INTRODUCTION TO THREE-NEUTRINO MIXING PARAMETERS LISTINGS

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Introduction and Notation: With the exception of the LSND anomaly, current accelerator, reactor, solar and atmospheric neutrino data can be described within the framework of a 3×3 mixing matrix between the flavor eigenstates ν_e, ν_μ, and ν_τ and mass eigenstates ν_1, ν_2 and ν_3. (See Eq. (13.32) of the Review "Neutrino Mass, Mixing and Flavor Change" by B. Kayser.) Whether or not this is the ultimately correct framework, it is currently widely used to parametrize neutrino mixing data and to plan new experiments.

The mass differences are called Δm^2_{21} and Δm^2_{32} following Eq. (13.30) in the review. In these Listings, we assume that $\Delta m^2_{31} \sim \Delta m^2_{32}$, although in the future, experiments may be precise enough to measure these separately. The angles, as specified in Eq. (13.31) of the review, are labeled θ_{12}, θ_{23}, and θ_{13}. The CP violating phase is called δ, but that does not yet appear in the Listings. The familiar two neutrino form for oscillations is given in Eqs. (13.18) and (13.19). Despite the fact that the mixing angles have been measured to be much larger than in the quark sector, the two-neutrino form is often a very good approximation and is used in many situations. This is possible thanks to the existence of two small numbers, $\Delta m^2_{21}/\Delta m^2_{32} \ll 1$, $\sin^2(2\theta_{13}) < 0.13$.

The angles appear in the equations below in many forms. They most often appear as $\sin^2(2\theta)$. The Listings currently use this convention.

Accelerator neutrino experiments: Ignoring the small Δm^2_{21} scale, CP violation, and matter effects, the equations for the probability of appearance in an accelerator oscillation experiment are:

$$P(\nu_\mu \to \nu_\tau) = \sin^2(2\theta_{23}) \cos^4(\theta_{13}) \sin^2(\Delta m^2_{32} L/4E) \quad (1)$$

$$P(\nu_\mu \to \nu_e) = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2(\Delta m^2_{32} L/4E) \quad (2)$$

$$P(\nu_e \to \nu_\mu) = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2(\Delta m^2_{32} L/4E) \quad (3)$$

$$P(\nu_e \to \nu_\tau) = \sin^2(2\theta_{13}) \cos^2(\theta_{23}) \sin^2(\Delta m^2_{32} L/4E) \quad (4)$$

For the case of negligible θ_{13}, these probabilities vanish except for $\mathrm{P}(\nu_\mu \rightarrow \nu_\tau)$, which then takes the familiar two-neutrino form.

New long-baseline experiments are being planned to search for non-zero θ_{13} through $P(\nu_\mu \rightarrow \nu_e)$. Including the CP violating terms and low mass scale, the equation for neutrino oscillation in vacuum is:

$$
\begin{aligned}
P(\nu_\mu \rightarrow \nu_e) &= P1 + P2 + P3 + P4 \\
P1 &= \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2(\Delta m^2_{32} L/4E) \\
P2 &= \cos^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2(\Delta m^2_{21} L/4E) \\
P3 &= -/+ J \sin(\delta) \sin(\Delta m^2_{32} L/4E) \\
P4 &= J \cos(\delta) \cos(\Delta m^2_{32} L/4E)
\end{aligned}
\qquad (5)
$$

where

$$
\begin{aligned}
J = \cos(\theta_{13}) \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \sin(\Delta m^2_{32} L/4E) \\
\sin(\Delta m^2_{21} L/4E)
\end{aligned}
\qquad (6)
$$

is the "Jarlskog Invariant" for the lepton sector, and the sign in the 3rd term is negative for neutrinos and positive for anti-neutrinos. For most new proposed long-baseline accelerator experiments, P2 can safely be neglected, but depending on the values of θ_{13} and δ, the other three terms could be comparable. Also, depending on the distance and the mass hierarchy, matter effects will need to be included.

Reactor neutrino experiments: Nuclear reactors are prolific sources of $\bar{\nu}_e$ with an energy near 4 MeV. The oscillation probability can be expressed

$$
\begin{aligned}
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2(\Delta m^2_{21} L/4E) \\
- \sin^2(2\theta_{13}) \sin^2(\Delta m^2_{32} L/4E)
\end{aligned}
\qquad (7)
$$

For short distances (L<5 km), it is a good approximation to ignore the second term on the right, and this takes the familiar two-neutrino form with θ_{13} and Δm^2_{32}. For long distances and small θ_{13}, the last term oscillates rapidly and averages to zero for an experiment with finite energy resolution, leading to the familiar two-neutrino form with θ_{12} and Δm^2_{21}.

Solar and Atmospheric neutrino experiments: Solar neutrino experiments are sensitive to ν_e disappearance and have allowed the measurement of θ_{12} and Δm^2_{21}. They are also sensitive to θ_{13}. In the discussion after Eq. (13.21), we identify $\Delta m^2_{\odot} = \Delta m^2_{21}$ and $\theta_{\odot} = \theta_{12}$.

Atmospheric neutrino experiments are primarily sensitive to ν_μ disappearance through $\nu_\mu \to \nu_\tau$ oscillations, and have allowed the measurement of θ_{23} and Δm^2_{32}. In Eqs. (13.24) and (13.25), we identify $\Delta m^2_{atm} = \Delta m^2_{32} \sim \Delta m^2_{31}$ and $\theta_{atm} = \theta_{23}$. Despite the large ν_e component of the atmospheric neutrino flux, it is difficult to measure Δm^2_{21} effects. This is because of a cancellation between $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\mu$ together, with the fact that the ratio of ν_μ and ν_e atmospheric fluxes, which arise from sequential π and μ decay, is near 2.