

## 13. QUARK MODEL

Revised April 2000 by C. Amsler (Univ. of Zürich) and C.G. Wohl (LBNL).

### 13.1. Quantum numbers of the quarks

Each quark has spin 1/2 and baryon number 1/3. Table 13.1 gives the additive quantum numbers (other than baryon number) of the three generations of quarks. Our convention is that the *flavor* of a quark ( $l_z$ , S, C, B, or T) has the same sign as its *charge*. With this convention, any flavor carried by a *charged* meson has the same sign as its charge; *e.g.*, the strangeness of the  $K^+$  is +1, the bottomness of the  $B^+$  is +1, and the charm *and* strangeness of the  $D_s^-$  are each -1.

By convention, each quark is assigned positive parity. Then each antiquark has negative parity.

**Table 13.1:** Additive quantum numbers of the quarks.

Property \ Quark	$d$	$u$	$s$	$c$	$b$	$t$
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
$l_z$ – isospin $z$ -component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	+1

### 13.2. Mesons: $q\bar{q}$ states

Nearly all known mesons are bound states of a quark  $q$  and an antiquark  $\bar{q}'$  (the flavors of  $q$  and  $q'$  may be different). If the orbital angular momentum of the  $q\bar{q}'$  state is  $L$ , then the parity  $P$  is  $(-1)^{L+1}$ . A state  $q\bar{q}'$  of a quark and its own antiquark is also an eigenstate of charge conjugation, with  $C = (-1)^{L+S}$ , where the spin  $S$  is 0 or 1. The  $L = 0$  states are the pseudoscalars,  $J^P = 0^-$ , and the vectors,  $J^P = 1^-$ . Assignments for many of the known mesons are given in Table 13.2. States in the “normal” spin-parity series,  $P = (-1)^J$ , must, according to the above, have  $S = 1$  and hence  $CP = +1$ . Thus mesons with normal spin-parity and  $CP = -1$  are forbidden in the  $q\bar{q}'$  model. The  $J^{PC} = 0^{- -}$  state is forbidden as well. Mesons with such  $J^{PC}$  may exist, but would lie outside the  $q\bar{q}'$  model.

The nine possible  $q\bar{q}'$  combinations containing  $u$ ,  $d$ , and  $s$  quarks group themselves into an octet and a singlet:

$$3 \otimes \bar{3} = 8 \oplus 1 \quad (13.1)$$

States with the same  $IJ^P$  and additive quantum numbers can mix. (If they are eigenstates of charge conjugation, they must also have the same value of  $C$ .) Thus the  $I = 0$  member of the ground-state pseudoscalar octet mixes with the corresponding pseudoscalar singlet to produce the  $\eta$  and  $\eta'$ . These appear as members of a nonet, which is shown as the middle plane in Fig. 13.1(a). Similarly, the ground-state vector nonet appears as the middle plane in Fig. 13.1(b).

A fourth quark such as charm can be included in this scheme by extending the symmetry to SU(4), as shown in Fig. 13.1. Bottom extends the symmetry to SU(5); to draw the multiplets would require four dimensions.

For the pseudoscalar mesons, the Gell-Mann-Okubo formula is

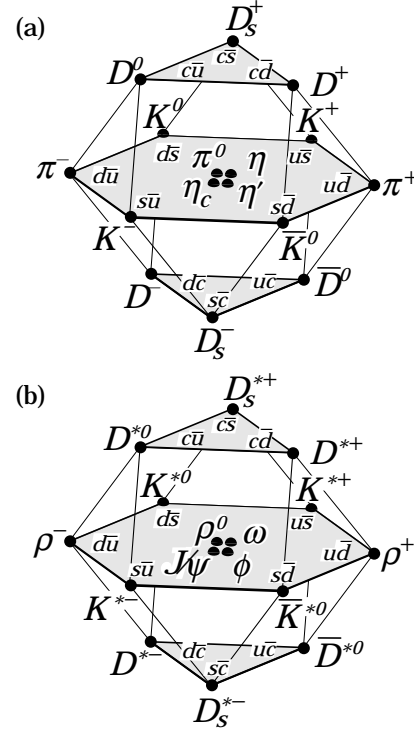
$$m_\eta^2 = \frac{1}{3}(4m_K^2 - m_\pi^2), \quad (13.2)$$

assuming no octet-singlet mixing. However, the octet  $\eta_8$  and singlet  $\eta_1$  mix because of SU(3) breaking. In general, the mixing angle is

mass dependent and becomes complex for resonances of finite width. Neglecting this, the physical states  $\eta$  and  $\eta'$  are given in terms of a mixing angle  $\theta_P$  by

$$\eta = \eta_8 \cos \theta_P - \eta_1 \sin \theta_P \quad (13.3a)$$

$$\eta' = \eta_8 \sin \theta_P + \eta_1 \cos \theta_P. \quad (13.3b)$$



**Figure 13.1:** SU(4) 16-plets for the (a) pseudoscalar and (b) vector mesons made of  $u$ ,  $d$ ,  $s$ , and  $c$  quarks. The nonets of light mesons occupy the central planes, to which the  $c\bar{c}$  states have been added. The neutral mesons at the centers of these planes are mixtures of  $u\bar{u}$ ,  $d\bar{d}$ ,  $s\bar{s}$ , and  $c\bar{c}$  states.

These combinations diagonalize the mass-squared matrix

$$M^2 = \begin{pmatrix} M_{11}^2 & M_{18}^2 \\ M_{18}^2 & M_{88}^2 \end{pmatrix}, \quad (13.4)$$

where  $M_{88}^2 = \frac{1}{3}(4m_K^2 - m_\pi^2)$ . It follows that

$$\tan^2 \theta_P = \frac{M_{88}^2 - m_\eta^2}{m_{\eta'}^2 - M_{88}^2}. \quad (13.5)$$

The sign of  $\theta_P$  is meaningful in the quark model. If

$$\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \quad (13.6a)$$

$$\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}, \quad (13.6b)$$

then the matrix element  $M_{18}^2$ , which is due mostly to the strange quark mass, is negative. From the relation

$$\tan \theta_P = \frac{M_{88}^2 - m_\eta^2}{M_{18}^2}, \quad (13.7)$$

we find that  $\theta_P < 0$ . However, caution is suggested in the use of the  $\eta$ - $\eta'$  mixing-angle formulas, as they are extremely sensitive to SU(3)

**Table 13.2:** Suggested  $q\bar{q}$  quark-model assignments for most of the known mesons. Some assignments, especially for the  $0^{++}$  multiplet and for some of the higher multiplets, are controversial. Mesons in bold face are included in the Meson Summary Table. Of the light mesons in the Summary Table, the  $f_0(1500)$ ,  $f_1(1510)$ ,  $f_2(1950)$ ,  $f_2(2300)$ ,  $f_2(2340)$ , and one of the two peaks in the  $\eta(1440)$  entry are not in this table. Within the  $q\bar{q}$  model, it is especially hard to find a place for the first two of these  $f$  mesons and for one of the  $\eta(1440)$  peaks. See the “Note on Non- $q\bar{q}$  Mesons” at the end of the Meson Listings.

$N\ 2S+1L_J$	$J^{PC}$	$u\bar{d}, u\bar{u}, d\bar{d}$ $I = 1$	$u\bar{u}, d\bar{d}, s\bar{s}$ $I = 0$	$c\bar{c}$ $I = 0$	$b\bar{b}$ $I = 0$	$\bar{s}u, \bar{s}d$ $I = 1/2$	$c\bar{u}, c\bar{d}$ $I = 1/2$	$c\bar{s}$ $I = 0$	$\bar{b}u, \bar{b}d$ $I = 1/2$	$\bar{b}s$ $I = 0$	$\bar{b}c$ $I = 0$
$1^1S_0$	$0^{-+}$	$\pi$	$\eta, \eta'$	$\eta_c$		$K$	$D$	$D_s$	$B$	$B_s$	$B_c$
$1^3S_1$	$1^{--}$	$\rho$	$\omega, \phi$	$J/\psi(1S)$	$\Upsilon(1S)$	$K^*(892)$	$D^*(2010)$	$D_s^*$	$B^*$	$B_s^*$	
$1^1P_1$	$1^{+-}$	$b_1(1235)$	$h_1(1170), h_1(1380)$	$h_c(1P)$		$K_{1B}^\dagger$	$D_1(2420)$	$D_{s1}(2536)$			
$1^3P_0$	$0^{++}$	$a_0(1450)^*$	$f_0(1370)^*, f_0(1710)^*$	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$K_0^*(1430)$					
$1^3P_1$	$1^{++}$	$a_1(1260)$	$f_1(1285), f_1(1420)$	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$	$K_{1A}^\dagger$					
$1^3P_2$	$2^{++}$	$a_2(1320)$	$f_2(1270), f_2'(1525)$	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$K_2^*(1430)$	$D_2^*(2460)$				
$1^1D_2$	$2^{-+}$	$\pi_2(1670)$	$\eta_2(1645), \eta_2(1870)$			$K_2(1770)$					
$1^3D_1$	$1^{--}$	$\rho(1700)$	$\omega(1650)$	$\psi(3770)$		$K^*(1680)^\ddagger$					
$1^3D_2$	$2^{--}$					$K_2(1820)$					
$1^3D_3$	$3^{--}$	$\rho_3(1690)$	$\omega_3(1670), \phi_3(1850)$			$K_3^*(1780)$					
$1^3F_4$	$4^{++}$	$a_4(2040)$	$f_4(2050), f_4(2220)$			$K_4^*(2045)$					
$2^1S_0$	$0^{-+}$	$\pi(1300)$	$\eta(1295), \eta(1440)$	$\eta_c(2S)$		$K(1460)$					
$2^3S_1$	$1^{--}$	$\rho(1450)$	$\omega(1420), \phi(1680)$	$\psi(2S)$	$\Upsilon(2S)$	$K^*(1410)^\ddagger$					
$2^3P_2$	$2^{++}$		$f_2(1810), f_2(2010)$		$\chi_{b2}(2P)$	$K_2^*(1980)$					
$3^1S_0$	$0^{-+}$	$\pi(1800)$	$\eta(1760)$			$K(1830)$					

\* See our scalar minireview in the Particle Listings. The candidates for the  $I = 1$  states are  $a_0(980)$  and  $a_0(1450)$ , while for  $I = 0$  they are:  $f_0(400-1200)$ ,  $f_0(980)$ ,  $f_0(1370)$ , and  $f_0(1710)$ . The light scalars are problematic, since there may be two poles for one  $q\bar{q}$  state and  $a_0(980)$ ,  $f_0(980)$  may be  $K\bar{K}$  bound states.

† The  $K_{1A}$  and  $K_{1B}$  are nearly equal ( $45^\circ$ ) mixes of the  $K_1(1270)$  and  $K_1(1400)$ .

‡The  $K^*(1410)$  could be replaced by the  $K^*(1680)$  as the  $2^3S_1$  state.

If we allow  $M_{88}^2 = \frac{1}{3}(4m_K^2 - m_\pi^2)(1 + \Delta)$ , the mixing angle is determined by

$$\tan^2 \theta_P = 0.0319(1 + 17\Delta) \quad (13.8)$$

$$\theta_P = -10.1^\circ(1 + 8.5\Delta) \quad (13.9)$$

to first order in  $\Delta$ . A small breaking of the Gell-Mann-Okubo relation can produce a major modification of  $\theta_P$ .

For the vector mesons,  $\pi \rightarrow \rho$ ,  $K \rightarrow K^*$ ,  $\eta \rightarrow \phi$ , and  $\eta' \rightarrow \omega$ , so that

$$\phi = \omega_8 \cos \theta_V - \omega_1 \sin \theta_V \quad (13.10)$$

$$\omega = \omega_8 \sin \theta_V + \omega_1 \cos \theta_V. \quad (13.11)$$

For “ideal” mixing,  $\phi = s\bar{s}$ , so  $\tan \theta_V = 1/\sqrt{2}$  and  $\theta_V = 35.3^\circ$ . Experimentally,  $\theta_V$  is near  $35^\circ$ , the sign being determined by a formula like that for  $\tan \theta_P$ . Following this procedure we find the mixing angles given in Table 13.3.

**Table 13.3:** Singlet-octet mixing angles for several nonets, neglecting possible mass dependence and imaginary parts. The sign conventions are given in the text. The values of  $\theta_{\text{quad}}$  are obtained from the equations in the text, while those for  $\theta_{\text{lin}}$  are obtained by replacing  $m^2$  by  $m$  throughout. Of the two isosinglets in a nonet, the mostly octet one is listed first.

$J^{PC}$	Nonet members	$\theta_{\text{quad}}$	$\theta_{\text{lin}}$
$0^{-+}$	$\pi, K, \eta, \eta'$	$-10^\circ$	$-23^\circ$
$1^{--}$	$\rho, K^*(892), \phi, \omega$	$39^\circ$	$36^\circ$
$2^{++}$	$a_2(1320), K_2^*(1430), f_2'(1525), f_2(1270)$	$28^\circ$	$26^\circ$
$3^{--}$	$\rho_3(1690), K_3^*(1780), \phi_3(1850), \omega_3(1670)$	$29^\circ$	$28^\circ$



which contains the nucleon and  $\Delta(1232)$ , consists only of the  $(56, 0_0^+)$  supermultiplet. The  $N = 1$  band consists only of the  $(70, 1_1^-)$  multiplet and contains the negative-parity baryons with masses below about 1.9 GeV. The  $N = 2$  band contains five supermultiplets:  $(56, 0_2^+)$ ,  $(70, 0_2^+)$ ,  $(56, 2_2^+)$ ,  $(70, 2_2^+)$ , and  $(20, 1_2^+)$ . Baryons belonging to the  $(20, 1_2^+)$  supermultiplet are not ever likely to be observed, since a coupling from the ground-state baryons requires a two-quark excitation. Selection rules are similarly responsible for the fact that many other baryon resonances have not been observed [4].

In Table 13.4, quark-model assignments are given for many of the established baryons whose  $SU(6) \otimes O(3)$  compositions are relatively unmixed. We note that the unestablished resonances  $\Sigma(1480)$ ,  $\Sigma(1560)$ ,  $\Sigma(1580)$ ,  $\Sigma(1770)$ , and  $\Xi(1620)$  in our Baryon Particle Listings are too low in mass to be accommodated in most quark models [4,5].

**Table 13.4:** Quark-model assignments for many of the known baryons in terms of a flavor-spin  $SU(6)$  basis. Only the dominant representation is listed. Assignments for some states, especially for the  $\Lambda(1810)$ ,  $\Lambda(2350)$ ,  $\Xi(1820)$ , and  $\Xi(2030)$ , are merely educated guesses. For assignments of the charmed baryons, see the “Note on Charmed Baryons” in the Particle Listings.

$J^P$	$(D, L_N^P)$	$S$	Octet members			Singlets
$1/2^+$	$(56, 0_0^+)$	$1/2$	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$
$1/2^+$	$(56, 0_2^+)$	$1/2$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(?)$
$1/2^-$	$(70, 1_1^-)$	$1/2$	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(?)$ $\Lambda(1405)$
$3/2^-$	$(70, 1_1^-)$	$1/2$	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$ $\Lambda(1520)$
$1/2^-$	$(70, 1_1^-)$	$3/2$	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(?)$
$3/2^-$	$(70, 1_1^-)$	$3/2$	$N(1700)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$
$5/2^-$	$(70, 1_1^-)$	$3/2$	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(?)$
$1/2^+$	$(70, 0_2^+)$	$1/2$	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$ $\Lambda(?)$
$3/2^+$	$(56, 2_2^+)$	$1/2$	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$
$5/2^+$	$(56, 2_2^+)$	$1/2$	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$
$7/2^-$	$(70, 3_3^-)$	$1/2$	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$ $\Lambda(2100)$
$9/2^-$	$(70, 3_3^-)$	$3/2$	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$
$9/2^+$	$(56, 4_4^+)$	$1/2$	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$
Decuplet members						
$3/2^+$	$(56, 0_0^+)$	$3/2$	$\Delta(1232)$	$\Sigma(1385)$	$\Xi(1530)$	$\Omega(1672)$
$1/2^-$	$(70, 1_1^-)$	$1/2$	$\Delta(1620)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$
$3/2^-$	$(70, 1_1^-)$	$1/2$	$\Delta(1700)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$
$5/2^+$	$(56, 2_2^+)$	$3/2$	$\Delta(1905)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$
$7/2^+$	$(56, 2_2^+)$	$3/2$	$\Delta(1950)$	$\Sigma(2030)$	$\Xi(?)$	$\Omega(?)$
$11/2^+$	$(56, 4_4^+)$	$3/2$	$\Delta(2420)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$

The quark model for baryons is extensively reviewed in Ref. 6 and 7.

### 13.4. Dynamics

Many specific quark models exist, but most contain the same basic set of dynamical ingredients. These include:

- i) A confining interaction, which is generally spin-independent.
- ii) A spin-dependent interaction, modeled after the effects of gluon exchange in QCD. For example, in the  $S$ -wave states, there is a spin-spin hyperfine interaction of the form

$$H_{HF} = -\alpha_S M \sum_{i>j} (\vec{\sigma} \lambda_a)_i (\vec{\sigma} \lambda_a)_j, \quad (13.19)$$

where  $M$  is a constant with units of energy,  $\lambda_a$  ( $a = 1, \dots, 8$ ) is the set of  $SU(3)$  unitary spin matrices, defined in Sec. 32, on “ $SU(3)$  Isoscalar Factors and Representation Matrices,” and the sum runs over constituent quarks or antiquarks. Spin-orbit interactions, although allowed, seem to be small.

- iii) A strange quark mass somewhat larger than the up and down quark masses, in order to split the  $SU(3)$  multiplets.
- iv) In the case of isoscalar mesons, an interaction for mixing  $q\bar{q}$  configurations of different flavors (*e.g.*,  $u\bar{u} \leftrightarrow d\bar{d} \leftrightarrow s\bar{s}$ ), in a manner which is generally chosen to be flavor independent.

These four ingredients provide the basic mechanisms that determine the hadron spectrum.

#### References:

1. F.E. Close, in *Quarks and Nuclear Forces* (Springer-Verlag, 1982), p. 56.
2. Particle Data Group, Phys. Lett. **111B** (1982).
3. R.H. Dalitz and L.J. Reinders, in *Hadron Structure as Known from Electromagnetic and Strong Interactions, Proceedings of the Hadron '77 Conference* (Veda, 1979), p. 11.
4. N. Isgur and G. Karl, Phys. Rev. **D18**, 4187 (1978); *ibid.* **D19**, 2653 (1979); *ibid.* **D20**, 1191 (1979); K.-T. Chao, N. Isgur, and G. Karl, Phys. Rev. **D23**, 155 (1981).
5. C.P. Forsyth and R.E. Cutkosky, Z. Phys. **C18**, 219 (1983).
6. A.J.G. Hey and R.L. Kelly, Phys. Reports **96**, 71 (1983). Also see S. Gasiorowicz and J.L. Rosner, Am. J. Phys. **49**, 954 (1981).
7. N. Isgur, Int. J. Mod. Phys. **E1**, 465 (1992); G. Karl, Int. J. Mod. Phys. **E1**, 491 (1992).