15. GRAND UNIFIED THEORIES

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15.1. Grand Unification

15.1.1. *Standard Model: An Introduction*:

In spite of all the successes of the Standard Model [SM], it is unlikely to be the final theory. It leaves many unanswered questions. Why the local gauge interactions $SU(3)_C \times SU(2)_L \times U(1)_Y$, and why 3 families of quarks and leptons? Moreover, why does one family consist of the states $[Q, u^c, d^c; L, e^c]$ transforming as $[(3, 2, 1/3)$, $(\bar{3}, 1, -4/3)$, $(\bar{3}, 1, 2/3)$; $(1, 2, -1)$, $(1, 1, 2)]$, where $Q = (u, d)$, and $L = (\nu, e)$ are $SU(2)_L$ doublets, and u^c, d^c, e^c are charge conjugate $SU(2)_L$ singlet fields with the $U(1)_Y$ quantum numbers given? [We use the convention that electric charge $Q_{EM} = T_{3L} + Y/2$ and all fields are left-handed.] Note the SM gauge interactions of quarks and leptons are completely fixed by their gauge charges. Thus, if we understood the origin of this charge quantization, we would also understand why there are no fractionally charged hadrons. Finally, what is the origin of quark and lepton masses, or the apparent hierarchy of family masses and quark mixing angles? Perhaps if we understood this, we would also know the origin of CP violation, the solution to the strong CP problem, the origin of the cosmological matter-antimatter asymmetry, or the nature of dark matter.

The SM has 19 arbitrary parameters; their values are chosen to fit the data. Three arbitrary gauge couplings: g_3, g, g' (where g, g' are the $SU(2)_L$, $U(1)_Y$ couplings, respectively) or equivalently, $\alpha_s = (g_3^2/4\pi)$, $\alpha_{EM} = (e^2/4\pi)$ $(e = g \ \sin\theta_W)$, and $\sin^2\theta_W = (g')^2/(g^2 + (g')^2)$. In addition, there are 13 parameters associated with the 9 charged fermion masses and the four mixing angles in the CKM matrix. The remaining 3 parameters are v, λ [the Higgs VEV (vacuum expectation value) and quartic coupling] (or equivalently, M_Z, m_h^0), and the QCD θ parameter. In addition, data from neutrino oscillation experiments provide convincing evidence for neutrino masses. With 3 light Majorana neutrinos, there are at least 9 additional parameters in the neutrino sector; 3 masses, 3 mixing angles and 3 phases. In summary, the SM has too many arbitrary parameters, and leaves open too many unresolved questions to be considered complete. These are the problems which grand unified theories hope to address.

15.1.2. *Charge Quantization*:

In the Standard Model, quarks and leptons are on an equal footing; both fundamental particles without substructure. It is now clear that they may be two faces of the same coin; unified, for example, by extending QCD (or $SU(3)_C$) to include leptons as the fourth color, $SU(4)_C$ [1]. The complete Pati-Salam gauge group is $SU(4)_C \times SU(2)_L \times SU(2)_R$, with the states of one family $[(Q, L), (Q^c, L^c)]$ transforming as $[(4, 2, 1)$, $(\bar{4}, 1, \bar{2})]$, where $Q^c = (d^c, u^c)$, $L^c = (e^c, \nu^c)$ are doublets under $SU(2)_R$. Electric charge is now given by the relation $Q_{EM} = T_{3L} + T_{3R} + 1/2(B-L)$, and $SU(4)_C$ contains the subgroup $SU(3)_C \times (B-L)$ where B (L) is baryon (lepton) number. Note ν^c has no SM quantum numbers and is thus completely "sterile." It is introduced to complete the $SU(2)_R$ lepton doublet. This additional state is desirable when considering neutrino masses.

Although quarks and leptons are unified with the states of one family forming two irreducible representations of the gauge group, there are still 3 independent gauge couplings (two if one also imposes parity, *i.e.*, $L \leftrightarrow R$ symmetry). As a result, the three low-energy gauge couplings are still independent arbitrary parameters. This difficulty is resolved by embedding the SM gauge group into the simple unified gauge group, Georgi-Glashow SU(5), with one universal gauge coupling α_G defined at the grand unification scale M_G [2]. Quarks and leptons still sit in two irreducible representations, as before, with a $\mathbf{10} = [Q, u^c, e^c]$ and $\bar{\mathbf{5}} = [d^c, L]$. Nevertheless, the three low energy gauge couplings are now determined in terms of two independent parameters : α_G and M_G. Hence, there is one prediction.

In order to break the electroweak symmetry at the weak scale and give mass to quarks and leptons, Higgs doublets are needed which can sit in either a $\mathbf{5_H}$ or $\bar{\mathbf{5}}_\mathbf{H}$. The additional 3 states are color triplet Higgs scalars. The couplings of these color triplets violate baryon and lepton number, and nucleons decay via the exchange of a single color triplet Higgs scalar. Hence, in order not to violently disagree with the non-observation of nucleon decay, their mass must be greater than $\sim 10^{10-11}$ GeV. Moreover, in supersymmetric GUTs, in order to cancel anomalies, as well as give mass to both up and down quarks, both Higgs multiplets $\mathbf{5_H}$, $\bar{\mathbf{5}}_\mathbf{H}$ are required. As we shall discuss later, nucleon decay now constrains the color triplet Higgs states in a SUSY GUT to have mass significantly greater than M_G.

Complete unification is possible with the symmetry group SO(10), with one universal gauge coupling α_G, and one family of quarks and leptons sitting in the 16-dimensional-spinor representation $\mathbf{16} = [\mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}]$ [3]. The SU(5) singlet $\mathbf{1}$ is identified with ν^c. In Table 15.1 we present the states of one family of quarks and leptons, as they appear in the $\mathbf{16}$. It is an amazing and perhaps even profound fact that all the states of a single family of quarks and leptons can be represented digitally as a set of 5 zeros and/or ones or equivalently as the tensor product of 5 "spin" 1/2 states with $\pm = |\pm \frac{1}{2}>$ and with the condition that we have an even number of $|->$ spins. The first three "spins" correspond to $\mathrm{SU}(3)_C$ color quantum numbers, while the last two are $\mathrm{SU}(2)_L$ weak quantum numbers. In fact, an $\mathrm{SU}(3)_C$ rotation just raises one color index and lowers another, thereby changing colors $\{r,\ b,\ y\}$. Similarly an $\mathrm{SU}(2)_L$ rotation raises one weak index and lowers another, thereby flipping the weak isospin from up to down or vice versa. In this representation, weak hypercharge Y is given by the simple relation $Y = 2/3(\sum \text{ color spins}) - (\sum \text{ weak spins})$. SU(5) rotations [not in the Standard Model] then raise (or lower) a color index, while at the same time lowering (or raising) a weak index. It is easy to see that such rotations can mix the states $\{Q,\ u^c,\ e^c\}$ and $\{d^c, L\}$ among themselves, and ν^c is a singlet. The new SO(10) rotations [not in SU(5)] are then given by either raising or lowering any two spins. For example, by lowering the two weak indices, ν^c rotates into e^c, etc.

SO(10) has two inequivalent maximal breaking patterns: $\mathrm{SO}(10) \rightarrow \mathrm{SU}(5) \times \mathrm{U}(1)_X$ and $\mathrm{SO}(10) \rightarrow \mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. In the first case, we obtain Georgi-Glashow SU(5) if Q_{EM} is given in terms of SU(5) generators alone, or so-called flipped SU(5) [4] if Q_{EM} is partly in $\mathrm{U}(1)_X$. In the latter case, we have the Pati-Salam symmetry. If

Table 15.1: The quantum numbers of the **16** dimensional representation of SO(10).

State	Y	Color	Weak
$\boldsymbol{\nu}^c$	0	$+ + +$	$+ +$
$\boldsymbol{e}^c$	2	$+ + +$	$- -$
$\boldsymbol{u}_r$	1/3	$- + +$	$+ -$
$\boldsymbol{d}_r$	1/3	$- + +$	$- +$
$\boldsymbol{u}_b$	1/3	$+ - +$	$+ -$
$\boldsymbol{d}_b$	1/3	$+ - +$	$- +$
$\boldsymbol{u}_y$	1/3	$+ + -$	$+ -$
$\boldsymbol{d}_y$	1/3	$+ + -$	$- +$
$\boldsymbol{u}_r^c$	$-4/3$	$+ - -$	$+ +$
$\boldsymbol{u}_b^c$	$-4/3$	$- + -$	$+ +$
$\boldsymbol{u}_y^c$	$-4/3$	$- - +$	$+ +$
$\boldsymbol{d}_r^c$	2/3	$+ - -$	$- -$
$\boldsymbol{d}_b^c$	2/3	$- + -$	$- -$
$\boldsymbol{d}_y^c$	2/3	$- - +$	$- -$
$\boldsymbol{\nu}$	-1	$- - -$	$+ -$
$\boldsymbol{e}$	-1	$- - -$	$- +$

SO(10) breaks directly to the SM at M_G, then we retain the prediction for gauge coupling unification. However, more possibilities for breaking (hence more breaking scales and more parameters) are available in SO(10). Nevertheless with one breaking pattern SO(10) $\rightarrow$ SU(5) $\rightarrow$ SM, where the last breaking scale is M_G, the predictions from gauge coupling unification are preserved. The Higgs multiplets in minimal SO(10) are contained in the fundamental $\mathbf{10_H} = [\mathbf{5_H},\ \bar{\mathbf{5}}_\mathbf{H}]$ representation. Note, only in SO(10) does the gauge symmetry distinguish quark and lepton multiplets from Higgs multiplets.

Finally, larger symmetry groups have been considered. For example, $E(6)$ has a fundamental representation **27**, which under SO(10) transforms as a $[\mathbf{16} + \mathbf{10} + \mathbf{1}]$. The breaking pattern $E(6) \rightarrow \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ is also possible. With the additional permutation symmetry $Z(3)$ interchanging the three SU(3)s, we obtain so-called "trinification [5]," with a universal gauge coupling. The latter breaking pattern has been used in phenomenological analyses of the heterotic string [6]. However, in larger symmetry groups, such as $E(6)$, SU(6), *etc.*, there are now many more states which have not been observed and must be removed from the effective low-energy theory. In particular, three families of **27**s in $E(6)$ contain three Higgs type multiplets transforming as **10**s of SO(10). This makes these larger symmetry groups unattractive starting points for model building.

15.1.3. *String Theory and Orbifold GUTs*:

Orbifold compactification of the heterotic string [7–9], and recent field theoretic constructions known as orbifold GUTs [10], contain grand unified symmetries realized in 5 and 6 dimensions. However, upon compactifying all but four of these extra dimensions, only the MSSM is recovered as a symmetry of the effective four dimensional field theory.[1] These theories can retain many of the nice features of four dimensional SUSY GUTs, such as charge quantization, gauge coupling unification and sometimes even Yukawa unification; while at the same time resolving some of the difficulties of 4d GUTs, in particular problems with unwieldy Higgs sectors necessary for spontaneously breaking the GUT symmetry, problems with doublet-triplet Higgs splitting or rapid proton decay. We will comment further on the corrections to the four dimensional GUT picture due to orbifold GUTs in the following sections.

15.1.4. *Gauge coupling unification*:

The biggest paradox of grand unification is to understand how it is possible to have a universal gauge coupling g_G in a grand unified theory [GUT], and yet have three unequal gauge couplings at the weak scale with $g_3 > g > g'$. The solution is given in terms of the concept of an effective field theory [EFT] [16]. The GUT symmetry is spontaneously broken at the scale M_G, and all particles not in the SM obtain mass of order M_G. When calculating Green's functions with external energies $E \gg M_G$, we can neglect the mass of all particles in the loop and hence all particles contribute to the renormalization group running of the universal gauge coupling. However, for $E \ll M_G$, one can consider an effective field theory including only the states with mass $< E \ll M_G$. The gauge symmetry of the EFT is $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, and the three gauge couplings renormalize independently. The states of the EFT include only those of the SM; 12 gauge bosons, 3 families of quarks and leptons, and one or more Higgs doublets. At M_G, the two effective theories [the GUT itself is most likely

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[1] Also, in recent years there has been a great deal of progress in constructing three and four family models in Type IIA string theory with intersecting D6 branes [11]. Although these models can incorporate SU(5) or a Pati-Salam symmetry group in four dimensions, they typically have problems with gauge coupling unification. In the former case this is due to charged exotics which affect the RG running, while in the latter case the $\mathrm{SU}(4)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ symmetry never unifies. Note, heterotic string theory models also exist whose low energy effective 4d field theory is a SUSY GUT [12]. These models have all the virtues and problems of 4d GUTs. Finally, many heterotic string models have been constructed with the standard model gauge symmetry in 4d and no intermediate GUT symmetry in less than 10d. Recently some minimal 3 family supersymmetric models have been constructed [13,14]. These theories may retain some of the symmetry relations of GUTs, however the unification scale would typically be the string scale, of order 5×10^{17} GeV, which is inconsistent with low energy data. A way out of this problem was discovered in the context of the strongly coupled heterotic string, defined in an effective 11 dimensions [15]. In this case the 4d Planck scale (which controls the value of the string scale) now unifies with the GUT scale.

the EFT of a more fundamental theory defined at a higher scale] must give identical results; hence we have the boundary conditions $g_3 = g_2 = g_1 \equiv g_G$, where at any scale $\mu < M_G$, we have $g_2 \equiv g$ and $g_1 = \sqrt{5/3}\, g'$. Then using two low-energy couplings, such as $\alpha_s(M_Z)$, $\alpha_{EM}(M_Z)$, the two independent parameters α_G, M_G can be fixed. The third gauge coupling, $\sin^2\theta_W$ in this case, is then predicted. This was the procedure up until about 1991 [17,18]. Subsequently, the uncertainties in $\sin^2\theta_W$ were reduced tenfold. Since then, $\alpha_{EM}(M_Z)$, $\sin^2\theta_W$ have been used as input to predict α_G, M_G, and $\alpha_s(M_Z)$ [19].

We emphasize that the above boundary condition is only valid when using one-loop-renormalization group [RG] running. With precision electroweak data, however, it is necessary to use two-loop-RG running. Hence, one must include one-loop-threshold corrections to gauge coupling boundary conditions at both the weak and GUT scales. In this case, it is always possible to define the GUT scale as the point where $\alpha_1(M_G) = \alpha_2(M_G) \equiv \tilde{\alpha}_G$ and $\alpha_3(M_G) = \tilde{\alpha}_G\,(1+\epsilon_3)$. The threshold correction ϵ_3 is a logarithmic function of all states with mass of order M_G and $\tilde{\alpha}_G = \alpha_G + \Delta$, where α_G is the GUT coupling constant above M_G, and Δ is a one-loop-threshold correction. To the extent that gauge coupling unification is perturbative, the GUT threshold corrections are small and calculable. This presumes that the GUT scale is sufficiently below the Planck scale or any other strong coupling extension of the GUT, such as a strongly coupled string theory.

Supersymmetric grand unified theories [SUSY GUTs] are an extension of non-SUSY GUTs [20]. The key difference between SUSY GUTs and non-SUSY GUTs is the low-energy effective theory. The low-energy effective field theory in a SUSY GUT is assumed to satisfy $N = 1$ supersymmetry down to scales of order the weak scale, in addition to the SM gauge symmetry. Hence, the spectrum includes all the SM states, plus their supersymmetric partners. It also includes one pair (or more) of Higgs doublets; one to give mass to up-type quarks, and the other to down-type quarks and charged leptons. Two doublets with opposite hypercharge Y are also needed to cancel fermionic triangle anomalies. Finally, it is important to recognize that a low-energy SUSY-breaking scale (the scale at which the SUSY partners of SM particles obtain mass) is necessary to solve the gauge hierarchy problem.

Simple non-SUSY SU(5) is ruled out, initially by the increased accuracy in the measurement of $\sin^2\theta_W$, and by early bounds on the proton lifetime (see below) [18]. However, by now LEP data [19] has conclusively shown that SUSY GUTs is the new Standard Model; by which we mean the theory used to guide the search for new physics beyond the present SM (see Fig. 15.1). SUSY extensions of the SM have the property that their effects decouple as the effective SUSY-breaking scale is increased. Any theory beyond the SM must have this property simply because the SM works so well. However, the SUSY-breaking scale cannot be increased with impunity, since this would reintroduce a gauge hierarchy problem. Unfortunately there is no clear-cut answer to the question, "When is the SUSY-breaking scale too high?" A conservative bound would suggest that the third generation squarks and sleptons must be lighter than about 1 TeV, in order that the one-loop corrections to the Higgs mass from Yukawa interactions remain of order the Higgs mass bound itself.

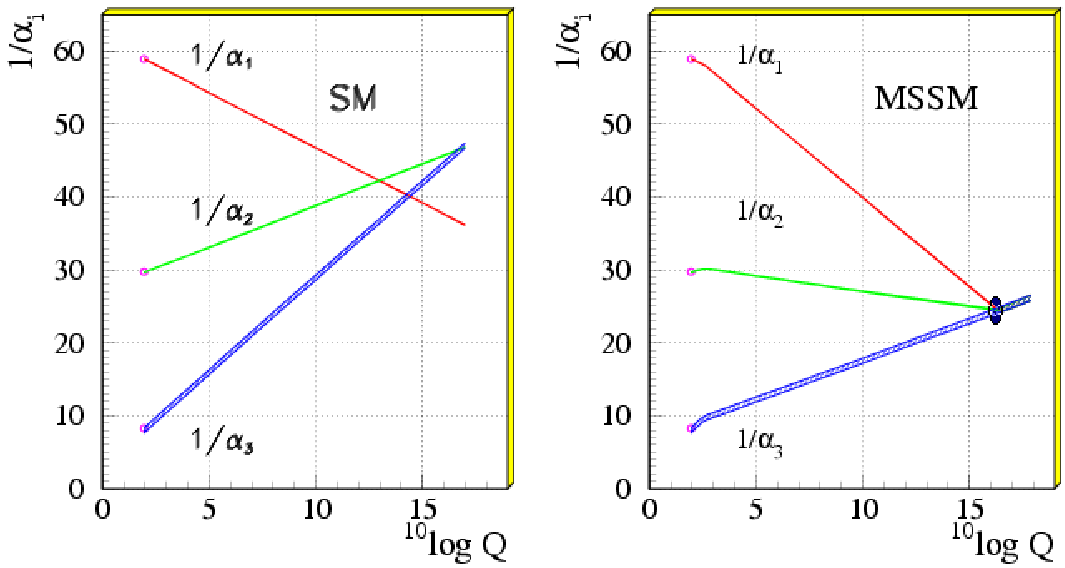

Figure 15.1: Gauge coupling unification in non-SUSY GUTs on the left vs. SUSY GUTs on the right using the LEP data as of 1991. Note, the difference in the running for SUSY is the inclusion of supersymmetric partners of standard model particles at scales of order a TeV (Fig. taken from Ref. 21). Given the present accurate measurements of the three low energy couplings, in particular $\alpha_s(M_Z)$, GUT scale threshold corrections are now needed to precisely fit the low energy data. The dark blob in the plot on the right represents these model dependent corrections.

At present, gauge coupling unification within SUSY GUTs works extremely well. Exact unification at M_G, with two-loop-RG running from M_G to M_Z, and one-loop-threshold corrections at the weak scale, fits to within 3 σ of the present precise low-energy data. A small threshold correction at M_G ($\epsilon_3 \sim -3$ to -4%) is sufficient to fit the low-energy data precisely [22–24].[2] This may be compared to non-SUSY GUTs, where the fit misses by $\sim 12\ \sigma$, and a precise fit requires new weak-scale states in incomplete GUT multiplets, or multiple GUT-breaking scales.[3]

[2] This result implicitly assumes universal GUT boundary conditions for soft SUSY-breaking parameters at M_G. In the simplest case, we have a universal gaugino mass $M_{1/2}$, a universal mass for squarks and sleptons m_{16}, and a universal Higgs mass m_{10}, as motivated by SO(10). In some cases, threshold corrections to gauge coupling unification can be exchanged for threshold corrections to soft SUSY parameters. See for example, Ref. 25 and references therein.

[3] Non-SUSY GUTs with a more complicated breaking pattern can still fit the data. For example, non-SUSY SO(10) $\rightarrow$ SU(4)$_C \times$ SU(2)$_L \times$ SU(2)$_R$ $\rightarrow$SM, with the second

Following the analysis of Ref. 24 let us try to understand the need for the GUT threshold correction and its order of magnitude. The renormalization group equations relate the low energy gauge coupling constants $\alpha_i(M_Z),\ i = 1, 2, 3$ to the value of the unification scale Λ_U and the GUT coupling α_U by the expression

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_U} + \frac{b_i}{2\pi} \log\left(\frac{\Lambda_U}{M_Z}\right) + \delta_i \tag{15.1}$$

where Λ_U is the GUT scale evaluated at one loop and the threshold corrections, δ_i, are given by $\delta_i = \delta_i^{(2)} + \delta_i^{(l)} + \delta_i^{(g)}$ with $\delta_i^{(2)}$ representing two loop running effects, $\delta_i^{(l)}$ the light threshold corrections at the SUSY breaking scale and $\delta_i^{(g)} = \delta_i^{(h)} + \delta_i^{(b)}$ representing GUT scale threshold corrections. Note, in this analysis, the two loop RG running is treated on the same footing as weak and GUT scale threshold corrections. One then obtains the prediction

$$(\alpha_3(M_Z) - \alpha_3^{LO}(M_Z))/\alpha_3^{LO}(M_Z) = -\alpha_3^{LO}(M_Z)\ \delta_s \tag{15.2}$$

where $\alpha_3^{LO}(M_Z)$ is the leading order one loop RG result and $\delta_s = \frac{1}{7}(5\delta_1 - 12\delta_2 + 7\delta_3)$ is the net threshold correction. [A similar formula applies at the GUT scale with the GUT threshold correction, ϵ_3, given by $\epsilon_3 = -\tilde{\alpha}_G\ \delta_s^{(g)}$.] Given the experimental inputs [28]:

$$\begin{aligned} \alpha_{em}^{-1}(M_Z) &= 127.906 \pm 0.019 \\ sin^2\theta_W(M_Z) &= 0.2312 \pm 0.0002 \\ \alpha_3(M_Z) &= 0.1187 \pm 0.0020 \end{aligned} \tag{15.3}$$

and taking into account the light threshold corrections, assuming an ensemble of 10 SUSY spectra [24](corresponding to the Snowmass benchmark points), we have

$$\alpha_3^{LO}(M_Z) \approx 0.118 \tag{15.4}$$

and

$$\begin{aligned} \delta_s^{(2)} &\approx -0.82 \\ \delta_s^{(l)} &\approx -0.50 + \frac{19}{28\pi} \log\frac{M_{SUSY}}{M_Z}. \end{aligned}$$

For $M_{SUSY} = 1$ TeV, we have $\delta_s^{(2)} + \delta_s^{(l)} \approx -0.80$. Since the one loop result $\alpha_3^{LO}(M_Z)$ is very close to the experimental value, we need $\delta_s \approx 0$ or equivalently, $\delta_s^{(g)} \approx 0.80$. This corresponds, at the GUT scale, to $\epsilon_3 \approx -3\%$.[4]

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breaking scale of order an intermediate scale, determined by light neutrino masses using the see-saw mechanism, can fit the low-energy data for gauge couplings [26], and at the same time survive nucleon decay bounds [27], discussed in the following section.

[4] In order to fit the low energy data for gauge coupling constants we require a relative shift in $\alpha_3(M_G)$ of order 3% due to GUT scale threshold corrections. If these GUT scale

In four dimensional SUSY GUTs, the threshold correction ϵ_3 receives a positive contribution from Higgs doublets and triplets.[5] Thus a larger, negative contribution must come from the GUT breaking sector of the theory. This is certainly possible in specific SO(10) [29] or SU(5) [30] models, but it is clearly a significant constraint on the 4d GUT sector of the theory. In five or six dimensional orbifold GUTs, on the other hand, the "GUT scale" threshold correction comes from the Kaluza-Klein modes between the compactification scale, M_c, and the effective cutoff scale M_*.[6] Thus, in orbifold GUTs, gauge coupling unification at two loops is only consistent with the low energy data with a fixed value for M_c and M_*.[7] Typically, one finds $M_c < M_G = 3 \times 10^{16}$ GeV, where M_G is the 4d GUT scale. Since the grand unified gauge bosons, responsible for nucleon decay, get mass at the compactification scale, the result $M_c < M_G$ for orbifold GUTs has significant consequences for nucleon decay.

A few final comments are in order. We do not consider the scenario of split supersymmetry [33] in this review. In this scenario squarks and sleptons have mass at a scale $\tilde{m} \gg M_Z$, while gauginos and Higgsinos have mass of order the weak scale. Gauge coupling unification occurs at a scale of order 10^{16} GeV, *provided that the scale* $\tilde{m}$ *lies in the range* $10^3 - 10^{11}$ *GeV* [34]. A serious complaint concerning the split SUSY scenario is that it does not provide a solution to the gauge hierarchy problem. Moreover, it is only consistent with grand unification if it also postulates an "intermediate" scale, $\tilde{m}$, for scalar masses. In addition, it is in conflict with $b - \tau$ Yukawa unification, unless $\tan\beta$ is fine-tuned to be close to 1 [34].[8]

We have also neglected to discuss non-supersymmetric GUTs in four dimensions which still survive once one allows for several scales of GUT symmetry breaking [26]. Finally,

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corrections were not present, however, weak scale threshold corrections of order 9% (due to the larger value of α_3 at M_Z) would be needed to resolve the discrepancy with the data for exact gauge coupling unification at M_G. Leaving out the fact that any consistent GUT necessarily contributes threshold corrections at the GUT scale, it is much more difficult to find the necessary larger corrections at the weak scale. For example, we need $M_{SUSY} \approx 40$ TeV for the necessary GUT scale threshold correction to vanish.

5 Note, the Higgs contribution is given by $\epsilon_3 = \frac{3\tilde{\alpha}_G}{5\pi} \log|\frac{\tilde{M}_t \, \gamma}{M_G}|$ where $\tilde{M}_t$ is the effective color triplet Higgs mass (setting the scale for dimension 5 baryon and lepton number violating operators) and $\gamma = \lambda_b/\lambda_t$ at M_G. Since $\tilde{M}_t$ is necessarily greater than M_G, the Higgs contribution to ϵ_3 is positive.

6 In string theory, the cutoff scale is the string scale.

7 It is interesting to note that a ratio $M_*/M_c \sim 100$, needed for gauge coupling unification to work in orbifold GUTs is typically the maximum value for this ratio consistent with perturbativity [31]. In addition, in orbifold GUTs brane-localized gauge kinetic terms may destroy the successes of gauge coupling unification. However, for values of $M_*/M_c = M_* \pi R \gg 1$ the unified bulk gauge kinetic terms can dominate over the brane-localized terms [32].

8 $b - \tau$ Yukawa unification only works for $\tilde{m} < 10^4$ for $\tan\beta \geq 1.5$. This is because the effective theory between the gaugino mass scale and $\tilde{m}$ includes only one Higgs doublet, as in the standard model. In this case, the large top quark Yukawa coupling tends to increase

it has been shown that non-supersymmetric GUTs in warped 5 dimensional orbifolds can be consistent with gauge coupling unification, assuming that the right-handed top quark and the Higgs doublets are composite-like objects with a compositeness scale of order a TeV [36].

15.1.5. *Nucleon Decay*:

Baryon number is necessarily violated in any GUT [37]. In SU(5), nucleons decay via the exchange of gauge bosons with GUT scale masses, resulting in dimension-6 baryon-number-violating operators suppressed by $(1/M_G^2)$. The nucleon lifetime is calculable and given by $\tau_N \propto M_G^4/(\alpha_G^2\ m_p^5)$. The dominant decay mode of the proton (and the baryon-violating decay mode of the neutron), via gauge exchange, is $p \to e^+\ \pi^0$ $(n \to e^+\ \pi^-)$. In any simple gauge symmetry, with one universal GUT coupling and scale $(\alpha_G,\ M_G)$, the nucleon lifetime from gauge exchange is calculable. Hence, the GUT scale may be directly observed via the extremely rare decay of the nucleon. Experimental searches for nucleon decay began with the Kolar Gold Mine, Homestake, Soudan, NUSEX, Frejus, HPW, and IMB detectors [17]. The present experimental bounds come from Super-Kamiokande and Soudan II. We discuss these results shortly. Non-SUSY GUTs are also ruled out by the non-observation of nucleon decay [18]. In SUSY GUTs, the GUT scale is of order 3×10^{16} GeV, as compared to the GUT scale in non-SUSY GUTs, which is of order 10^{15} GeV. Hence, the dimension-6 baryon-violating operators are significantly suppressed in SUSY GUTs [20] with $\tau_p \sim 10^{34-38}$ yrs.

However, in SUSY GUTs, there are additional sources for baryon-number violation—dimension-4 and -5 operators [38]. Although the notation does not change, when discussing SUSY GUTs, all fields are implicitly bosonic superfields, and the operators considered are the so-called F terms, which contain two fermionic components, and the rest scalars or products of scalars. Within the context of SU(5), the dimension-4 and -5 operators have the form $(\mathbf{10}\ \bar{\mathbf{5}}\ \bar{\mathbf{5}}) \supset (u^c\ d^c\ d^c) + (Q\ L\ d^c) + (e^c\ L\ L)$, and $(\mathbf{10}\ \mathbf{10}\ \mathbf{10}\ \bar{\mathbf{5}}) \supset (Q\ Q\ Q\ L) + (u^c\ u^c\ d^c\ e^c) + B$ and L conserving terms, respectively. The dimension-4 operators are renormalizable with dimensionless couplings; similar to Yukawa couplings. On the other hand, the dimension-5 operators have a dimensionful coupling of order $(1/M_G)$.

The dimension-4 operators violate baryon number or lepton number, respectively, but not both. The nucleon lifetime is extremely short if both types of dimension-4 operators are present in the low-energy theory. However, both types can be eliminated by requiring R parity. In SU(5), the Higgs doublets reside in a $\mathbf{5_H}$, $\bar{\mathbf{5}}_\mathbf{H}$, and R parity distinguishes the $\bar{\mathbf{5}}$ (quarks and leptons) from $\bar{\mathbf{5}}_\mathbf{H}$ (Higgs). R parity [39] (or more precisely, its cousin, family reflection symmetry) (see Dimopoulos and Georgi [20] and DRW [40]) takes $F \to -F$, $H \to H$ with $F = \{\mathbf{10},\ \bar{\mathbf{5}}\}$, $H = \{\bar{\mathbf{5}}_\mathbf{H},\ \mathbf{5_H}\}$. This forbids the dimension-4 operator $(\mathbf{10}\ \bar{\mathbf{5}}\ \bar{\mathbf{5}})$, but allows the Yukawa couplings of the form $(\mathbf{10}\ \bar{\mathbf{5}}\ \bar{\mathbf{5}}_\mathbf{H})$ and $(\mathbf{10}\ \mathbf{10}\ \mathbf{5_H})$. It also forbids the dimension-3, lepton-number-violating operator $(\bar{\mathbf{5}}\ \mathbf{5_H}) \supset (L\ H_u)$, with a coefficient with dimensions of mass which, like the μ parameter,

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the ratio λ_b/λ_τ as one runs down in energy below $\tilde{m}$. This is opposite to what happens in MSSM where the large top quark Yukawa coupling decreases the ratio λ_b/λ_τ [35].

could be of order the weak scale and the dimension-5, baryon-number-violating operator $(\mathbf{10}\ \mathbf{10}\ \mathbf{10}\ \bar{\mathbf{5}}_{\mathbf{H}}) \supset (Q\ Q\ Q\ H_d) + \cdots$.

Note, in the MSSM, it is possible to retain R-parity-violating operators at low energy, as long as they violate either baryon number or lepton number only, but not both. Such schemes are natural if one assumes a low-energy symmetry, such as lepton number, baryon number, or a baryon parity [41]. However, these symmetries cannot be embedded in a GUT. Thus, in a SUSY GUT, only R parity can prevent unwanted dimension four operators. Hence, by naturalness arguments, R parity must be a symmetry in the effective low-energy theory of any SUSY GUT. This does not mean to say that R parity is guaranteed to be satisfied in any GUT.

Note also, R parity distinguishes Higgs multiplets from ordinary families. In SU(5), Higgs and quark/lepton multiplets have identical quantum numbers; while in $E(6)$, Higgs and families are unified within the fundamental **27** representation. Only in SO(10) are Higgs and ordinary families distinguished by their gauge quantum numbers. Moreover, the Z(4) center of SO(10) distinguishes **10**s from **16**s, and can be associated with R parity [42].

In SU(5), dimension-5 baryon-number-violating operators may be forbidden at tree level by additional symmetries. These symmetries are typically broken, however, by the VEVs responsible for the color triplet Higgs masses. Consequently, these dimension-5 operators are generically generated via color triplet Higgsino exchange. Hence, the color triplet partners of Higgs doublets must necessarily obtain mass of order the GUT scale. The dominant decay modes from dimension-5 operators are $p \to K^+\ \bar{\nu}\ \ (n \to K^0\ \bar{\nu})$. This is due to a simple symmetry argument; the operators $(Q_i\ Q_j\ Q_k\ L_l)$, $(u_i^c\ u_j^c\ d_k^c\ e_l^c)$ (where $i,\ j,\ k,\ l = 1, 2, 3$ are family indices, and color and weak indices are implicit) must be invariant under $\mathrm{SU}(3)_C$ and $\mathrm{SU}(2)_L$. As a result, their color and weak doublet indices must be anti-symmetrized. However, since these operators are given by bosonic superfields, they must be totally symmetric under interchange of all indices. Thus, the first operator vanishes for $i = j = k$, and the second vanishes for $i = j$. Hence, a second or third generation member must exist in the final state [40].

Recent Super-Kamiokande bounds on the proton lifetime severely constrain these dimension-6 and dimension-5 operators with $\tau_{(p\to e^+\pi^0)} > 5.0 \times 10^{33}$ yrs (79.3 ktyr exposure), $\tau_{(n\to e^+\pi^-)} > 5 \times 10^{33}$ yrs (61 ktyr), and $\tau_{(p\to K^+\bar{\nu})} > 1.6 \times 10^{33}$ yrs (79.3 ktyr), $\tau_{(n\to K^0\bar{\nu})} > 1.7 \times 10^{32}$ yrs (61 ktyr) at (90% CL) based on the listed exposures [43]. These constraints are now sufficient to rule out minimal SUSY SU(5) [44]. [9] Non-minimal Higgs sectors in SU(5) or SO(10) theories still survive [23,30]. The upper bound on the proton lifetime from these theories is approximately a factor of 5 above the experimental bounds. They are, however, being pushed to their theoretical limits. Hence, if SUSY GUTs are correct, nucleon decay should be seen soon.

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[9] This conclusion relies on the mild assumption that the three-by-three matrices diagonalizing squark and slepton mass matrices are not so different from their fermionic partners. It has been shown that if this caveat is violated, then dimension five proton decay in minimal SUSY $SU(5)$ may not be ruled out [45].

Is there a way out of this conclusion? Orbifold GUTs and string theories, see Sect. 15.1.3, contain grand unified symmetries realized in higher dimensions. In the process of compactification and GUT symmetry breaking, color triplet Higgs states are removed (projected out of the massless sector of the theory). In addition, the same projections typically rearrange the quark and lepton states so that the massless states which survive emanate from different GUT multiplets. In these models, proton decay due to dimension 5 operators can be severely suppressed or eliminated completely. However, proton decay due to dimension 6 operators may be enhanced, since the gauge bosons mediating proton decay obtain mass at the compactification scale, M_c, which is less than the 4d GUT scale (see the discussion at the end of Section 15.1.4), or suppressed, if the states of one family come from different irreducible representations. Which effect dominates is a model dependent issue. In some complete 5d orbifold GUT models [47,24] the lifetime for the decay $\tau(p \to e^+ \pi^0)$ can be near the excluded bound of 5×10^{33} years with, however, large model dependent and/or theoretical uncertainties. In other cases, the modes $p \to K^+ \bar{\nu}$ and $p \to K^0 \mu^+$ may be dominant [24]. To summarize, in either 4d or orbifold string/field theories, nucleon decay remains a premier signature for SUSY GUTs. Moreover, the observation of nucleon decay may distinguish extra-dimensional orbifold GUTs from four dimensional ones.

Before concluding the topic of baryon-number violation, consider the status of $\Delta B = 2$ neutron- anti-neutron oscillations. Generically, the leading operator for this process is the dimension-9 six-quark operator $G_{(\Delta B=2)}$ $(u^c\ d^c\ d^c\ u^c\ d^c\ d^c)$, with dimensionful coefficient $G_{(\Delta B=2)} \sim 1/M^5$. The present experimental bound $\tau_{n-\overline{n}} \geq 0.86 \times 10^8$ sec. at 90% CL [48] probes only up to the scale $M \leq 10^6$ GeV. For $M \sim M_G$, $n - \overline{n}$ oscillations appear to be unobservable for any GUT (for a recent discussion see Ref. 49).

15.1.6. *Yukawa coupling unification*:

15.1.6.1. *3rd generation, b–τ or t–b–τ unification:*

If quarks and leptons are two sides of the same coin, related by a new grand unified gauge symmetry, then that same symmetry relates the Yukawa couplings (and hence the masses) of quarks and leptons. In SU(5), there are two independent renormalizable Yukawa interactions given by $\lambda_t\ (\mathbf{10\ 10\ 5_H}) + \lambda\ (\mathbf{10\ \bar{5}\ \bar{5}_H})$. These contain the SM interactions $\lambda_t\ (\mathbf{Q\ u^c\ H_u}) + \lambda\ (\mathbf{Q\ d^c\ H_d}\ +\ \mathbf{e^c\ L\ H_d})$. Hence, at the GUT scale, we have the tree-level relation, $\lambda_b = \lambda_\tau \equiv \lambda$ [35]. In SO(10), there is only one independent renormalizable Yukawa interaction given by $\lambda\ (\mathbf{16\ 16\ 10_H})$, which gives the tree-level relation, $\lambda_t = \lambda_b = \lambda_\tau \equiv \lambda$ [50,51]. Note, in the discussion above, we assume the minimal Higgs content, with Higgs in **5**, $\bar{\mathbf{5}}$ for SU(5) and **10** for SO(10). With Higgs in higher-dimensional representations, there are more possible Yukawa couplings. [58–60]

In order to make contact with the data, one now renormalizes the top, bottom, and τ Yukawa couplings, using two-loop-RG equations, from M_G to M_Z. One then obtains the running quark masses $m_t(M_Z) = \lambda_t(M_Z)\ v_u$, $m_b(M_Z) = \lambda_b(M_Z)\ v_d$, and $m_\tau(M_Z) = \lambda_\tau(M_Z)\ v_d$, where $< H_u^0 > \equiv v_u = \sin\beta\ v/\sqrt{2}$, $< H_d^0 > \equiv v_d = \cos\beta\ v/\sqrt{2}$, $v_u/v_d \equiv \tan\beta$, and $v \sim 246$ GeV is fixed by the Fermi constant, G_μ.

Including one-loop-threshold corrections at M_Z, and additional RG running, one finds the top, bottom, and τ-pole masses. In SUSY, $b-\tau$ unification has two possible solutions, with $\tan\beta \sim 1$ or $40-50$. The small $\tan\beta$ solution is now disfavored by the LEP limit, $\tan\beta > 2.4$ [52]. [10] The large $\tan\beta$ limit overlaps the SO(10) symmetry relation.

When $\tan\beta$ is large, there are significant weak-scale threshold corrections to down quark and charged lepton masses, from either gluino and/or chargino loops [54]. Yukawa unification (consistent with low energy data) is only possible in a restricted region of SUSY parameter space with important consequences for SUSY searches [55].

15.1.6.2. *Three families:*

Simple Yukawa unification is not possible for the first two generations, of quarks and leptons. Consider the SU(5) GUT scale relation $\lambda_b = \lambda_\tau$. If extended to the first two generations, one would have $\lambda_s = \lambda_\mu$, $\lambda_d = \lambda_e$, which gives $\lambda_s/\lambda_d = \lambda_\mu/\lambda_e$. The last relation is a renormalization group invariant, and is thus satisfied at any scale. In particular, at the weak scale, one obtains $m_s/m_d = m_\mu/m_e$, which is in serious disagreement with the data, namely $m_s/m_d \sim 20$ and $m_\mu/m_e \sim 200$. An elegant solution to this problem was given by Georgi and Jarlskog [56]. Of course, a three-family model must also give the observed CKM mixing in the quark sector. Note, although there are typically many more parameters in the GUT theory above M_G, it is possible to obtain effective low-energy theories with many fewer parameters making strong predictions for quark and lepton masses.

It is important to note that grand unification alone is not sufficient to obtain predictive theories of fermion masses and mixing angles. Other ingredients are needed. In one approach additional global family symmetries are introduced (non-abelian family symmetries can significantly reduce the number of arbitrary parameters in the Yukawa matrices). These family symmetries constrain the set of effective higher dimensional fermion mass operators. In addition, sequential breaking of the family symmetry is correlated with the hierarchy of fermion masses. Three-family models exist which fit all the data, including neutrino masses and mixing [57]. In a completely separate approach for $SO(10)$ models, the Standard Model Higgs bosons are contained in the higher dimensional Higgs representations including the **10**, $\overline{\mathbf{126}}$ and/or **120**. Such theories have been shown to make predictions for neutrino masses and mixing angles [58–60].

15.1.7. *Neutrino Masses*:

Atmospheric and solar neutrino oscillations require neutrino masses. Adding three "sterile" neutrinos ν^c with the Yukawa coupling λ_ν $(\nu^{\mathbf{c}}\, \mathbf{L}\, \mathbf{H_u})$, one easily obtains three massive Dirac neutrinos with mass $m_\nu = \lambda_\nu\, v_u$. [11] However, in order to obtain a tau neutrino with mass of order 0.1 eV, one needs $\lambda_{\nu_\tau}/\lambda_\tau \leq 10^{-10}$. The see-saw mechanism, on the other hand, can naturally explain such small neutrino masses [61,62]. Since ν^c has no SM quantum numbers, there is no symmetry (other than global lepton number)

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[10] However, this bound disappears if one takes $M_{SUSY} = 2$ TeV and $m_t = 180$ GeV [53].

[11] Note, these "sterile" neutrinos are quite naturally identified with the right-handed neutrinos necessarily contained in complete families of $SO(10)$ or Pati-Salam.

which prevents the mass term $\frac{1}{2}\,\nu^c\,M\,\nu^c$. Moreover, one might expect $M \sim M_G$. Heavy "sterile" neutrinos can be integrated out of the theory, defining an effective low-energy theory with only light active Majorana neutrinos, with the effective dimension-5 operator $\frac{1}{2}\,(\mathbf{L}\,\mathbf{H_u})\,\lambda_\nu^T\,M^{-1}\,\lambda_\nu\,(\mathbf{L}\,\mathbf{H_u})$. This then leads to a 3×3 Majorana neutrino mass matrix $\mathbf{m} = m_\nu^T\,M^{-1}\,m_\nu$.

Atmospheric neutrino oscillations require neutrino masses with $\Delta m_\nu^2 \sim 3\times 10^{-3}$ eV^2 with maximal mixing, in the simplest two-neutrino scenario. With hierarchical neutrino masses, $m_{\nu_\tau} = \sqrt{\Delta m_\nu^2} \sim 0.055$ eV. Moreover, via the "see-saw" mechanism, $m_{\nu_\tau} = m_t(m_t)^2/(3M)$. Hence, one finds $M \sim 2\times 10^{14}$ GeV; remarkably close to the GUT scale. Note we have related the neutrino-Yukawa coupling to the top-quark-Yukawa coupling $\lambda_{\nu_\tau} = \lambda_t$ at M_G, as given in SO(10) or SU(4) $\times$ SU(2)$_L$ $\times$ SU(2)$_R$. However, at low energies they are no longer equal, and we have estimated this RG effect by $\lambda_{\nu_\tau}(M_Z) \approx \lambda_t(M_Z)/\sqrt{3}$.

15.1.8. *Selected Topics*:

15.1.8.1. *Magnetic Monopoles:*

In the broken phase of a GUT, there are typically localized classical solutions carrying magnetic charge under an unbroken U(1) symmetry [63]. These magnetic monopoles with mass of order M_G/α_G are produced during the GUT phase transition in the early universe. The flux of magnetic monopoles is experimentally found to be less than $\sim 10^{-16}$ cm^{-2} s^{-1} sr^{-1} [64]. Many more are predicted however, hence the GUT monopole problem. In fact, one of the original motivations for an inflationary universe is to solve the monopole problem by invoking an epoch of rapid inflation after the GUT phase transition [65]. This would have the effect of diluting the monopole density as long as the reheat temperature is sufficiently below M_G. Other possible solutions to the monopole problem include: sweeping them away by domain walls [66], U(1) electromagnetic symmetry breaking at high temperature [67] or GUT symmetry non-restoration [68]. Parenthetically, it was also shown that GUT monopoles can catalyze nucleon decay [69]. A significantly lower bound on the monopole flux can then be obtained by considering X-ray emission from radio pulsars due to monopole capture and the subsequent nucleon decay catalysis [70].

15.1.8.2. *Baryogenesis via Leptogenesis:*

Baryon-number-violating operators in SU(5) or SO(10) preserve the global symmetry $B-L$. Hence, the value of the cosmological $B-L$ density is an initial condition of the theory, and is typically assumed to be zero. On the other hand, anomalies of the electroweak symmetry violate $B+L$ while also preserving $B-L$. Hence, thermal fluctuations in the early universe, via so-called sphaleron processes, can drive $B+L$ to zero, washing out any net baryon number generated in the early universe at GUT temperatures [71].

One way out of this dilemma is to generate a net $B-L$ dynamically in the early universe. We have just seen that neutrino oscillations suggest a new scale of physics of order 10^{14} GeV. This scale is associated with heavy Majorana neutrinos with mass

M. If in the early universe, the decay of the heavy neutrinos is out of equilibrium and violates both lepton number and CP, then a net lepton number may be generated. This lepton number will then be partially converted into baryon number via electroweak processes [72].

15.1.8.3. *GUT symmetry breaking:*

The grand unification symmetry is necessarily broken spontaneously. Scalar potentials (or superpotentials) exist whose vacua spontaneously break SU(5) and SO(10). These potentials are ad hoc (just like the Higgs potential in the SM), and, therefore it is hoped that they may be replaced with better motivated sectors. Gauge coupling unification now tests GUT-breaking sectors, since it is one of the two dominant corrections to the GUT threshold correction ϵ_3. The other dominant correction comes from the Higgs sector and doublet-triplet splitting. This latter contribution is always positive $\epsilon_3 \propto \ln(M_T/M_G)$ (where M_T is an effective color triplet Higgs mass), while the low-energy data requires $\epsilon_3 < 0$. Hence, the GUT-breaking sector must provide a significant (of order -8%) contribution to ϵ_3 to be consistent with the Super-K bound on the proton lifetime [23,29,30,57].

In string theory (and GUTs in extra-dimensions), GUT breaking may occur due to boundary conditions in the compactified dimensions [7,10]. This is still ad hoc. The major benefits are that it does not require complicated GUT-breaking sectors.

15.1.8.4. *Doublet-triplet splitting:*

The Minimal Supersymmetric Standard Model has a μ problem: why is the coefficient of the bilinear Higgs term in the superpotential $\mu\ (\mathbf{H_u}\, \mathbf{H_d})$ of order the weak scale when, since it violates no low-energy symmetry, it could be as large as M_G? In a SUSY GUT, the μ problem is replaced by the problem of *doublet-triplet* splitting—giving mass of order M_G to the color triplet Higgs, and mass μ to the Higgs doublets. Several mechanisms for natural doublet-triplet splitting have been suggested, such as the sliding singlet, missing partner or missing VEV [73], and pseudo-Nambu-Goldstone boson mechanisms. Particular examples of the missing partner mechanism for SU(5) [30], the missing VEV mechanism for SO(10) [23,57], and the pseudo-Nambu-Goldstone boson mechanism for SU(6) [74], have been shown to be consistent with gauge coupling unification and proton decay. There are also several mechanisms for explaining why μ is of order the SUSY-breaking scale [75]. Finally, for a recent review of the μ problem and some suggested solutions in SUSY GUTs and string theory, see Refs. [76,9] and references therein.

Once again, in string theory (and orbifold GUTs), the act of breaking the GUT symmetry via orbifolding projects certain states out of the theory. It has been shown that it is possible to remove the color triplet Higgs while retaining the Higgs doublets in this process. Hence the doublet-triplet splitting problem is finessed. As discussed earlier (see Section 15.1.5), this has the effect of eliminating the contribution of dimension 5 operators to nucleon decay.

15.2. Conclusion

Grand unification of the strong and electroweak interactions requires that the three low energy gauge couplings unify (up to small threshold corrections) at a unique scale, M_G. Supersymmetric grand unified theories provide, by far, the most predictive and economical framework allowing for perturbative unification.

The three pillars of SUSY GUTs are:

- gauge coupling unification at $M_G \sim 3 \times 10^{16}$ GeV;
- low-energy supersymmetry [with a large SUSY desert], and
- nucleon decay.

The first prediction has already been verified (see Fig. 15.1). Perhaps the next two will soon be seen. Whether or not Yukawa couplings unify is more model dependent. Nevertheless, the "digital" 16-dimensional representation of quarks and leptons in SO(10) is very compelling, and may yet lead to an understanding of fermion masses and mixing angles.

In any event, the experimental verification of the first three pillars of SUSY GUTs would forever change our view of Nature. Moreover, the concomitant evidence for a vast SUSY desert would expose a huge lever arm for discovery. For then it would become clear that experiments probing the TeV scale could reveal physics at the GUT scale and perhaps beyond. Of course, some questions will still remain: Why do we have three families of quarks and leptons? How is the grand unified symmetry and possible family symmetries chosen by Nature? At what scale might stringy physics become relevant? *Etc.*

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