CPT INVARIANCE TESTS IN NEUTRAL KAON DECAY

Written November 2007 by M. Antonelli (LNF-INFN, Frascati) and G. D'Ambrosio (Naples U., INFN).

CPT theorem is based on three assumptions: quantum field theory, locality, and Lorentz invariance, and thus it is a fundamental probe of our basic understanding of particle physics. Strangeness oscillation in $K^0 - \overline{K}^0$ system, described by the equation

$$i\frac{d}{dt}\begin{bmatrix} K^0 \\ \overline{K}^0 \end{bmatrix} = [M - i\Gamma/2]\begin{bmatrix} K^0 \\ \overline{K}^0 \end{bmatrix} ,$$

where M and Γ are hermitian matrices (see PDG review [1], references [2,3], and KLOE paper [4] for notations and previous literature), allows a very accurate test of CPT symmetry; indeed since CPT requires $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, the mass and width eigenstates, $K_{S,L}$, have a CPT-violating piece, δ, in addition to the usual CPT-conserving parameter ϵ:

$$K_{S,L} = \frac{1}{\sqrt{2\left(1 + |\epsilon_{S,L}|^2\right)}}\left[\left(1 + \epsilon_{S,L}\right)K^0 + \left(1 - \epsilon_{S,L}\right)\overline{K}^0\right]$$

$$\epsilon_{S,L} = \frac{-i\Im\left(M_{12}\right) - \frac{1}{2}\Im\left(\Gamma_{12}\right) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}\left(\Gamma_{11} - \Gamma_{22}\right)\right]}{m_L - m_S + i(\Gamma_S - \Gamma_L)/2}$$
$$\equiv \epsilon \mp \delta. \tag{1}$$

Using the phase convention $\Im(\Gamma_{12}) = 0$, we determine the phase of ϵ to be $\varphi_{SW} \equiv \arctan\frac{2(m_L - m_S)}{\Gamma_S - \Gamma_L}$. Imposing unitarity to an arbitrary combination of K^0 and $\overline{K}^0$ wave functions, we obtain the Bell-Steinberger relation [5] connecting CP and CPT violation in the mass matrix to CP and CPT violation in the decay; in fact, neglecting $\mathcal{O}(\epsilon)$ corrections to the coefficient of the CPT-violating parameter, δ, we can write [4]

$$\left[\frac{\Gamma_S + \Gamma_L}{\Gamma_S - \Gamma_L} + i\tan\phi_{\rm SW}\right]\left[\frac{\Re(\epsilon)}{1 + |\epsilon|^2} - i\Im(\delta)\right] =$$
$$\frac{1}{\Gamma_S - \Gamma_L}\sum_f A_L(f)A_S^*(f), \tag{2}$$

where $A_{L,S}(f) \equiv A(K_{L,S} \to f)$. We stress that this relation is phase-convention-independent. The advantage of the neutral kaon system is that only a few decay modes give significant contributions to the r.h.s. in Eq. (2); in fact, defining for the hadronic modes

$$\alpha_i \equiv \frac{1}{\Gamma_S} \langle \mathcal{A}_L(i) \mathcal{A}_S^*(i) \rangle = \eta_i \ \mathcal{B}(K_S \to i),$$
$$i = \pi^0\pi^0, \pi^+\pi^-(\gamma), 3\pi^0, \pi^0\pi^+\pi^-(\gamma), \tag{3}$$

the recent data from CPLEAR, KLOE, KTeV, and NA48 have led to the following determinations (the analysis described in Ref. 4 has been updated by using the recent measurements of K_L branching ratios from KTeV [6] and NA48 [7,8])

$$\alpha_{\pi^+\pi^-} = \Big((1.112 \pm 0.013) + i(1.061 \pm 0.014) \Big) \times 10^{-3} ,$$
$$\alpha_{\pi^0\pi^0} = \Big((0.493 \pm 0.007) + i(0.471 \pm 0.007) \Big) \times 10^{-3} ,$$
$$\alpha_{\pi^+\pi^-\pi^0} = \Big((0 \pm 2) + i(0 \pm 2) \Big) \times 10^{-6},$$
$$|\alpha_{\pi^0\pi^0\pi^0}| < 7 \times 10^{-6} \quad \text{at } 95\% \text{ CL} . \tag{4}$$

The semileptonic contribution to the right-handed side of Eq. (2) requires the determination of several observables: we define [2,3]

$$\mathcal{A}(K^0 \to \pi^- l^+ \nu) = \mathcal{A}_0 (1 - y) ,$$
$$\mathcal{A}(K^0 \to \pi^+ l^- \nu) = \mathcal{A}_0^* (1 + y^*)(x_+ - x_-)^* ,$$
$$\mathcal{A}(\overline{K}^0 \to \pi^+ l^- \nu) = \mathcal{A}_0^* (1 + y^*) ,$$
$$\mathcal{A}(\overline{K}^0 \to \pi^- l^+ \nu) = \mathcal{A}_0 (1 - y)(x_+ + x_-) , \tag{5}$$

where x_+ (x_-) describes the violation of the $\Delta S = \Delta Q$ rule in CPT-conserving (violating) decay amplitudes, and y parametrizes CPT violation for $\Delta S = \Delta Q$ transitions. Taking advantage of their tagged $K^0(\overline{K}^0)$ beams, CPLEAR has measured $\Im(x_+)$, $\Re(x_-)$, $\Im(\delta)$, and $\Re(\delta)$ [10]. These determinations have been improved in Ref. 4 by including the information $A_S - A_L = 4[\Re(\delta) + \Re(x_-)]$, where $A_{L,S}$ are the K_L and K_S semileptonic charge asymmetries, respectively, from the PDG [11] and KLOE [12]. Here we are also including the T-violating asymmetry measurement from CPLEAR [13].

Table 1: Values, errors, and correlation coefficients for $\Re(\delta)$, $\Im(\delta)$, $\Re(x_-)$, $\Im(x_+)$, and $A_S + A_L$ obtained from a combined fit, including KLOE [4] and CPLEAR [13].

	value	Correlations coefficients				
$\Re(\delta)$	$(3.0 \pm 2.3) \times 10^{-4}$	1				
$\Im(\delta)$	$(-0.66 \pm 0.65) \times 10^{-2}$	− 0.21	1			
$\Re(x_-)$	$(-0.30 \pm 0.21) \times 10^{-2}$	− 0.21	−0.60	1		
$\Im(x_+)$	$(0.02 \pm 0.22) \times 10^{-2}$	− 0.38	−0.14	0.47	1	
$A_S + A_L$	$(-0.40 \pm 0.83) \times 10^{-2}$	− 0.10	−0.63	0.99	0.43	1

The value $A_S + A_L$ in Table 1 can be directely included in the semileptonic contributions to the Bell Steinberger relations in Eq. (2)

$$\begin{aligned} &\sum_{\pi\ell\nu} \langle \mathcal{A}_L(\pi\ell\nu)\mathcal{A}_S^*(\pi\ell\nu)\rangle \\ &\qquad = 2\Gamma(K_L \to \pi\ell\nu)\Big((\Re(\epsilon) - \Re(y) - i(\Im(x_+) + \Im(\delta))\Big) \\ &\qquad = 2\Gamma(K_L \to \pi\ell\nu)\Big((A_S + A_L)/4 - i(\Im(x_+) + \Im(\delta))\Big) \ . \end{aligned} \tag{6}$$

Defining

$$\alpha_{\pi\ell\nu} \equiv \frac{1}{\Gamma_S}\sum_{\pi\ell\nu}\langle \mathcal{A}_L(\pi\ell\nu)\mathcal{A}_S^*(\pi\ell\nu)\rangle + 2i\frac{\tau_{K_S}}{\tau_{K_L}}\mathcal{B}(K_L \to \pi\ell\nu)\Im(\delta) \ , \tag{7}$$

we find:

$$\alpha_{\pi\ell\nu} = \Big((-0.2 \pm 0.5) + i(0.1 \pm 0.5)\Big) \times 10^{-5} \, .$$

Inserting the values of the α parameters into Eq. (2), we find

$$\begin{aligned} \Re(\epsilon) &= (161.2 \pm 0.6) \times 10^{-5}, \\ \Im(\delta) &= (-0.6 \pm 1.9) \times 10^{-5} \, . \end{aligned} \tag{8}$$

The complete information on Eq. (8) is given in Table 2.

Table 2: Summary of results: values, errors, and correlation coefficients for $\Re(\epsilon)$, $\Im(\delta)$, $\Re(\delta)$, and $\Re(x_-)$.

	value	Correlations coefficients			
$\Re(\epsilon)$	$(161.2 \pm 0.6) \times 10^{-5}$	1			
$\Im(\delta)$	$(-0.6 \pm 1.9) \times 10^{-5}$	-0.26	1		
$\Re(\delta)$	$(2.5 \pm 2.3) \times 10^{-4}$	-0.08	-0.09	1	
$\Re(x_-)$	$(-4.2 \pm 1.7) \times 10^{-3}$	-0.13	0.17	-0.43	1

Now the agreement with CPT conservation, $\Im(\delta) = \Re(\delta) = \Re(x_-) = 0$, is at 30% C.L.

The allowed region in the $\Re(\epsilon) - \Im(\delta)$ plane at 68% CL and 95% C.L. is shown in the top panel of Fig. 1.

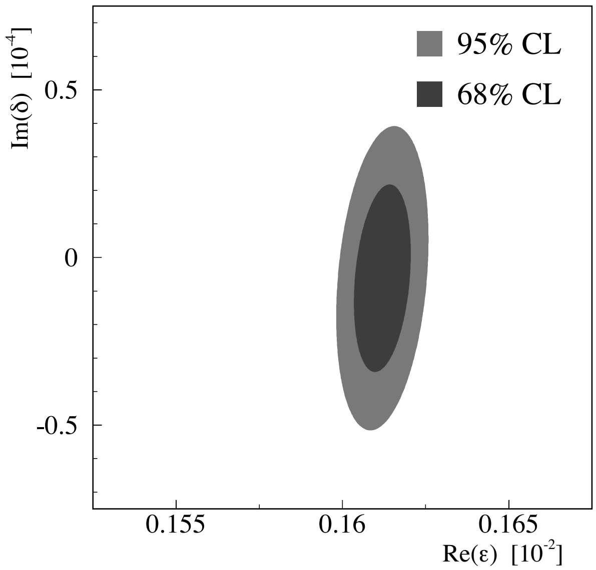
The process giving the largest contribution to the size of the allowed region is $K_L \to \pi^+\pi^-$, through the uncertainty on ϕ_{+-}.

The limits on $\Im(\delta)$ and $\Re(\delta)$ can be used to constrain the $K^0 - \overline{K}^0$ mass and width difference

$$\delta = \frac{i(m_{K^0} - m_{\overline{K}^0}) + \frac{1}{2}(\Gamma_{K^0} - \Gamma_{\overline{K}^0})}{\Gamma_S - \Gamma_L} \cos\phi_{SW} e^{i\phi_{SW}} [1 + \mathcal{O}(\epsilon)] \, .$$

The allowed region in the $\mathit{\Delta} M = (m_{K^0} - m_{\overline{K}^0}), \mathit{\Delta}\Gamma = (\Gamma_{K^0} - \Gamma_{\overline{K}^0})$ plane is shown in the bottom panel of Fig. 1. As a result, we improve on the previous limits (see for instance, P. Bloch in Ref. 11) and in the limit $\Gamma_{K^0} - \Gamma_{\overline{K}^0} = 0$ we obtain

$$-5.1 \times 10^{-19} \text{ GeV} < m_{K^0} - m_{\overline{K}^0} < 5.1 \times 10^{-19} \text{ GeV at } 95 \text{ \% C.L.}$$



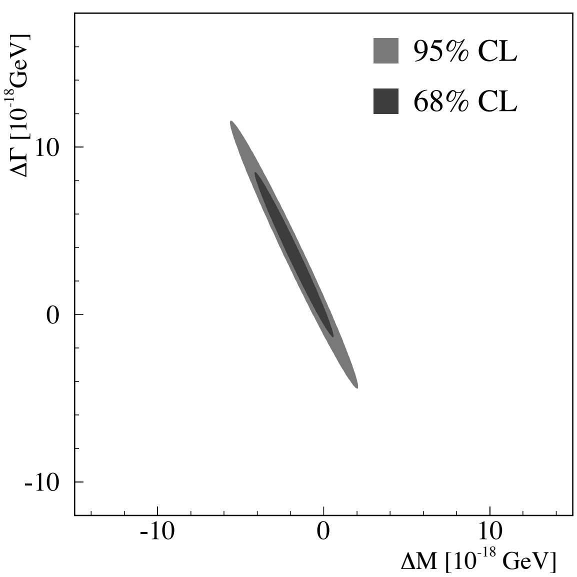


Figure 1: Top: allowed region at 68% and 95% C.L. in the $\Re(\epsilon)$, $\Im(\delta)$ plane. Bottom: allowed region at 68% and 95% C.L. in the $\Delta M, \Delta\Gamma$ plane.

References

1. See the "CP Violation in Meson Decays," in this *Review*.
2. L. Maiani, "CP And CPT Violation In Neutral Kaon Decays," L. Maiani, G. Pancheri, and N. Paver, *The Second Daphne Physics Handbook*, Vol. 1, 2.
3. G. D'Ambrosio, G. Isidori, and A. Pugliese, "CP and CPT mesurements at DAΦNE," L. Maiani, G. Pancheri, and N. Paver, *The Second Daphne Physics Handbook*, Vol. 1, 2.
4. F. Ambrosino *et al.*, [KLOE Collaboration], JHEP **0612**, 011 (2006) [`arXiv:hep-ex/0610034`].
5. J. S. Bell and J. Steinberger, In Wolfenstein, L. (ed.): *CP violation,* 42-57. (In *Oxford International Symposium Conference on Elementary Particles*, September 1965, 195-208, 221-222). (See Book Index).
6. T. Alexopoulos *et al.*, [KTeV Collaboration], Phys. Rev. **D70**, 092006 (1998).
7. A. Lai *et al.*, [NA48 Collaboration], Phys. Lett. **B645**, 26 (2007);
   A. Lai *et al.*, [NA48 Collaboration], Phys. Lett. **B602**, 41 (2004).
8. We thank G. Isidori and M. Palutan for their contribution to the original analysis [4] performed with KLOE data.
9. A. Angelopoulos *et al.*, [CPLEAR Collaboration], Phys. Reports **374**, 165 (2003).
10. A. Angelopoulos *et al.*, [CPLEAR Collaboration], Phys. Lett. **B444**, 52 (1998).
11. W. M. Yao *et al.*, [Particle Data Group], J. Phys. **G33**, 1 (2006).
12. F. Ambrosino *et al.*, [KLOE Collaboration], Phys. Lett. **B636**, 173 (2006) [`arXiv:hep-ex/0601026`].
13. P. Bloch, M. Fidecaro, private communication of the data in a finer binning format; A. Angelopoulos *et al.*, [CPLEAR Collaboration], Phys. Lett. **B444**, 43 (1998).