital for our quantitative understanding of the $(\Sigma\pi, N\overline{K})$ system in this region. This will lead to better knowledge of kaonic coupling strengths and to more reliable dispersion-theoretic arguments concerning strange-particle processes.

The present status of the $\Lambda(1405)$ thus depends heavily on theoretical arguments, a somewhat unsatisfactory basis for a four-star rating. Nevertheless, there is no known reason to doubt its existence or quantum numbers. The 3-quark model for baryons has been broadly successful in accounting for all of the $L^P = 1^-$ excited baryonic states (CAPSTICK 89), apart from the relatively large mass separation between the $\Lambda(1405)$ and $\Lambda(1520)$. Quark model builders have no reservations about accepting the $\Lambda(1405)$ as a 3-quark state. However, calculations with broken-chiral-symmetric models, which combine internal 3-quark configurations with external meson-baryon states (*e.g.*, VEIT 85, KAISER 95) end up with descriptions of the $\Lambda(1405)$ dominated by the meson-baryon terms in the wavefunctions. Models using meson-baryon potentials readily fit its mass, and give ΔE negative, as is found empirically. The problem is not so much one of "either (a) or (b)," but rather how to achieve "both (a) and (b)." Theoreticians have not yet been able to deal with the full coupled-channels system, with qqq and $qqqq\overline{q}$ configurations (at the least) being treated on the same footing. On the experimental side, better statistics are needed, both above and below the pK^- threshold. To disentangle the physics, the $I = 1$ channels also need more attention. For example, low-energy pK_L^0 interactions have not been studied at all in the last 25 years.