30. Probability

Table 30.1. Some common probability density functions, with corresponding characteristic functions and means and variances. In the Table, $\Gamma(k)$ is the gamma function, equal to $(k-1)!$ when k is an integer.

Distribution	Probability density function f (variable; parameters)	Characteristic function $\phi(u)$	Mean	Variance σ^2
Uniform	$f(x;a,b) = \begin{cases} 1/(b-a) & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$	$\dfrac{e^{ibu}-e^{iau}}{(b-a)iu}$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$
Binomial	$f(r;N,p) = \dfrac{N!}{r!(N-r)!}\; p^r q^{N-r}$ $r = 0,1,2,\ldots,N\;;\quad 0 \le p \le 1\;;\quad q = 1-p$	$(q+pe^{iu})^N$	Np	Npq
Poisson	$f(n;\nu) = \dfrac{\nu^n e^{-\nu}}{n!}\;;\quad n = 0,1,2,\ldots\;;\quad \nu > 0$	$\exp[\nu(e^{iu}-1)]$	ν	ν
Normal (Gaussian)	$f(x;\mu,\sigma^2) = \dfrac{1}{\sigma\sqrt{2\pi}}\; \exp(-(x-\mu)^2/2\sigma^2)$ $-\infty < x < \infty\;;\quad -\infty < \mu < \infty\;;\quad \sigma > 0$	$\exp(i\mu u - \frac{1}{2}\sigma^2 u^2)$	μ	σ^2
Multivariate Gaussian	$f(\boldsymbol{x};\boldsymbol{\mu},V) = \dfrac{1}{(2\pi)^{n/2}\sqrt{\lvert V\rvert}}$ $\times \exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T V^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right]$ $-\infty < x_j < \infty;\quad -\infty < \mu_j < \infty;\quad \det V > 0$	$\exp\left[i\boldsymbol{\mu}\cdot\boldsymbol{u} - \frac{1}{2}\boldsymbol{u}^T V \boldsymbol{u}\right]$	$\boldsymbol{\mu}$	V_{jk}
χ^2	$f(z;n) = \dfrac{z^{n/2-1}e^{-z/2}}{2^{n/2}\Gamma(n/2)}\;;\quad z \ge 0$	$(1-2iu)^{-n/2}$	n	$2n$
Student's t	$f(t;n) = \dfrac{1}{\sqrt{n\pi}}\,\dfrac{\Gamma[(n+1)/2]}{\Gamma(n/2)}\left(1+\dfrac{t^2}{n}\right)^{-(n+1)/2}$ $-\infty < t < \infty\;;\quad n$ not required to be integer	—	0 for $n \ge 2$	$n/(n-2)$ for $n \ge 3$
Gamma	$f(x;\lambda,k) = \dfrac{x^{k-1}\lambda^k e^{-\lambda x}}{\Gamma(k)}\;;\quad 0 < x < \infty\;;$ k not required to be integer	$(1-iu/\lambda)^{-k}$	k/λ	k/λ^2