V_{ud}, V_{us}, THE CABIBBO ANGLE, AND CKM UNITARITY

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The Cabibbo-Kobayashi-Maskawa (CKM) [1,2] three-generation quark mixing matrix written in terms of the Wolfenstein parameters (λ, A, ρ, η) [3] nicely illustrates the orthonormality constraint of unitarity and central role played by λ.

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \ . \quad (1)$$

That cornerstone is a carryover from the two-generation Cabibbo angle, $\lambda = \sin(\theta_{\mathrm{Cabibbo}}) = V_{us}$. Its value is a critical ingredient in determinations of the other parameters and in tests of CKM unitarity.

Unfortunately, the precise value of λ has been somewhat controversial in the past, with kaon decays suggesting [4] $\lambda \simeq 0.220$, while hyperon decays [5] and indirect determinations via nuclear β-decays imply a somewhat larger $\lambda \simeq 0.225 - 0.230$. That discrepancy is often discussed in terms of a deviation from the unitarity requirement

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \quad (2)$$

For many years, using a value of V_{us} derived from $K \to \pi e \nu$ (K_{e3}) decays, that sum was consistently 2–2.5 sigma below unity, a potential signal [6] for new physics effects. Below, we discuss the current status of V_{ud}, V_{us}, and their associated unitarity test in Eq. (2). (Since $|V_{ub}|^2 \simeq 1 \times 10^{-5}$ is negligibly small, it is ignored in this discussion.)

V_{ud}

The value of V_{ud} has been obtained from superallowed nuclear, neutron, and pion decays. Currently, the most precise

determination of V_{ud} comes from superallowed nuclear beta-decays [6] $(0^+ \rightarrow 0^+$ transitions). Measuring their half-lives, t, and Q values which give the decay rate factor, f, leads to a precise determination of V_{ud} via the master formula [7–9]

$$|V_{ud}|^2 = \frac{2984.48(5)\,\sec}{ft(1 + \mathrm{RC})} \tag{3}$$

where RC denotes the entire effect of electroweak radiative corrections, nuclear structure, and isospin violating nuclear effects. RC is nucleus-dependent, ranging from about $+3.0\%$ to $+3.6\%$ for the best measured superallowed decays. The most recent analysis of Hardy and Towner [10, 11] gives a weighted average (with errors combined in quadrature) of

$$V_{ud} = 0.97425(22) \text{ (superallowed)} , \tag{4}$$

which, assuming unitarity, corresponds to $\lambda = 0.2255(10)$. The new average value of V_{ud} is shifted upward compared to our 2007 value of $0.97418(27)$ primarily because of improvements in the experimental ft values and nuclear isospin breaking corrections employed. We note, however, that the possibility of additional nuclear coulombic corrections has been raised recently [12].

Combined measurements of the neutron lifetime, τ_n, and the ratio of axial-vector/vector couplings, $g_A \equiv G_A/G_V$, via neutron decay asymmetries can also be used to determine V_{ud}:

$$|V_{ud}|^2 = \frac{4908.7(1.9)\,\sec}{\tau_n(1 + 3g_A^2)}, \tag{5}$$

where the error stems from uncertainties in the electroweak radiative corrections [8] due to hadronic loop effects. Those effects have been recently updated and their error was reduced by about a factor of 2 [9], leading to a ±0.0002 theoretical uncertainty in V_{ud} (common to all V_{ud} extractions). Using the world averages from this *Review*

$$\tau_n^{\mathrm{ave}} = 885.7(8) \text{ sec}$$
$$g_A^{\mathrm{ave}} = 1.2695(29) \tag{6}$$

leads to

$$V_{ud} = 0.9746(4)_{\tau_n}(18)_{g_A}(2)_{\mathrm{RC}} \tag{7}$$

with the error dominated by g_A uncertainties (which have been expanded due to experimental inconsistencies). We note that a recent precise measurement [13] of $\tau_n = 878.5(7)(3)$ sec is also inconsistent with the world average from this *Review* and would lead to a considerably larger $V_{ud} = 0.9786(4)(18)(2)$. Alternatively, accepting the recent shorter lifetime measurement as correct, and employing it along with the value of V_{ud} in Eq. (4), leads to $g_A = 1.2763(7)$, which is outside of the range of Eq. (6) but in good accord with the most recent direct measurements of g_A [14]. Future neutron studies are expected to resolve these inconsistencies and significantly reduce the uncertainties in g_A and τ_n, potentially making them the best way to determine V_{ud}.

The recently completed PIBETA experiment at PSI measured the very small $(\mathcal{O}(10^{-8}))$ branching ratio for $\pi^+ \to \pi^o e^+ \nu_e$ with about $\pm 1/2\%$ precision. Their result gives [15]

$$V_{ud} = 0.9749(26) \left[\frac{BR(\pi^+ \to e^+ \nu_e(\gamma))}{1.2352 \times 10^{-4}} \right]^{\frac{1}{2}} \quad (8)$$

which is normalized using the very precisely determined theoretical prediction for $BR(\pi^+ \to e^+ \nu_e(\gamma)) = 1.2352(5) \times 10^{-4}$ [7], rather than the experimental branching ratio from this *Review* of $1.230(4) \times 10^{-4}$ which would lower the value to $V_{ud} = 0.9728(30)$. Theoretical uncertainties in that determination are very small; however, much higher statistics would be required to make this approach competitive with others.

V_{us}

$|V_{us}|$ may be determined from kaon decays, hyperon decays, and tau decays. Previous determinations have most often used $K\ell 3$ decays:

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta_K^\ell + \delta_{SU2}) C^2 |V_{us}|^2 f_+^2(0) I_K^\ell. \quad (9)$$

Here, ℓ refers to either e or μ, G_F is the Fermi constant, M_K is the kaon mass, S_{EW} is the short-distance radiative correction, δ_K^ℓ is the mode-dependent long-distance radiative correction, $f_+(0)$ is the calculated form factor at zero momentum transfer for the $\ell\nu$ system, and I_K^ℓ is the phase-space integral, which

depends on measured semileptonic form factors. For charged kaon decays, δ_{SU2} is the deviation from one of the ratio of $f_+(0)$ for the charged to neutral kaon decay; it is zero for the neutral kaon. C^2 is 1 (1/2) for neutral (charged) kaon decays. Most determinations of $|V_{us}|$ have been based only on $K \to \pi e\nu$ decays; $K \to \pi\mu\nu$ decays have not been used because of large uncertainties in I_K^μ. The experimental measurements are the semileptonic decay widths (based on the semileptonic branching fractions and lifetime) and form factors (allowing calculation of the phase space integrals). Theory is needed for S_{EW}, δ_K^ℓ, δ_{SU2}, and $f_+(0)$.

Many new measurements during the last few years have resulted in a significant shift in V_{us}. Most importantly, recent measurements of the $K \to \pi e\nu$ branching fractions are significantly different than earlier PDG averages, probably as a result of inadequate treatment of radiation in older experiments. This effect was first observed by BNL E865 [16] in the charged kaon system and then by KTeV [17,18] in the neutral kaon system; subsequent measurements were made by KLOE [19–22], NA48 [23–25], and ISTRA+ [26]. Current averages (*e.g.*, by the PDG [27] or Flavianet [28]) of the semileptonic branching fractions are based only on recent, high-statistics experiments where the treatment of radiation is clear. In addition to measurements of branching fractions, new measurements of lifetimes [29] and form factors [30–34], have resulted in improved precision for all of the experimental inputs to V_{us}. Precise measurements of form factors for $K_{\mu3}$ decay now make it possible to use both semileptonic decay modes to extract V_{us}.

Following the analysis of the Flavianet group [28], one finds the values of $|V_{us}|f_+(0)$ in Table 1. The average of these measurements gives

$$f_+(0)|V_{us}| = 0.21664(48). \qquad (10)$$

Figure 1 shows a comparison of these results with the PDG evaluation from 2002 [35], as well as $f_+(0)(1-|V_{ud}|^2-|V_{ub}|^2)^{1/2}$, the expectation for $f_+(0)|V_{us}|$ assuming unitarity, based on $|V_{ud}| = 0.9742 \pm 0.0003$, $|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$, and the

lattice calculation of $f_+(0) = 0.9644 \pm 0.0049$ [36](Lattice calculations of $f_+(0)$ have improved significantly in recent years, and therefore replace the classic calculation of Leutwyler and Roos [37].) Combining the result in Eq. (10) with the above value of $f_+(0)$ gives

$$|V_{us}| = \lambda = 0.2246 \pm 0.0012. \tag{11}$$

Table 1: $|V_{us}|f_+(0)$ from $K_{\ell 3}$.

Decay Mode	$|V_{us}|f_+(0)$
$K^{\pm}e3$	0.2173 ± 0.0008
$K^{\pm}\mu3$	0.2176 ± 0.0011
$K_L e3$	0.2163 ± 0.0006
$K_L \mu3$	0.2168 ± 0.0007
$K_S e3$	0.2154 ± 0.0013
Average	0.2166 ± 0.0005

A value of V_{us} can also be obtained from a comparison of the radiative inclusive decay rates for $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$ combined with a lattice gauge theory calculation of f_K/f_π via [42]

$$\frac{|V_{us}|f_K}{|V_{ud}|f_\pi} = 0.2387(4) \left[\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}\right]^{\frac{1}{2}} \tag{12}$$

with the small error coming from electroweak radiative corrections. Employing

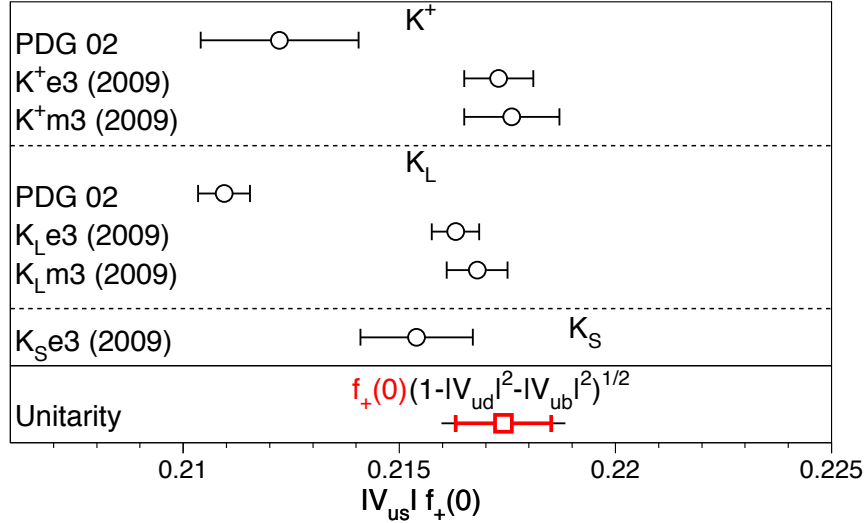$$\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))} = 1.3337(46), \tag{13}$$

which averages in the KLOE result [43], $B(K \to \mu\nu(\gamma)) = 63.66(9)(15)\%$ and [44]

$$f_K/f_\pi = 1.189(7) \tag{14}$$

along with the value of V_{ud} in Eq. (4) leads to

$$|V_{us}| = 0.2259(5)(13). \tag{15}$$

Figure 1: Comparison of determinations of $|V_{us}|f_+(0)$ from this review (labeled 2009), from the PDG 2002, and with the prediction from unitarity using $|V_{ud}|$ and the lattice calculation of $f_+(0)$ [36]. For $f_+(0)(1 - |V_{ud}|^2 - |V_{ub}|^2)^{1/2}$, the inner error bars are from the quoted uncertainty in $f_+(0)$; the total uncertainties include the $|V_{ud}|$ and $|V_{ub}|$ errors.

It should be mentioned that hyperon decay fits suggest [5]

$$|V_{us}| = 0.2250(27) \text{ Hyperon Decays} \qquad (16)$$

modulo SU(3) breaking effects that could shift that value up or down. We note that a recent representative effort [45] that incorporates SU(3) breaking found $V_{us} = 0.226(5)$. Similarly, inclusive strangeness changing tau decays give [46]

$$|V_{us}| = 0.2208(34) \text{ Tau Decays} \qquad (17)$$

where the central value depends on the strange quark mass. However, a recent BaBar study [47] of $\tau \to K\nu / \tau \to \pi\nu$ using the lattice value of f_K / f_π from Eq. (14) finds $V_{us} = 0.2255(24)$, in good agreement with other determinations.

Employing the value of V_{ud} in Eq. (4) and $V_{us} = 0.2252(9)$, the average of the $K\ell 3$ (Eq. (11)) and $K\mu 2$ (Eq. (15) determinations of V_{us}, leads to the unitarity consistency check

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(4)(4). \qquad (18)$$

where the first error is the uncertainty from $|V_{ud}|^2$ and the second error is the uncertainty from $|V_{us}|^2$.

CKM Unitarity Constraints

The current good experimental agreement with unitarity, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(6)$, provides strong confirmation of Standard Model radiative corrections (which range between 3-4% depending on the nucleus used) at better than the 50 sigma level [48]. In addition, it implies constraints on "New Physics" effects at both the tree and quantum loop levels. Those effects could be in the form of contributions to nuclear beta decays, K decays and/or muon decays, with the last of these providing normalization via the muon lifetime [49], which is used to obtain the Fermi constant, $G_\mu = 1.166371(6) \times 10^{-5} \text{GeV}^{-2}$.

In the following sections, we illustrate the implications of CKM unitarity for (1) exotic muon decays [50](beyond ordinary muon decay $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$) and (2) new heavy quark mixing V_{uD} [51]. Other examples in the literature [52,53] include Z_χ boson quantum loop effects, supersymmetry, leptoquarks, compositeness etc.

Exotic Muon Decays

If additional lepton flavor violating decays such as $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$ (wrong neutrinos) occur, they would cause confusion in searches for neutrino oscillations at, for example, muon storage rings/neutrino factories or other neutrino sources from muon decays. Calling the rate for all such decays Γ(exotic μ decays), they should be subtracted before the extraction of G_μ and normalization of the CKM matrix. Since that is not done and unitarity works, one has (at one-sided 95% CL)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - BR(\text{exotic } \mu \text{ decays}) \geq 0.9989 \qquad (19)$$

or

$$BR(\text{exotic } \mu \text{ decays}) < 0.001 . \qquad (20)$$

This bound is a factor of 10 better than the direct experimental bound on $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$.

New Heavy Quark Mixing

Heavy D quarks naturally occur in fourth quark generation models and some heavy quark "new physics" scenarios such as E_6 grand unification. Their mixing with ordinary quarks gives rise to V_{ud} which is constrained by unitarity (one sided 95% CL)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{uD}|^2 > 0.9989$$
$$|V_{uD}| < 0.03 \ . \tag{21}$$

A similar constraint applies to heavy neutrino mixing and the couplings $V_{\mu N}$ and V_{eN}.

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