12. CP VIOLATION

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The symmetries C (particle-antiparticle interchange) and P (space inversion) hold for strong and electromagnetic interactions. After the discovery of large C and P violation in the weak interactions, it appeared that the product CP was a good symmetry. In 1964 CP violation was observed in K^0 decays at a level given by the parameter $\epsilon \approx 2.3 \times 10^{-3}$. Larger CP-violation effects are anticipated in B^0 decays.

12.1. CP violation in Kaon decay

CP violation has been observed in the semi-leptonic decays $K_L^0 \to \pi^\mp \ell^\pm \nu$ and in the nonleptonic decay $K_L^0 \to 2\pi$. The experimental numbers that have been measured are

$$\delta = \frac{\Gamma(K_L^0 \to \pi^- \ell^+ \nu) - \Gamma(K_L^0 \to \pi^+ \ell^- \nu)}{\Gamma(K_L^0 \to \pi^- \ell^+ \nu) + \Gamma(K_L^0 \to \pi^+ \ell^- \nu)} \tag{12.1a}$$

$$\begin{aligned} \eta_{+-} &= A(K_L^0 \to \pi^+\pi^-)/A(K_S^0 \to \pi^+\pi^-) \\ &= |\eta_{+-}|\, e^{i\phi_{+-}} \qquad (12.1b) \\ \eta_{00} &= A(K_L^0 \to \pi^0\pi^0)/A(K_S^0 \to \pi^0\pi^0) \\ &= |\eta_{00}|\, e^{i\phi_{00}}\ . \qquad (12.1c) \end{aligned}$$

CP violation can occur either in the K^0–$\overline{K}^0$ mixing or in the decay amplitudes. Assuming CPT invariance, the mass eigenstates of the K^0–$\overline{K}^0$ system can be written

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle\ , \qquad |K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle\ . \tag{12.2}$$

If CP invariance held, we would have $q = p$ so that K_S would be CP even and K_L CP odd. (We define $|\overline{K}^0\rangle$ as $CP\ |K^0\rangle$). CP violation in K^0–$\overline{K}^0$ mixing is then given by the parameter $\widetilde{\epsilon}$ where

$$\frac{p}{q} = \frac{(1+\widetilde{\epsilon})}{(1-\widetilde{\epsilon})}\ . \tag{12.3}$$

CP violation can also occur in the decay amplitudes

$$A\big(K^0 \to \pi\pi(I)\big) = A_I e^{i\delta_I}\ , \qquad A\big(\overline{K}^0 \to \pi\pi(I)\big) = A_I^* e^{i\delta_I}\ , \tag{12.4}$$

where I is the isospin of $\pi\pi$, δ_I is the final-state phase shift, and A_I would be real if CP invariance held. The CP-violating observables are usually expressed in terms of ϵ and ϵ' defined by

$$\eta_{+-} = \epsilon + \epsilon'\ , \qquad \eta_{00} = \epsilon - 2\epsilon'\ , \tag{12.5a}$$

One can then show [1]

$$\epsilon = \widetilde{\epsilon} + i\ \big(\mathrm{Im}\ A_0/\mathrm{Re}\ A_0\big)\ , \tag{12.5b}$$

$$\sqrt{2}\epsilon' = ie^{i(\delta_2-\delta_0)}(\mathrm{Re}\ A_2/\mathrm{Re}\ A_0)\ \big(\mathrm{Im}\ A_2/\mathrm{Re}\ A_2 - \mathrm{Im}\ A_0/\mathrm{Re}\ A_0\big)\ , \tag{12.5c}$$

$$\delta = 2\mathrm{Re}\ \epsilon/(1+|\epsilon|^2) \approx 2\mathrm{Re}\ \epsilon\ . \tag{12.5d}$$

In Eq. (12.5c) small corrections of order $\epsilon' \times \mathrm{Re}\ (A_2/A_0)$ are neglected and Eq. (12.5d) assumes the $\Delta S = \Delta Q$ rule.

The quantities Im A_0, Im A_2, and Im ϵ depend on the choice of phase convention since one can change the phases of K^0 and $\overline{K}^0$ by a transformation of the strange quark state $|s\rangle \to |s\rangle\ e^{i\alpha}$; of course, observables are unchanged. It is possible by a choice of phase convention to set $\mathrm{Im}\,A_0$ or $\mathrm{Im}\,A_2$ or $\mathrm{Im}\,\widetilde{\epsilon}$ to zero, but none of these is zero with the usual phase conventions in the Standard Model. The choice $\mathrm{Im}\,A_0 = 0$ is called the Wu-Yang phase convention [2] in which case $\epsilon = \widetilde{\epsilon}$. The value of ϵ' is independent of phase convention and a nonzero value demonstrates CP violation in the decay amplitudes, referred to as direct CP violation. The possibility that direct CP violation is essentially zero and that CP violation occurs only in the mixing matrix was referred to as the superweak theory [3].

By applying CPT invariance and unitarity the phase of ϵ is given approximately by

$$\phi(\epsilon)\ \approx\ \tan^{-1}\frac{2(m_{K_L} - m_{K_S})}{\Gamma_{K_S} - \Gamma_{K_L}} = 43.49 \pm 0.08^\circ \tag{12.6a}$$

while Eq. (12.5c) gives

$$\phi(\epsilon') = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 48 \pm 4^\circ\ , \tag{12.6b}$$

where the numerical value is based on an analysis of π–π scattering [4]. The approximation in Eq. (12.6a) depends on the assumption that direct CP violation is very small in all K^0 decays. This is expected to be good to a few tenths of a degree as indicated by the small value of ϵ' and of η_{+-0}, the CP violation parameter in the decay $K_S \to \pi^+\pi^-\pi^0$ [5], although limits on η_{000} are still poor. The relation in Eq. (12.6a) is exact in the superweak theory so this is sometimes called the superweak phase. An important point for the analysis is that $\cos[\phi(\epsilon') - \phi(\epsilon)] \simeq 1$. The consequence is that only two real quantities need be measured, the magnitude of ϵ and the value of (ϵ'/ϵ) including its sign. The measured quantity $|\eta_{00}/\eta_{+-}|^2$, which is very close to unity, is given to a good approximation by

$$|\eta_{00}/\eta_{+-}|^2 \approx 1 - 6\mathrm{Re}\ (\epsilon'/\epsilon) \approx 1 - 6\epsilon'/\epsilon\ . \tag{12.7}$$

From the experimental measurements, one finds

$$\epsilon = (2.271 \pm 0.017) \times 10^{-3}\ , \tag{12.8a}$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \epsilon'/\epsilon = (2.1 \pm 0.5) \times 10^{-3}\ , \tag{12.8b}$$

$$\phi_{+-} = 43.5 \pm 0.5^\circ\ , \tag{12.8c}$$

$$\phi_{00}–\phi_{+-} = -0.1 \pm 0.8\ , \tag{12.8d}$$

$$\delta = (3.33 \pm 0.14) \times 10^{-3}\ . \tag{12.8e}$$

Direct CP violation, as indicated by ϵ'/ϵ, is expected in the Standard Model; most calcuations [6] give a somewhat smaller value, but they have a large uncertainty. The value of δ agrees with Eq. (12.5d). The values of ϕ_{+-} and $\phi_{00} - \phi_{+-}$ are used to set limits on CPT violation. [See Tests of Conservation Laws.]

In the Standard Model, CP violation arises as a result of a single phase entering the CKM matrix (Sec. 11). As a result in what is now the standard phase convention, two elements have large phases, $V_{ub} \sim e^{-i\gamma}$, $V_{td} \sim e^{-i\beta}$. Because these elements have small magnitudes and involve the third generation, CP violation in the K^0 system is small. On the other hand, large effects are expected in the B^0 system, which is a major motivation for B factories.

12.2. CP violation in B decay

CP violation in the B^0 system can be observed by comparing B^0 and $\overline{B}^0$ decays [7]. For a final CP eigenstate a, the decay rate has a time dependence given by

$$\begin{aligned} \Gamma_a \sim e^{-\Gamma t}\Big(&\big[1+|\lambda_a|^2\big] \pm \big[1-|\lambda_a|^2\big]\ \cos(\Delta M t) \\ &\mp \mathrm{Im}\ \lambda_a\ \ \sin(\Delta M t)\Big) \end{aligned} \tag{12.9}$$

where the top sign is for B^0 and the bottom for $\overline{B}^0$ and

$$\lambda_a = (q_B/p_B)\ \overline{A}_a/A_a\ . \tag{12.10}$$

The quantities p_B and q_B come from the analogue for B^0 of Eq. (12.2), and $A_a(\overline{A}_a)$ is the decay amplitude to state a for $B^0(\overline{B}^0)$. However, for B^0 the eigenstates are expected to have a negligible lifetime difference and are only distinguished by the mass difference ΔM; also as a consequence $|q_B/p_B| \approx 1$ so that $\widetilde{\epsilon}_B$ is purely imaginary.

If only one quark weak transition contributes to the decay, $|\overline{A}_a/A_a| = 1$ so that $|\lambda_a| = 1$ and the $\cos(\Delta Mt)$ term vanishes. In this case, the difference between B^0 and $\overline{B}^0$ decays is given by the $\sin(\Delta Mt)$ term with the asymmetry coefficient

$$a_a = \frac{\Gamma_a(t) - \overline{\Gamma}_a(t)}{\left(\Gamma_a(t) + \overline{\Gamma}_a(t)\right)\sin(\Delta Mt)} = \eta_a \sin\left(2(\phi_M + \phi_D)\right) , \qquad (12.11)$$

where $2\phi_M$ is the phase of the B^0–$\overline{B}^0$ mixing, ϕ_D is the weak phase of the decay transition, and η_a is the CP eigenvalue of a.

For $B^0(\overline{B}^0) \to \psi K_S$ from the transition $b \to c\overline{c}s$, one finds in the Standard Model that the asymmetry is given directly in terms of a CKM phase with no hadronic uncertainty:

$$a_{\psi K_S} = -\sin 2\beta \; . \qquad (12.12)$$

From the constraints on the CKM matrix (Sec. 11) $\sin 2\beta$ is predicted to be between 0.3 and 0.9. A significantly different value could be a sign of new physics.

A second decay of interest is B^0 $(\overline{B}^0) \to \pi^+\pi^-$ from the transition $b \to u\overline{u}d$ with

$$a_{\pi\pi} = \sin 2(\beta + \gamma) \; . \qquad (12.13)$$

While either of these asymmetries could be ascribed to B^0–$\overline{B}^0$ mixing (q_B/p_B or $\widetilde{\epsilon}_B$), the difference between the two asymmetries is evidence for direct CP violation. From Eq. (12.10) it is seen that this corresponds to a phase difference between $A_{\psi K_S}$ and $A_{\pi^+\pi^-}$. Thus this is analogous to ϵ'. In the standard phase convention, 2β in Eqs. (12.12) and (12.13) arises from B^0–$\overline{B}^0$ mixing whereas the γ in Eq. (12.13) comes from V_{ub} in the transition $b \to u\overline{u}d$. The result in Eq. (12.13) may have a sizeable correction due to what is called a penguin diagram. This is a one-loop graph producing $b \to d +$ gluon with a W and a quark, predominantly the t quark, in the loop. This leads to an amplitude proportional to $V_{tb}^*V_{td}$, which has a weak phase different from that of the original tree amplitude proportional to $V_{ub}V_{ud}^*$. There are several methods to approximately determine this correction using additional measurements [8].

CP violation in the decay amplitude is also revealed by the $\cos(\Delta Mt)$ term in Eq. (12.9) or by a difference in rates of B^+ and B^- to charge-conjugate states. These effects, however, require two contributing amplitudes to the decay (such as a tree amplitude plus a penguin) and also require final-state interaction phases. Predicted effects are very uncertain and are generally small [9].

In the case of the B_s system, the mass difference ΔM is much larger than for B^0 and has not yet been measured. As a result, it will be difficult to isolate the $\sin(\Delta Mt)$ term to measure asymmetries. Furthermore, in the Standard Model with the standard phase convention, ϕ_M is very small so that decays due to $b \to c\overline{c}s$, yielding $B_s \to \psi\eta'$, would have zero asymmetry. Decays due to $b \to u\overline{u}d$, yielding $B_s \to \rho^0 K_S$, would have an asymmetry $\sin 2\gamma$ in the tree approximation. The width difference $\Delta\Gamma$ is also expected to be much larger for B_s so that $\Delta\Gamma/\Gamma$ might be as large as 0.15. In this case, there might be a possibility of detecting CP violation as in the case of K^0 by observing the B_s states with different lifetimes decaying into the same CP eigenstate [10].

For further details, see the notes on CP violation in the K_L^0, K_S^0, and B^0 Particle Listings of this *Review.*

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