## SUPERSYMMETRY, PART I (THEORY)

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***I.1. Introduction:*** Supersymmetry (SUSY) is a generalization of the space-time symmetries of quantum field theory that transforms fermions into bosons and vice versa. The existence of such a non-trivial extension of the Poincaré symmetry of ordinary quantum field theory was initially surprising, and its form is highly constrained by theoretical principles [1]. Supersymmetry also provides a framework for the unification of particle physics and gravity [2–5], which is governed by the Planck energy scale, $M_{\rm P} \approx 10^{19}$ GeV (where the gravitational interactions become comparable in magnitude to the gauge interactions). In particular, it is possible that supersymmetry will ultimately explain the origin of the large hierarchy of energy scales from the $W$ and $Z$ masses to the Planck scale [6–10]. This is the so-called *gauge hierarchy*. The stability of the gauge hierarchy in the presence of radiative quantum corrections is not possible to maintain in the Standard Model, but can be maintained in supersymmetric theories.

If supersymmetry were an exact symmetry of nature, then particles and their superpartners (which differ in spin by half a unit) would be degenerate in mass. Since superpartners have not (yet) been observed, supersymmetry must be a broken symmetry. Nevertheless, the stability of the gauge hierarchy can still be maintained if the supersymmetry breaking is *soft* [11,12], and the corresponding supersymmetry-breaking mass parameters are no larger than a few TeV. In particular, soft-supersymmetry-breaking terms of the Lagrangian are either linear, quadratic, or cubic in the fields, with some restrictions elucidated in Ref. 11. The impact of such terms becomes negligible at energy scales much larger than the size of the supersymmetry-breaking masses. The most interesting theories of this type are theories of "low-energy" (or "weak-scale") supersymmetry, where the effective scale of supersymmetry breaking is tied to the scale of electroweak symmetry breaking [7–10]. The latter is characterized by the Standard Model Higgs vacuum expectation value, $v \simeq 246$ GeV.

Although there are no unambiguous experimental results (at present) that require the existence of new physics at the TeV-scale, expectations of the latter are primarily based on three theoretical arguments. First, a *natural* explanation (*i.e.*, one that is stable with respect to quantum corrections) of the gauge hierarchy demands new physics at the TeV-scale [10]. Second, the unification of the three Standard Model gauge couplings at a very high energy close to the Planck scale is possible if new physics beyond the Standard Model (which modifies the running of the gauge couplings above the electroweak scale) is present. The minimal supersymmetric extension of the Standard Model, where supersymmetric masses lie below a few TeV, provides simple example of successful gauge coupling unification [13]. Third, the existence of dark matter, which makes up approximately one quarter of the energy density of the universe, cannot be explained within the Standard Model of particle physics [14]. Remarkably, a stable weakly-interacting massive particle (WIMP) whose mass and interaction rate are governed by new physics associated with the TeV-scale can be consistent with the observed density of dark matter (this is the so-called *WIMP miracle*, which is reviewed in Ref. 15). The lightest supersymmetric particle is a promising (although not the unique) candidate for the dark matter [16,17]. Further aspects of dark matter can be found in Ref. 18.

***I.2. Structure of the MSSM:*** The minimal supersymmetric extension of the Standard Model (MSSM) consists of taking the fields of the two-Higgs-doublet extension of the Standard Model and adding the corresponding supersymmetric partners [19,20]. The corresponding field content of the MSSM and their gauge quantum numbers are shown in Table 1. The electric charge $Q = T_3 + \frac{1}{2}Y$ is determined in terms of the third component of the weak isospin ($T_3$) and the U(1) hypercharge ($Y$).

The gauge super-multiplets consist of the gluons and their *gluino* fermionic superpartners, and the SU(2)×U(1) gauge bosons and their *gaugino* fermionic superpartners. The Higgs multiplets consist of two complex doublets of Higgs fields, their *higgsino* fermionic superpartners, and the corresponding antiparticle fields. The matter super-multiplets consist of three generations of left-handed and right-handed quarks and lepton fields, their scalar superpartners (squark and slepton fields), and the corresponding antiparticle fields. The enlarged Higgs sector of the MSSM constitutes the minimal structure needed to guarantee the cancellation of anomalies from the introduction of the higgsino superpartners. Moreover, without a second Higgs doublet, one cannot generate mass for both "up"-type and

**Table 1:** The fields of the MSSM and their SU(3)×SU(2)×U(1) quantum numbers are listed. Only one generation of quarks and leptons is exhibited. For each lepton, quark, and Higgs supermultiplet, there is a corresponding anti-particle multiplet of charge-conjugated fermions and their associated scalar partners.

| Field Content of the MSSM | | | | | |
|---|---|---|---|---|---|
| Super-Multiplets | Boson Fields | Fermionic Partners | SU(3) | SU(2) | U(1) |
| gluon/gluino | $g$ | $\widetilde{g}$ | 8 | 1 | 0 |
| gauge/ | $W^{\pm}$ , $W^0$ | $\widetilde{W}^{\pm}$ , $\widetilde{W}^0$ | 1 | 3 | 0 |
| gaugino | $B$ | $\widetilde{B}$ | 1 | 1 | 0 |
| slepton/ | $(\widetilde{\nu}, \widetilde{e}^-)_L$ | $(\nu, e^-)_L$ | 1 | 2 | −1 |
| lepton | $\tilde{e}^-_R$ | $e^-_R$ | 1 | 1 | −2 |
| squark/ | $(\widetilde{u}_L, \widetilde{d}_L)$ | $(u, d)_L$ | 3 | 2 | 1/3 |
| quark | $\widetilde{u}_R$ | $u_R$ | 3 | 1 | 4/3 |
| | $\widetilde{d}_R$ | $d_R$ | 3 | 1 | −2/3 |
| Higgs/ | $(H^0_d\, , \, H^-_d)$ | $(\widetilde{H}^0_d, \widetilde{H}^-_d)$ | 1 | 2 | −1 |
| higgsino | $(H^+_u\, , \, H^0_u)$ | $(\widetilde{H}^+_u, \widetilde{H}^0_u)$ | 1 | 2 | 1 |

"down"-type quarks (and charged leptons) in a way consistent with the supersymmetry [21–23].

A general supersymmetric Lagrangian is determined by three functions of the superfields (composed of the fields of the super-multiplets): the superpotential, the Kähler potential, and the gauge kinetic-energy function [5]. For *renormalizable* globally supersymmetric theories, minimal forms for the latter two functions are required in order to generate canonical kinetic energy terms for all the fields. A renormalizable superpotential, which is at most cubic in the superfields, yields supersymmetric Yukawa couplings and mass terms. A combination of gauge invariance and supersymmetry produces couplings of gaugino fields to matter (or Higgs) fields and their corresponding superpartners. The (renormalizable) MSSM Lagrangian is then constructed by including all possible supersymmetric interaction terms (of dimension four or less) that satisfy SU(3)×SU(2)×U(1) gauge invariance and $B$–$L$ conservation (where $B$ =baryon number and $L$ =lepton number).

Finally, the most general soft-supersymmetry-breaking terms are added [11,12,24]. To generate nonzero neutrino masses, extra structure is needed as discussed in Section I.8.

***I.2.1. R-parity and the lightest supersymmetric particle:*** As a consequence of $B$–$L$ invariance, the MSSM possesses a multiplicative R-parity invariance, where $R = (-1)^{3(B-L)+2S}$ for a particle of spin $S$ [25]. Note that this implies that all the ordinary Standard Model particles have even R parity, whereas the corresponding supersymmetric partners have odd R parity. The conservation of R parity in scattering and decay processes has a crucial impact on supersymmetric phenomenology. For example, starting from an initial state involving ordinary (R-even) particles, it follows that supersymmetric particles must be produced in pairs. In general, these particles are highly unstable and decay into lighter states. However, R-parity invariance also implies that the lightest supersymmetric particle (LSP) is absolutely stable, and must eventually be produced at the end of a decay chain initiated by the decay of a heavy unstable supersymmetric particle.

In order to be consistent with cosmological constraints, a stable LSP is almost certainly electrically and color neutral [26]. (There are some model circumstances in which a colored gluino LSP is allowed [27], but we do not consider this possibility further here.) Consequently, the LSP in an R-parity-conserving theory is weakly interacting with ordinary matter, *i.e.*, it behaves like a stable heavy neutrino and will escape collider detectors without being directly observed. Thus, the canonical signature for conventional R-parity-conserving supersymmetric theories is missing (transverse) energy, due to the escape of the LSP. Moreover, as noted at the end of Section I, the LSP is a promising candidate for dark matter [16,17].

***I.2.2. The goldstino and gravitino:*** In the MSSM, supersymmetry breaking is accomplished by including the most general renormalizable soft-supersymmetry-breaking terms consistent with the SU(3)×SU(2)×U(1) gauge symmetry and R-parity invariance. These terms parameterize our ignorance of the fundamental mechanism of supersymmetry breaking. If supersymmetry breaking occurs spontaneously, then a massless

Goldstone fermion called the *goldstino* ($\widetilde{G}_{1/2}$) must exist. The goldstino would then be the LSP, and could play an important role in supersymmetric phenomenology [28].

However, the goldstino degrees of freedom are physical only in models of spontaneously-broken global supersymmetry. If supersymmetry is a local symmetry, then the theory must incorporate gravity; the resulting theory is called supergravity [29]. In models of spontaneously-broken supergravity, the goldstino is "absorbed" by the *gravitino* ($\widetilde{G}$) [sometimes called $\widetilde{g}_{3/2}$ in the older literature], the spin-3/2 superpartner of the graviton [30]. By this super-Higgs mechanism, the goldstino is removed from the physical spectrum and the gravitino acquires a mass ($m_{3/2}$). In processes with center-of-mass energy $E \gg m_{3/2}$, the goldstino–gravitino equivalence theorem [31] states that the interactions of the helicity $\pm\frac{1}{2}$ gravitino (whose properties approximate those of the goldstino) dominate those of the helicity $\pm\frac{3}{2}$ gravitino. The interactions of gravitinos with with other light fields can be described by a low-energy effective Lagrangian that is determined by fundamental principles (see, *e.g.*, Ref. 32).

***I.2.3. Hidden sectors and the structure of supersymmetry breaking*** **[24]:** It is very difficult (perhaps impossible) to construct a realistic model of spontaneously-broken low-energy supersymmetry where the supersymmetry breaking arises solely as a consequence of the interactions of the particles of the MSSM. An alternative scheme posits a theory consisting of at least two distinct sectors: a *hidden* sector consisting of particles that are completely neutral with respect to the Standard Model gauge group, and a *visible* sector consisting of the particles of the MSSM. There are no renormalizable tree-level interactions between particles of the visible and hidden sectors. Supersymmetry breaking is assumed to originate in the hidden sector, and its effects are transmitted to the MSSM by some mechanism (often involving the mediation by particles that comprise an additional *messenger* sector). Two theoretical scenarios have been examined in detail: gravity-mediated and gauge-mediated supersymmetry breaking.

Supergravity models provide a natural mechanism for transmitting the supersymmetry breaking of the hidden sector to the particle spectrum of the MSSM. In models of *gravity-mediated* supersymmetry breaking, gravity is the messenger of supersymmetry breaking [33–35]. More precisely, supersymmetry breaking is mediated by effects of gravitational strength (suppressed by inverse powers of the Planck mass). In this scenario, the gravitino mass is of order the electroweak-symmetry-breaking scale, while its couplings are roughly gravitational in strength [2,36]. Such a gravitino typically plays no role in supersymmetric phenomenology at colliders (except perhaps indirectly in the case where the gravitino is the LSP [37]) .

In *gauge-mediated* supersymmetry breaking, gauge forces transmit the supersymmetry breaking to the MSSM. A typical structure of such models involves a hidden sector where supersymmetry is broken, a messenger sector consisting of particles (messengers) with SU(3)×SU(2)×U(1) quantum numbers, and the visible sector consisting of the fields of the MSSM [38–40]. The direct coupling of the messengers to the hidden sector generates a supersymmetry-breaking spectrum in the messenger sector. Finally, supersymmetry breaking is transmitted to the MSSM via the virtual exchange of the messengers. In models of *direct gauge mediation*, the supersymmetry-breaking sector includes fields that carry Standard Model quantum numbers, in which case no separate messenger sector is required [41].

The gravitino mass in models of gauge-mediated supersymmetry breaking is typically in the eV range (although in some cases it can be as large as a GeV), which implies that $\widetilde{G}$ is the LSP. In particular, the gravitino is a potential dark matter candidate (for a recent review and guide to the literature, see Ref. 17). The couplings of the helicity $\pm\frac{1}{2}$ components of $\widetilde{G}$ to the particles of the MSSM (which approximate those of the goldstino, cf. Section I.2.3) are significantly stronger than gravitational strength and amenable to experimental collider analyses.

The concept of a hidden sector is more general than supersymmetry. *Hidden valley* models [42] posit the existence of a hidden sector of new particles and interactions that are very weakly coupled to particles of the Standard Model. The impact of a hidden valley on supersymmetric phenomenology at colliders can be significant if the LSP lies in the valley sector [43].

***I.2.4. Supersymmetry and extra dimensions:***

Approaches to supersymmetry breaking have also been developed in the context of theories in which the number of space dimensions is greater than three. In particular, a number of supersymmetry-breaking mechanisms have been proposed that are inherently extra-dimensional [44]. The size of the extra dimensions can be significantly larger than $M_{\rm P}^{-1}$; in some cases on the order of $(\text{TeV})^{-1}$ or even larger [45,46].

For example, in one approach, the fields of the MSSM live on some brane (a lower-dimensional manifold embedded in a higher-dimensional spacetime), while the sector of the theory that breaks supersymmetry lives on a second-separated brane. Two examples of this approach are anomaly-mediated supersymmetry breaking of Ref. 47, and gaugino-mediated supersymmetry breaking of Ref. 48; in both cases supersymmetry breaking is transmitted through fields that live in the bulk (the higher-dimensional space between the two branes). This setup has some features in common with both gravity-mediated and gauge-mediated supersymmetry breaking (*e.g.*, a hidden and visible sector and messengers).

Alternatively, one can consider a higher-dimensional theory that is compactified to four spacetime dimensions. In this approach, supersymmetry is broken by boundary conditions on the compactified space that distinguish between fermions and bosons. This is the so-called Scherk-Schwarz mechanism [49]. The phenomenology of such models can be strikingly different from that of the usual MSSM [50]. All these extra-dimensional ideas clearly deserve further investigation, although they will not be discussed further here.

***I.2.5. Split-supersymmetry:*** If supersymmetry is not connected with the origin of the electroweak scale, string theory suggests that supersymmetry still plays a significant role in Planck-scale physics. However, it may still be possible that some remnant of the superparticle spectrum survives down to the TeV-scale or below. This is the idea of *split-supersymmetry* [51], in which supersymmetric scalar partners of the quarks and leptons are significantly heavier (perhaps by many orders of magnitude) than 1 TeV, whereas the fermionic partners of the gauge and Higgs bosons have masses on the order of 1 TeV or below (presumably protected by some chiral symmetry). With the exception of a single light neutral scalar whose properties are indistinguishable from those of the Standard Model Higgs boson, all other Higgs bosons are also taken to be very heavy.

The supersymmetry breaking required to produce such a scenario would destabilize the gauge hierarchy. In particular, split-supersymmetry cannot provide a natural explanation for the existence of the light Standard-Model-like Higgs boson, whose mass lies orders below the mass scale of the heavy scalars. Nevertheless, models of split-supersymmetry can account for the dark matter (which is assumed to be the LSP) and gauge coupling unification. Thus, there is some motivation for pursuing the phenomenology of such approaches [52]. One notable difference from the usual MSSM phenomenology is the existence of a long-lived gluino [53].

***I.3. Parameters of the MSSM:*** The parameters of the MSSM are conveniently described by considering separately the supersymmetry-conserving sector and the supersymmetry-breaking sector. A careful discussion of the conventions used in defining the tree-level MSSM parameters can be found in Ref. 54. For simplicity, consider first the case of one generation of quarks, leptons, and their scalar superpartners.

***I.3.1. The supersymmetric-conserving parameters:***

The parameters of the supersymmetry-conserving sector consist of: (i) gauge couplings: $g_s$, $g$, and $g'$, corresponding to the Standard Model gauge group SU(3)×SU(2)×U(1) respectively; (ii) a supersymmetry-conserving higgsino mass parameter $\mu$; and (iii) Higgs-fermion Yukawa coupling constants: $\lambda_u$, $\lambda_d$, and $\lambda_e$ (corresponding to the coupling of one generation of left- and right-handed quarks and leptons, and their superpartners to the Higgs bosons and higgsinos). Because there is no right-handed neutrino (and its superpartner) in the MSSM as defined here, one cannot introduce a Yukawa coupling $\lambda_\nu$.

***I.3.2. The supersymmetric-breaking parameters:***
The supersymmetry-breaking sector contains the following set of parameters: (i) gaugino Majorana masses $M_3$, $M_2$, and $M_1$ associated with the SU(3), SU(2), and U(1) subgroups of the Standard Model; (ii) five scalar squared-mass parameters for the squarks and sleptons, $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, and $M^2_{\widetilde{E}}$ [corresponding to the five electroweak gauge multiplets, *i.e.*, superpartners of $(u,d)_L$, $u^c_L$, $d^c_L$, $(\nu,\ e^-)_L$, and $e^c_L$, where the superscript $c$ indicates a charge-conjugated fermion and flavor indices are suppressed]; and (iii) Higgs-squark-squark and Higgs-slepton-slepton trilinear interaction terms, with coefficients $\lambda_u A_U$, $\lambda_d A_D$, and $\lambda_e A_E$ (which define the so-called "$A$-parameters"). It is traditional to factor out the Yukawa couplings in the definition of the $A$-parameters (originally motivated by a simple class of gravity-mediated supersymmetry-breaking models [2,4]). If the $A$-parameters defined in this way are parametrically of the same order (or smaller) as compared to other supersymmetry-breaking mass parameters, then only the $A$-parameters of the third generation will be phenomenologically relevant. Finally, we add: (iv) three scalar squared-mass parameters–two of which ($m_1^2$ and $m_2^2$) contribute to the diagonal Higgs squared-masses, given by $m_1^2+|\mu|^2$ and $m_2^2+|\mu|^2$, and a third which contributes to the off-diagonal Higgs squared-mass term, $m^2_{12} \equiv B\mu$ (which defines the "$B$-parameter").

The breaking of the electroweak symmetry SU(2)×U(1) to $\mathrm{U(1)_{EM}}$ is only possible after introducing the supersymmetry-breaking Higgs squared-mass parameters. Minimizing the resulting tree-level Higgs scalar potential, these three squared-mass parameters can be re-expressed in terms of the two Higgs vacuum expectation values, $v_d$ and $v_u$ (also called $v_1$ and $v_2$, respectively, in the literature), and the CP-odd Higgs mass $A^0$ (cf. Section I.5). Here, $v_d$ [$v_u$] is the vacuum expectation value of the neutral component of the Higgs field $H_d$ [$H_u$] that couples exclusively to down-type (up-type) quarks and leptons. Note that $v_d^2 + v_u^2 = 4m_W^2/g^2 \simeq (246\ \mathrm{GeV})^2$ is fixed by the $W$ mass and the gauge coupling, whereas the ratio

$$\tan\beta = v_u/v_d \tag{1}$$

is a free parameter of the model. By convention, the phases of the Higgs field are chosen such that $0 \leq \beta \leq \pi/2$. Equivalently, the tree-level conditions for the scalar potential minimum relate the diagonal and off-diagonal Higgs squared-masses in terms of $m_Z^2 = \frac{1}{4}(g^2 + g'^2)(v_d^2 + v_u^2)$, the angle $\beta$ and the CP-odd Higgs mass $m_A$:

$$\sin 2\beta = \frac{2m_{12}^2}{m_1^2 + m_2^2 + 2|\mu|^2} = \frac{2m_{12}^2}{m_A^2}\,, \tag{2}$$

$$\tfrac{1}{2}m_Z^2 = -|\mu|^2 + \frac{m_1^2 - m_2^2 \tan^2\beta}{\tan^2\beta - 1}\,. \tag{3}$$

Note that supersymmetry-breaking mass terms for the fermionic superpartners of scalar fields and non-holomorphic trilinear scalar interactions (*i.e.*, interactions that mix scalar fields and their complex conjugates) have not been included above in the soft-supersymmetry-breaking sector. These terms can potentially destabilize the gauge hierarchy [11] in models with a gauge-singlet superfield. The latter is not present in the MSSM; hence as noted in Ref. 12, these so-called non-standard soft-supersymmetry-breaking terms are benign. However, the coefficients of these terms (which have dimensions of mass) are expected to be significantly suppressed compared to the TeV-scale in a fundamental theory of supersymmetry-breaking. Consequently, we follow the usual approach and omit these terms from further consideration.

***I.3.3. MSSM-124:*** The total number of independent physical parameters that define the MSSM (in its most general form) is quite large, primarily due to the soft-supersymmetry-breaking sector. In particular, in the case of three generations of quarks, leptons, and their superpartners, $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, and $M^2_{\widetilde{E}}$ are hermitian $3\times 3$ matrices, and $A_U$, $A_D$, and $A_E$ are complex $3 \times 3$ matrices. In addition, $M_1$, $M_2$, $M_3$, $B$, and $\mu$ are, in general, complex. Finally, as in the Standard Model, the Higgs-fermion Yukawa couplings, $\lambda_f$ ($f = u$, $d$, and $e$), are complex $3 \times 3$ matrices that are related to the quark and lepton mass matrices via: $M_f = \lambda_f v_f/\sqrt{2}$, where $v_e \equiv v_d$ [with $v_u$ and $v_d$ as defined above Eq. (1)].

However, not all these parameters are physical. Some of the MSSM parameters can be eliminated by expressing interaction eigenstates in terms of the mass eigenstates, with an appropriate redefinition of the MSSM fields to remove unphysical degrees of freedom. The analysis of Ref. 55 shows that the MSSM possesses 124 independent parameters. Of these, 18 parameters correspond to Standard Model parameters (including the QCD vacuum angle $\theta_{\rm QCD}$), one corresponds to a Higgs sector parameter (the analogue of the Standard Model Higgs mass), and 105 are genuinely new parameters of the model. The latter include: five real parameters and three $CP$-violating phases in the gaugino/higgsino sector, 21 squark and slepton masses, 36 real mixing angles to define the squark and slepton mass eigenstates, and 40 $CP$-violating phases that can appear in squark and slepton interactions. The most general R-parity-conserving minimal supersymmetric extension of the Standard Model (without additional theoretical assumptions) will be denoted henceforth as MSSM-124 [56].

***I.4. The supersymmetric-particle spectrum:*** The supersymmetric particles (*sparticles*) differ in spin by half a unit from their Standard Model partners. The supersymmetric partners of the gauge and Higgs bosons are fermions, whose names are obtained by appending "ino" at the end of the corresponding Standard Model particle name. The gluino is the color-octet Majorana fermion partner of the gluon with mass $M_{\widetilde{g}} = |M_3|$. The supersymmetric partners of the electroweak gauge and Higgs bosons (the gauginos and higgsinos) can mix. As a result, the physical states of definite mass are model-dependent linear combinations of the charged and neutral gauginos and higgsinos, called *charginos* and *neutralinos*, respectively. Like the gluino, the neutralinos are also Majorana fermions, which provide for some distinctive phenomenological signatures [57,58]. The supersymmetric partners of the quarks and leptons are spin-zero bosons: the *squarks*, charged *sleptons*, and *sneutrinos*, respectively. A complete set of Feynman rules for the sparticles of the MSSM can be found in Ref. 59. The MSSM Feynman rules also are implicitly contained in a number of Feynman diagram and amplitude generation software packages (see *e.g.*, Refs. 60−62).

***I.4.1. The charginos and neutralinos:*** The mixing of the charged gauginos ($\widetilde{W}^{\pm}$) and charged higgsinos ($H_u^+$ and $H_d^-$) is described (at tree-level) by a 2×2 complex mass matrix [63–65]:

$$M_C \equiv \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g v_u \\ \frac{1}{\sqrt{2}} g v_d & \mu \end{pmatrix} . \tag{4}$$

To determine the physical chargino states and their masses, one must perform a singular value decomposition [66,67] of the complex matrix $M_C$:

$$U^* M_C V^{-1} = \mathrm{diag}(M_{\widetilde{\chi}_1^+}\,,\, M_{\widetilde{\chi}_2^+})\,, \tag{5}$$

where $U$ and $V$ are unitary matrices, and the right-hand side of Eq. (5) is the diagonal matrix of (non-negative) chargino masses. The physical chargino states are denoted by $\widetilde{\chi}_1^{\pm}$ and $\widetilde{\chi}_2^{\pm}$. These are linear combinations of the charged gaugino and higgsino states determined by the matrix elements of $U$ and $V$ [63–65]. The chargino masses correspond to the *singular values* [66] of $M_C$, *i.e.*, the positive square roots of the eigenvalues of $M_C^\dagger M_C$:

$$M^2_{\widetilde{\chi}_1^+,\widetilde{\chi}_2^+} = \tfrac{1}{2}\Big\{|\mu|^2 + |M_2|^2 + 2m_W^2 \mp \Big[\big(|\mu|^2 + |M_2|^2 + 2m_W^2\big)^2 - 4|\mu|^2|M_2|^2 - 4m_W^4 \sin^2 2\beta + 8m_W^2 \sin 2\beta\, \mathrm{Re}(\mu M_2)\Big]^{1/2}\Big\}, \tag{6}$$

where the states are ordered such that $M_{\widetilde{\chi}_1^+} \leq M_{\widetilde{\chi}_2^+}$.

It is convenient to choose a convention where $\tan\beta$ and $M_2$ are real and positive. Note that the relative phase of $M_2$ and $\mu$ is meaningful. (If $CP$-violating effects are neglected, then $\mu$ can be chosen real but may be either positive or negative.) The sign of $\mu$ is convention-dependent; the reader is warned that both sign conventions appear in the literature. The sign convention for $\mu$ in Eq. (4) is used by the LEP collaborations [68] in their plots of exclusion contours in the $M_2$ vs. $\mu$ plane derived from the non-observation of $e^+e^- \to \widetilde{\chi}_1^+\widetilde{\chi}_1^-$.

The mixing of the neutral gauginos ($\widetilde{B}$ and $\widetilde{W}^0$) and neutral higgsinos ($\widetilde{H}_d^0$ and $\widetilde{H}_u^0$) is described (at tree-level) by a $4\times 4$

complex symmetric mass matrix [63,64,69,70]:

$$M_N \equiv \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g'v_d & \frac{1}{2}g'v_u \\ 0 & M_2 & \frac{1}{2}gv_d & -\frac{1}{2}gv_u \\ -\frac{1}{2}g'v_d & \frac{1}{2}gv_d & 0 & -\mu \\ \frac{1}{2}g'v_u & -\frac{1}{2}gv_u & -\mu & 0 \end{pmatrix} . \qquad (7)$$

To determine the physical neutralino states and their masses, one must perform a Takagi-diagonalization [66,67,71,72] of the complex symmetric matrix $M_N$:

$$W^T M_N W = \mathrm{diag}(M_{\widetilde{\chi}_1^0}\,,\, M_{\widetilde{\chi}_2^0}\,,\, M_{\widetilde{\chi}_3^0}\,,\, M_{\widetilde{\chi}_4^0})\,, \qquad (8)$$

where $W$ is a unitary matrix and the right-hand side of Eq. (8) is the diagonal matrix of (non-negative) neutralino masses. The physical neutralino states are denoted by $\widetilde{\chi}_i^0$ ($i = 1, \ldots 4$), where the states are ordered such that $M_{\widetilde{\chi}_1^0} \leq M_{\widetilde{\chi}_2^0} \leq M_{\widetilde{\chi}_3^0} \leq M_{\widetilde{\chi}_4^0}$. The $\widetilde{\chi}_i^0$ are the linear combinations of the neutral gaugino and higgsino states determined by the matrix elements of $W$ (in Ref. 63, $W = N^{-1}$). The neutralino masses correspond to the singular values of $M_N$ (*i.e.*, the positive square roots of the eigenvalues of $M_N^\dagger M_N$). Exact formulae for these masses can be found in Refs. [69] and [73]. A numerical algorithm for determining the mixing matrix $W$ has been given by Ref. 74.

If a chargino or neutralino state approximates a particular gaugino or higgsino state, it is convenient to employ the corresponding nomenclature. Specifically, if $M_1$ and $M_2$ are small compared to $m_Z$ and $|\mu|$, then the lightest neutralino $\widetilde{\chi}_1^0$ would be nearly a pure *photino*, $\widetilde{\gamma}$, the supersymmetric partner of the photon. If $M_1$ and $m_Z$ are small compared to $M_2$ and $|\mu|$, then the lightest neutralino would be nearly a pure *bino*, $\widetilde{B}$, the supersymmetric partner of the weak hypercharge gauge boson. If $M_2$ and $m_Z$ are small compared to $M_1$ and $|\mu|$, then the lightest chargino pair and neutralino would constitute a triplet of roughly mass-degenerate pure *winos*, $\widetilde{W}^{\pm}$, and $\widetilde{W}_3^0$, the supersymmetric partners of the weak SU(2) gauge bosons. Finally, if $|\mu|$ and $m_Z$ are small compared to $M_1$ and $M_2$, then the lightest neutralino would be nearly a pure *higgsino*. Each of the above cases leads to a strikingly different phenomenology.

***I.4.2. The squarks, sleptons and sneutrinos:*** For a given fermion $f$, there are two supersymmetric partners, $\widetilde{f}_L$ and $\widetilde{f}_R$, which are scalar partners of the corresponding left- and right-handed fermion. (There is no $\widetilde{\nu}_R$ in the MSSM.) However, in general, $\widetilde{f}_L$ and $\widetilde{f}_R$ are not mass eigenstates, since there is $\widetilde{f}_L$–$\widetilde{f}_R$ mixing. For three generations of squarks, one must in general diagonalize $6 \times 6$ matrices corresponding to the basis $(\widetilde{q}_{iL}, \widetilde{q}_{iR})$, where $i = 1, 2, 3$ are the generation labels. For simplicity, only the one-generation case is illustrated in detail below. (The effects of second and third generation squark mixing can be significant and is treated in Ref. 75.)

Using the notation of the third family, the one-generation tree-level squark squared-mass matrix is given by [76]

$$M_F^2 = \begin{pmatrix} M_{\widetilde{Q}}^2 + m_q^2 + L_q & m_q X_q^* \\ m_q X_q & M_{\widetilde{R}}^2 + m_q^2 + R_q \end{pmatrix} , \qquad (9)$$

where

$$X_q \equiv A_q - \mu^* (\cot\beta)^{2T_{3q}} , \qquad (10)$$

and $T_{3q} = \frac{1}{2}$ $[-\frac{1}{2}]$ for $q = t$ $[b]$. The diagonal squared masses are governed by soft-supersymmetry-breaking squared masses $M_{\widetilde{Q}}^2$ and $M_{\widetilde{R}}^2 \equiv M_{\widetilde{U}}^2$ $[M_{\widetilde{D}}^2]$ for $q = t$ $[b]$, the corresponding quark masses $m_t$ $[m_b]$, and electroweak correction terms:

$$L_q \equiv (T_{3q} - e_q \sin^2\theta_W) m_Z^2 \cos 2\beta , \quad R_q \equiv e_q \sin^2\theta_W \, m_Z^2 \cos 2\beta , \qquad (11)$$

where $e_q = \frac{2}{3}$ $[-\frac{1}{3}]$ for $q = t$ $[b]$. The off-diagonal squared squark masses are proportional to the corresponding quark masses and depend on $\tan\beta$ [Eq. (1)], the soft-supersymmetry-breaking $A$-parameters and the higgsino mass parameter $\mu$. The signs of the $A$ and $\mu$ parameters are convention-dependent; other choices appear frequently in the literature. Due to the appearance of the *quark* mass in the off-diagonal element of the squark squared-mass matrix, one expects the $\widetilde{q}_L$–$\widetilde{q}_R$ mixing to be small, with the possible exception of the third generation, where mixing can be enhanced by factors of $m_t$ and $m_b \tan\beta$.

In the case of third generation $\widetilde{q}_L$–$\widetilde{q}_R$ mixing, the mass eigenstates (usually denoted by $\widetilde{q}_1$ and $\widetilde{q}_2$, with $m_{\widetilde{q}_1} < m_{\widetilde{q}_2}$) are determined by diagonalizing the $2 \times 2$ matrix $M_F^2$ given by

Eq. (9). The corresponding squared masses and mixing angle are given by [76]:

$$m^2_{\widetilde{q}_{1,2}} = \frac{1}{2}\left[\mathrm{Tr}\, M_F^2 \mp \sqrt{(\mathrm{Tr} M_F^2)^2 - 4 \det M_F^2}\right] ,$$

$$\sin 2\theta_{\widetilde{q}} = \frac{2m_q|X_q|}{m^2_{\widetilde{q}_2} - m^2_{\widetilde{q}_1}} \,. \tag{12}$$

The one-generation results above also apply to the charged sleptons, with the obvious substitutions: $q \to \tau$ with $T_{3\tau} = -\frac{1}{2}$ and $e_\tau = -1$, and the replacement of the supersymmetry-breaking parameters: $M^2_{\widetilde{Q}} \to M^2_{\widetilde{L}}$, $M^2_{\widetilde{D}} \to M^2_{\widetilde{E}}$, and $A_q \to A_\tau$. For the neutral sleptons, $\widetilde{\nu}_R$ does not exist in the MSSM, so $\widetilde{\nu}_L$ is a mass eigenstate.

In the case of three generations, the supersymmetry-breaking scalar-squared masses [$M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, $M^2_{\widetilde{L}}$, and $M^2_{\widetilde{E}}$] and the $A$-parameters that parameterize the Higgs couplings to up- and down-type squarks and charged sleptons (henceforth denoted by $A_U$, $A_D$, and $A_E$, respectively) are now $3 \times 3$ matrices as noted in Section I.3. The diagonalization of the $6 \times 6$ squark mass matrices yields $\widetilde{f}_{iL}$–$\widetilde{f}_{jR}$ mixing (for $i \neq j$). In practice, since the $\widetilde{f}_L$–$\widetilde{f}_R$ mixing is appreciable only for the third generation, this additional complication can often be neglected (although see Ref. 75 for examples in which the mixing between the second and third generations is relevant).

Radiative loop corrections will modify all tree-level results for masses quoted in this section. These corrections must be included in any precision study of supersymmetric phenomenology [77]. Beyond tree level, the definition of the supersymmetric parameters becomes convention-dependent. For example, one can define physical couplings or running couplings, which differ beyond the tree level. This provides a challenge to any effort that attempts to extract supersymmetric parameters from data. The Supersymmetry Les Houches Accord (SLHA) [78] has been adopted, which establishes a set of conventions for specifying generic file structures for supersymmetric model specifications and input parameters, supersymmetric mass and coupling spectra, and decay tables. These provide a universal interface between spectrum calculation programs, decay packages, and high energy physics event generators. Ultimately, these efforts will facilitate the reconstruction of the fundamental supersymmetric theory (and its breaking mechanism) from high-precision studies of supersymmetric phenomena at future colliders.

***I.5. The Higgs sector of the MSSM:*** Next, consider the MSSM Higgs sector [22,23,79]. Despite the large number of potential $CP$-violating phases among the MSSM-124 parameters, the tree-level MSSM Higgs sector is automatically $CP$-conserving. That is, unphysical phases can be absorbed into the definition of the Higgs fields such that $\tan\beta$ is a real parameter (conventionally chosen to be positive). Consequently, the physical neutral Higgs scalars are $CP$ eigenstates. The MSSM Higgs sector contains five physical spin-zero particles: a charged Higgs boson pair ($H^{\pm}$), two $CP$-even neutral Higgs bosons (denoted by $h^0$ and $H^0$ where $m_h < m_H$), and one $CP$-odd neutral Higgs boson ($A^0$).

***I.5.1 The Tree-level MSSM Higgs sector:*** The properties of the Higgs sector are determined by the Higgs potential, which is made up of quadratic terms [whose squared-mass coefficients were specified above Eq. (1)] and quartic interaction terms governed by dimensionless couplings. The quartic interaction terms are manifestly supersymmetric at tree level (although these are modified by supersymmetry-breaking effects at the loop level). In general, the quartic couplings arise from two sources: (i) the supersymmetric generalization of the scalar potential (the so-called "$F$-terms"), and (ii) interaction terms related by supersymmetry to the coupling of the scalar fields and the gauge fields, whose coefficients are proportional to the corresponding gauge couplings (the so-called "$D$-terms").

In the MSSM, $F$-term contributions to the quartic couplings are absent (although such terms may be present in extensions of the MSSM, *e.g.*, models with Higgs singlets). As a result, the strengths of the MSSM quartic Higgs interactions are fixed in terms of the gauge couplings. Due to the resulting constraint on the form of the two-Higgs-doublet scalar potential, all the tree-level MSSM Higgs-sector parameters depend only on two quantities: $\tan\beta$ [defined in Eq. (1)] and one Higgs mass usually taken to be $m_A$. From these two quantities, one can predict the values of the remaining Higgs boson masses, an angle $\alpha$ (which measures the component of the original $Y = \pm 1$ Higgs doublet states in the physical $CP$-even neutral scalars), and the Higgs boson self-couplings.

***I.5.2 The radiatively-corrected MSSM Higgs sector:*** When radiative corrections are incorporated, additional parameters of the supersymmetric model enter via virtual loops. The impact of these corrections can be significant [80]. For example, the tree-level MSSM-124 prediction for the upper bound of the lightest $CP$-even Higgs mass, $m_h \leq m_Z|\cos 2\beta| \leq m_Z$ [22,23], can be substantially modified when radiative corrections are included. The qualitative behavior of these radiative corrections can be most easily seen in the large top-squark mass limit, where in addition, both the splitting of the two diagonal entries and the two off-diagonal entries of the top-squark squared-mass matrix [Eq. (9)] are small in comparison to the average of the two top-squark squared masses, $M_{\rm S}^2 \equiv \frac{1}{2}(M_{\widetilde{t}_1}^2 + M_{\widetilde{t}_2}^2)$. In this case (assuming $m_A > m_Z$), the predicted upper bound for $m_h$ (which reaches its maximum at large $\tan\beta$) is approximately given by

$$m_h^2 \lesssim m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(M_{\rm S}^2/m_t^2\right) + \frac{X_t^2}{M_{\rm S}^2}\left(1 - \frac{X_t^2}{12M_{\rm S}^2}\right)\right], \quad (13)$$

where $X_t \equiv A_t - \mu\cot\beta$ is the top-squark mixing factor [see Eq. (9)].

A more complete treatment of the radiative corrections [81] shows that Eq. (13) somewhat overestimates the true upper bound of $m_h$. These more refined computations, which incorporate renormalization group improvement and the leading two-loop contributions, yield $m_h \lesssim 135$ GeV (with an accuracy of a few GeV) for $m_t = 175$ GeV and $M_S \lesssim 2$ TeV [81]. This Higgs-mass upper bound can be relaxed somewhat in non-minimal extensions of the MSSM, as noted in Section I.9.

In addition, one-loop radiative corrections can introduce $CP$-violating effects in the Higgs sector, which depend on some of the $CP$-violating phases among the MSSM-124 parameters [82]. Although these effects are more model-dependent, they can have a non-trivial impact on the Higgs searches at future colliders. A summary of the current MSSM Higgs mass limits can be found in Ref. 83.

***I.6. Restricting the MSSM parameter freedom:*** In Sections I.4 and I.5, we surveyed the parameters that comprise the MSSM-124. However, in its most general form, the MSSM-124 is not a phenomenologically-viable theory over most of its parameter space. This conclusion follows from the observation that a generic point in the MSSM-124 parameter space exhibits: (i) no conservation of the separate lepton numbers $L_e$, $L_\mu$, and $L_\tau$; (ii) unsuppressed flavor-changing neutral currents (FCNC's); and (iii) new sources of $CP$ violation that are inconsistent with the experimental bounds.

For example, the MSSM contains many new sources of $CP$ violation [84]. In particular, some combinations of the complex phases of the gaugino-mass parameters, the $A$-parameters, and $\mu$ must be less than on the order of $10^{-2}$–$10^{-3}$ (for a supersymmetry-breaking scale of 100 GeV) to avoid generating electric dipole moments for the neutron, electron, and atoms in conflict with observed data [85–87]. The non-observation of FCNC's [88–90] places additional strong constraints on the off-diagonal matrix elements of the squark and slepton soft-supersymmetry-breaking squared masses and $A$-parameters (see Section I.3.3). As a result of the phenomenological deficiencies listed above, almost the entire MSSM-124 parameter space is ruled out! This theory is viable only at very special "exceptional" regions of the full parameter space.

The MSSM-124 is also theoretically incomplete as it provides no explanation for the origin of the supersymmetry-breaking parameters (and in particular, why these parameters should conform to the exceptional points of the parameter space mentioned above). Moreover, there is no understanding of the choice of parameters that leads to the breaking of the electroweak symmetry. What is needed ultimately is a fundamental theory of supersymmetry breaking, which would provide a rationale for a set of soft-supersymmetry-breaking terms that is consistent with all phenomenological constraints.

The successful unification of the SU(3)×SU(2)×U(1) gauge couplings in supersymmetric grand unified theories [8,51,91,92] suggests the possibility that the high-energy structure of the theory may be considerable simpler than its low-energy realization. The desired phenomenological constraints of the low-energy theory can often be implemented by the dynamics which govern the more fundamental theory that resides at the high energy scale.

In this Section, we examine a number of theoretical frameworks that yield phenomenologically viable regions of the the general MSSM parameter space. The resulting supersymmetric particle spectrum is then a function of a relatively small number of input parameters. This is accomplished by imposing a simple structure on the soft-supersymmetry-breaking terms at a common high-energy scale $M_X$ (typically chosen to be the Planck scale, $M_{\rm P}$, the grand unification scale, $M_{\rm GUT}$, or the messenger scale, $M_{\rm mess}$). Using the renormalization group equations, one can then derive the low-energy MSSM parameters relevant for collider physics. The initial conditions (at the appropriate high-energy scale) for the renormalization group equations depend on the mechanism by which supersymmetry breaking is communicated to the effective low energy theory.

Examples of this scenario are provided by models of gravity-mediated and gauge-mediated supersymmetry breaking, to be discussed in more detail below. In some of these approaches, one of the diagonal Higgs squared-mass parameters is driven negative by renormalization group evolution [93]. In such models, electroweak symmetry breaking is generated radiatively, and the resulting electroweak symmetry-breaking scale is intimately tied to the scale of low-energy supersymmetry breaking.

***I.6.1. Gaugino mass unification:***

One prediction that arises in many grand unified supergravity models and gauge-mediated supersymmetry-breaking models is the unification of the (tree-level) gaugino mass parameters at some high-energy scale $M_{\rm X}$:

$$M_1(M_X) = M_2(M_X) = M_3(M_X) = m_{1/2}\,. \tag{14}$$

Consequently, the effective low-energy gaugino mass parameters (at the electroweak scale) are related:

$$M_3 = (g_s^2/g^2)M_2 \simeq 3.5 M_2\ , \qquad M_1 = (5g'^2/3g^2)M_2 \simeq 0.5 M_2. \tag{15}$$

In this case, the chargino and neutralino masses and mixing angles depend only on three unknown parameters: the gluino mass, $\mu$, and $\tan\beta$. If in addition $|\mu| \gg M_1 \gtrsim m_Z$, then the lightest neutralino is nearly a pure bino, an assumption often made in supersymmetric particle searches at colliders.

Although Eqs. (14) and (15) are often assumed in many phenomenological studies, a truly model-independent approach would take the gaugino mass parameters, $M_i$, to be independent parameters to be determined by experiment. For example, although LEP data yields a lower bound of 46 GeV on the mass of the lightest neutralino [94], an exactly massless neutralino *cannot* be ruled out today in a model-independent analysis [95].

It is possible that the tree-level masses for the gauginos are absent. In this case, the gaugino mass parameters arise at one-loop and do not satisfy Eq. (15). In supergravity, there exists a model-independent contribution to the gaugino mass whose origin can be traced to the super-conformal (super-Weyl) anomaly, which is common to all supergravity models [47]. Eq. (15) is then replaced (in the one-loop approximation) by:

$$M_i \simeq \frac{b_i g_i^2}{16\pi^2} m_{3/2}\,, \qquad (16)$$

where $m_{3/2}$ is the gravitino mass (assumed to be on the order of 1 TeV), and $b_i$ are the coefficients of the MSSM gauge beta-functions corresponding to the corresponding U(1), SU(2), and SU(3) gauge groups: $(b_1, b_2, b_3) = (\frac{33}{5}, 1, -3)$. Eq. (16) yields $M_1 \simeq 2.8M_2$ and $M_3 \simeq -8.3M_2$, which implies that the lightest chargino pair and neutralino comprise a nearly mass-degenerate triplet of winos, $\widetilde{W}^{\pm}$, $\widetilde{W}^0$ (c.f. Table 1), over most of the MSSM parameter space. (For example, if $|\mu| \gg m_Z$, then Eq. (16) implies that $M_{\widetilde{\chi}_1^{\pm}} \simeq M_{\widetilde{\chi}_1^0} \simeq M_2$ [96].)

The corresponding supersymmetric phenomenology differs significantly from the standard phenomenology based on Eq. (15), and is explored in detail in Ref. 97. Under certain theoretical assumptions on the structure of the Kähler potential (the so-called sequestered form introduced in Ref. 47), anomaly-mediated supersymmetry breaking also generates (approximate) flavor-diagonal squark and slepton mass matrices. This approach is called *anomaly-mediated* supersymmetry breaking (AMSB).

However in its simplest formulation, AMSB yields negative squared-mass contributions for the sleptons in the MSSM. It may be possible to cure this fatal flaw in approaches beyond the minimal supersymmetric model [98]. Alternatively, one can assume that anomaly-mediation is not the sole source of supersymmetry-breaking in the slepton sector.

Finally, it should be noted that the unification of gaugino masses (and scalar masses) can be accidental. In particular, the energy scale where unification takes place may not be directly related to any physical scale. This phenomenon has been called *mirage unification* and can occur in certain theories of fundamental supersymmetry-breaking [99].

***I.6.2. The constrained MSSM: mSUGRA, CMSSM, . . .*** In the *minimal* supergravity (mSUGRA) framework [2–4], a form of the Kähler potential is employed that yields minimal kinetic energy terms for the MSSM fields [100]. As a result, the soft-supersymmetry-breaking parameters at the high-energy scale $M_X$ take a particularly simple form in which the scalar squared masses and the $A$-parameters are flavor-diagonal and universal [34]:

$$
\begin{aligned}
&M^2_{\widetilde{Q}}(M_X) = M^2_{\widetilde{U}}(M_X) = M^2_{\widetilde{D}}(M_X) = m_0^2\mathbf{1}\,, \\
&M^2_{\widetilde{L}}(M_X) = M^2_{\widetilde{E}}(M_X) = m_0^2\mathbf{1}\,, \\
&m_1^2(M_X) = m_2^2(M_X) = m_0^2\,, \\
&A_U(M_X) = A_D(M_X) = A_E(M_X) = A_0\mathbf{1}\,, \qquad (17)
\end{aligned}
$$

where $\mathbf{1}$ is a $3 \times 3$ identity matrix in generation space. As in the Standard Model, this approach exhibits minimal flavor violation, whose unique source is the nontrivial flavor structure of the Higgs-fermion Yukawa couplings. The gaugino masses are also unified according to Eq. (14).

Renormalization group evolution is then used to derive the values of the supersymmetric parameters at the low-energy (electroweak) scale. For example, to compute squark masses, one must use the *low-energy* values for $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, and $M^2_{\widetilde{D}}$ in Eq. (9). Through the renormalization group running with boundary conditions specified in Eqs. (15) and (17), one can show that the low-energy values of $M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, and $M^2_{\widetilde{D}}$ depend primarily on $m_0^2$ and $m^2_{1/2}$. A number of useful approximate analytic expressions for superpartner masses in terms of the mSUGRA parameters can be found in Ref. 101.

In the mSUGRA approach, one typically finds that four flavors of squarks (with two squark eigenstates per flavor) and $\widetilde{b}_R$ are nearly mass-degenerate. The $\widetilde{b}_L$ mass and the diagonal $\widetilde{t}_L$ and $\widetilde{t}_R$ masses are reduced compared to the common squark mass of the first two generations. In addition, there are six flavors of nearly mass-degenerate sleptons (with two slepton eigenstates per flavor for the charged sleptons and one per flavor for the sneutrinos); the sleptons are expected to be somewhat lighter than the mass-degenerate squarks. Finally, third-generation squark masses and tau-slepton masses are sensitive to the strength of the respective $\widetilde{f}_L$–$\widetilde{f}_R$ mixing, as discussed below Eq. (9). The LSP is typically the lightest neutralino, $\widetilde{\chi}^0_1$, which is dominated by its bino component. In particular, mSUGRA parameter regimes in which the LSP is a chargino or the $\widetilde{\tau}_1$ (the lightest scalar superpartner of the $\tau$-lepton) are not phenomenologically viable.

One can count the number of independent parameters in the mSUGRA framework. In addition to 18 Standard Model parameters (excluding the Higgs mass), one must specify $m_0$, $m_{1/2}$, $A_0$, the Planck-scale values for $\mu$ and $B$-parameters (denoted by $\mu_0$ and $B_0$), and the gravitino mass $m_{3/2}$. Without additional model assumptions, $m_{3/2}$ is independent of the parameters that govern the mass spectrum of the superpartners of the Standard Model [34]. In principle, $A_0$, $B_0$, $\mu_0$, and $m_{3/2}$ can be complex, although in the mSUGRA approach, these parameters are taken (arbitrarily) to be real.

As previously noted, renormalization group evolution is used to compute the low-energy values of the mSUGRA parameters, which then fixes all the parameters of the low-energy MSSM. In particular, the two Higgs vacuum expectation values (or equivalently, $m_Z$ and $\tan\beta$) can be expressed as a function of the Planck-scale supergravity parameters. The simplest procedure is to remove $\mu_0$ and $B_0$ in favor of $m_Z$ and $\tan\beta$ [the sign of $\mu_0$, denoted sgn($\mu_0$) below, is not fixed in this process]. In this case, the MSSM spectrum and its interaction strengths are determined by five parameters:

$$m_0\,,\ A_0\,,\ m_{1/2}\,,\ \tan\beta\,,\ \text{and}\ \ \text{sgn}(\mu_0)\,, \qquad (18)$$

in addition to the 18 parameters of the Standard Model and an independent gravitino mass $m_{3/2}$. This framework is conventionally called the *constrained minimal sypersymmetric extension of the Standard Model* (CMSSM).

In the early literature, additional conditions were obtained by assuming a simplifying form for the hidden sector that provides the fundamental source of supersymmetry breaking. Two additional relations emerged among the mSUGRA parameters [100]: $B_0 = A_0 - m_0$ and $m_{3/2} = m_0$. These relations characterize a theory that was called minimal supergravity when first proposed. In the more recent literature, it has been more common to omit these extra conditions in defining the mSUGRA model (in which case the mSUGRA model and the CMSSM are synonymous). The authors of Ref. 102 advocate restoring the original nomenclature in which the mSUGRA model is defined with the extra conditions as originally proposed. Additional mSUGRA variations can be considered where different relations among the CMSSM parameters are imposed.

One can also relax the universality of scalar masses by decoupling the squared-masses of the Higgs bosons and the squarks/sleptons. This leads to the non-universal Higgs mass models (NUHM), thereby adding one or two new parameters to the CMSSM depending on whether the diagonal Higgs scalar squared-mass parameters ($m_1^2$ and $m_2^2$) are set equal (NUHM1) or taken to be independent (NUHM2) at the high energy scale $M_X^2$. Clearly, this modification preserves the minimal flavor violation of the mSUGRA approach. Nevertheless, the mSUGRA approach and its NUHM generalizations are probably too simplistic. Theoretical considerations suggest that the universality of Planck-scale soft-supersymmetry-breaking parameters is not generic [103]. In particular, effective operators at the Planck scale exist that do not respect flavor universality, and it is difficult to find a theoretical principle that would forbid them.

***I.6.3. Gauge-mediated supersymmetry breaking:*** In contrast to models of gravity-mediated supersymmetry breaking, the universality of the fundamental soft-supersymmetry-breaking squark and slepton squared-mass parameters is guaranteed in gauge-mediated supersymmetry breaking because the supersymmetry breaking is communicated to the sector of MSSM fields via gauge interactions [39,40]. In the minimal gauge-mediated supersymmetry-breaking (GMSB) approach, there is one effective mass scale, $\Lambda$, that determines all low-energy scalar and gaugino mass parameters through loop effects (while the resulting $A$-parameters are suppressed). In order that the resulting superpartner masses be on the order of 1 TeV or less, one must have $\Lambda \sim 100$ TeV. The origin of the $\mu$ and $B$-parameters is quite model-dependent, and lies somewhat outside the ansatz of gauge-mediated supersymmetry breaking. The simplest models of this type are even more restrictive than the CMSSM, with two fewer degrees of freedom. Benchmark reference points for GMSB models have been proposed in Ref. 104 to facilitate collider studies.

The minimal GMSB is not a fully realized model. The sector of supersymmetry-breaking dynamics can be very complex, and no complete model of gauge-mediated supersymmetry yet exists that is both simple and compelling. However, advances in the theory of dynamical supersymmetry breaking (which exploit the existence of metastable supersymmetry-breaking vacua in broad classes of models [105]) have generated new ideas and opportunities for model building. As a result, simpler models of successful gauge mediation of supersymmetry breaking have been achieved with the potential for overcoming a number of long-standing theoretical challenges [106]. In addition, model-independent techniques that encompass all known gauge mediation models have been recently formulated [107]. These methods are well-suited for a comprehensive analysis [108] of the phenomenological profile of gauge-mediated supersymmetry breaking.

It was noted in Section I.2 that the gravitino is the LSP in GMSB models. As a result, the next-to-lightest supersymmetric particle (NLSP) now plays a crucial role in the phenomenology of supersymmetric particle production and decays. Note that unlike the LSP, the NLSP can be charged. In GMSB models, the most likely candidates for the NLSP are $\widetilde{\chi}_1^0$ and $\widetilde{\tau}_R^{\pm}$. The NLSP will decay into its superpartner plus a gravitino (*e.g.*, $\widetilde{\chi}_1^0 \rightarrow \gamma\widetilde{G}$, $\widetilde{\chi}_1^0 \rightarrow Z\widetilde{G}$, or $\widetilde{\tau}_R^{\pm} \rightarrow \tau^{\pm}\widetilde{G}$), with lifetimes and branching ratios that depend on the model parameters.

Different choices for the identity of the NLSP and its decay rate lead to a variety of distinctive supersymmetric phenomenologies [40,109]. For example, a long-lived $\widetilde{\chi}_1^0$-NLSP that decays outside collider detectors leads to supersymmetric decay chains with missing energy in association with leptons and/or hadronic jets (this case is indistinguishable from the standard phenomenology of the $\widetilde{\chi}_1^0$-LSP). On the other hand, if $\widetilde{\chi}_1^0 \rightarrow \gamma\widetilde{G}$ is the dominant decay mode, and the decay occurs inside the detector, then nearly *all* supersymmetric particle decay chains would contain a photon. In contrast, in the case of a $\widetilde{\tau}_R^{\pm}$-NLSP, the $\widetilde{\tau}_R^{\pm}$ would either be long-lived or would decay inside the detector into a $\tau$-lepton plus missing energy.

***I.6.4. The phenomenological MSSM:*** Of course, any of the theoretical assumptions described in this Section could be wrong and must eventually be tested experimentally. To facilitate the exploration of MSSM phenomena in a more model-independent way while respecting the constraints noted at the beginning of this Section, the phenomenological MSSM (pMSSM) has been introduced [110].

The pMSSM is governed by 19 independent real parameters beyond the Standard Model, which include the three gaugino masses $M_1$, $M_2$ and $M_3$, the Higgs sector parameters $m_A$ and $\tan\beta$, the Higgsino mass parameter $\mu$, five squark and slepton squared-mass parameters for the degenerate first and second generations ($M^2_{\widetilde{Q}}$, $M^2_{\widetilde{U}}$, $M^2_{\widetilde{D}}$, ($M^2_{\widetilde{L}}$ and $M^2_{\widetilde{E}}$), the five corresponding squark and slepton squared-mass parameters for the third generation, and three third-generation $A$-parameters ($A_t$, $A_b$ and $A_\tau$). Note that the first and second generation $A$-parameters can be neglected as their phenomenological consequences are negligible. Search strategies at the LHC for the more general pMSSM have been examined in Ref. 111.

If supersymmetric phenomena are discovered, the measurements of (low-energy) supersymmetric parameters may eventually provide sufficient information to determine the organizing principle governing supersymmetry breaking and yield significant constraints on the values of the fundamental (high-energy) supersymmetric parameters. In particular, a number of sophisticated techniques have been recently developed for analyzing experimental data to test the viability of the particular supersymmetric framework and for measuring the fundamental model parameters and their uncertainties [112].

***I.7. Experimental data confronts the MSSM:***

Suppose some version of the MSSM satisfies the phenomenological constraints addressed in Section I.6. What are the expectations for the magnitude of the parameters that define such a model, and are these expectations consistent with present experimental data? For details on the constraints on supersymmetric particle masses from previous collider studies at LEP and the Tevatron and the most recent constraints from LHC data, see Ref. 94. Additional constraints arise from limits on the contributions of virtual supersymmetric particle exchange to a variety of Standard Model processes [88–90].

Recent LHC data has been especially effective in ruling out the existence of colored supersymmetric particles (primarily the gluino and the first two generations of squarks) with masses below about 1 TeV in the CMSSM [113]. However, such constraints are relaxed, in some cases by as much as a factor of two, in more generic frameworks of the MSSM [114].

***I.7.1 Naturalness constraints and the little hierarchy***

In Section I, weak-scale supersymmetry was motivated as a natural solution to the hierarchy problem, which could provide an understanding of the origin of the electroweak symmetry-breaking scale without a significant fine-tuning of the fundamental MSSM parameters. In this framework, the soft-supersymmetry-breaking masses must be generally of the order of 1 TeV or below [115]. This requirement is most easily seen in the determination of $m_Z$ by the scalar potential minimum condition. In light of Eq. (3), to avoid the fine-tuning of MSSM parameters, the soft-supersymmetry breaking squared-masses

$m_1^2$ and $m_2^2$ and the higgsino squared-mass $|\mu|^2$ should all be roughly of $\mathcal{O}(m_Z^2)$. Many authors have proposed quantitative measures of fine-tuning [115,116]. One of the simplest measures is the one given by Barbieri and Giudice [115],

$$\Delta_i \equiv \left|\frac{\partial \ln m_Z^2}{\partial \ln p_i}\right| , \qquad \Delta \equiv \max\ \Delta_i \,, \tag{19}$$

where the $p_i$ are the MSSM parameters at the high-energy scale $M_X$, which are set by the fundamental supersymmetry-breaking dynamics. The theory is more fine-tuned as $\Delta$ becomes larger.

One can apply the fine-tuning measure to any explicit model of supersymmetry-breaking. For example, in the approaches discussed in Section I.6, the $p_i$ are parameters of the model at the energy scale $M_X$ where the soft-supersymmetry breaking operators are generated by the dynamics of supersymmetry breaking. Renormalization group evolution then determines the values of the parameters appearing in Eq. (3) at the electroweak scale. In this way, $\Delta$ is sensitive to all the supersymmetry-breaking parameters of the model (see e.g. Ref. 117).

Consequently, there is a tension between the present experimental lower limits on the masses of colored supersymmetric particles [118] and the expectation that supersymmetry-breaking is associated with the electroweak symmetry-breaking scale. Moreover, this tension is exacerbated [119,120] by the experimental lower Higgs mass bound ($m_h \gtrsim 115$ GeV) [83], which is not far from the the MSSM upper bound ($m_h \lesssim 135$ GeV) [the dependence of the latter on the top-squark mass and mixing was noted in Section I.5.2]. If $M_{\rm SUSY}$ characterizes the scale of supersymmetric particle masses, then one would expect $\Delta \sim M_{\rm SUSY}^2/m_Z^2$. For example, if $M_{\rm SUSY} \sim 1$ TeV then there must be at least a $\Delta^{-1} \sim 1\%$ fine-tuning of the MSSM parameters to achieve the observed value of the $Z$ mass. This separation of the electroweak symmetry breaking and supersymmetry breaking scales is an example of the *little hierarchy problem* [119,121].

However, one must be very cautious when drawing conclusions about the viability of weak-scale supersymmetry to explain the origin of electroweak symmetry breaking. First, one must decide the largest tolerable value of $\Delta$ within the framework of weak-scale supersymmetry (should it be $\Delta \sim 10$? 100? 1000?). Second, the fine-tuning parameter $\Delta$ depends quite sensitively on the assumptions of the supersymmetry-breaking dynamics (e.g. the value of $M_X$ and relations among supersymmetry-breaking parameters in the fundamental high energy theory).

For example, in so-called focus point supersymmetry models [122], all squark masses can be as heavy as 5 TeV *without* significant fine-tuning. This can be attributed to a focusing behavior of the renormalization group evolution when certain relations hold among the high-energy values of the scalar squared-mass supersymmetry-breaking parameters. In this approach, the mass of the light CP-even Higgs boson can naturally be near its maximally allowed MSSM upper bound [123]. A recent reanalysis of focus-point and related models with modest fine-tuning in the context of CMSSM can be found in Ref. 124.

Among the colored superpartners, the third generation squarks generically have the most significant impact on the naturalness constraints [125], whereas their masses are the least constrained by LHC data. Hence, in the absence of any relation between third generation squarks and those of the first two generations, the naturalness constraints due to present LHC data can be considerably weaker than those obtained in the CMSSM. Indeed, models with first and second generation squark masses in the multi-TeV range do not generically require significant fine tuning. Such models have the added benefit that undesirable FCNCs mediated by squark exchange are naturally suppressed [126]. Other MSSM mass spectra that are compatible with moderate fine tuning have been investigated in Ref. 127. Moreover, one can also consider extensions of the MSSM in which the degree of fine-tuning is relaxed [128].

Finally, experimentally reported upper limits for supersymmetric particle masses are rarely model-independent. For example, mass limits for the gluino and the first and second generation squarks obtained under the assumption of the CMSSM can often be evaded in alternative or extended MSSM models, e.g., compressed supersymmetry [129] and stealth supersymmetry [130]. Moreover, experimental limits on the masses for the third generation squarks and color-neutral supersymmetric particles are less constrained than the masses of other colored supersymmetric states. The simplified models approach [131] is sometimes advertised as being more model-independent by focusing narrowly on a specific generic production process and decay chain. However this approach also depends on assumptions of the relative masses of the produced particle and decay products and the lack of interference from competing processes.

Thus, it is certainly premature in the first few years of the LHC era to conclude that weak scale supersymmetry is on the verge of exclusion.

### *I.7.2 Constraints from virtual exchange of supersymmetric particles*

There are a number of low-energy measurements that are sensitive to the effects of new physics through supersymmetric loop effects. For example, the virtual exchange of supersymmetric particles can contribute to the muon anomalous magnetic moment, $a_\mu \equiv \frac{1}{2}(g-2)_\mu$ [132], The Standard Model prediction for $a_\mu$ exhibits a $3.3\sigma$ deviation from the experimentally observed value [133], although a very recent theoretical re-analysis claims that the deviation exceeds $4\sigma$ [134].

The rare inclusive decay $b \to s\gamma$ also provides a sensitive probe to the virtual effects of new physics beyond the Standard Model. Experimental measurements of $B \to X_s + \gamma$ by the BELLE collaboration [135] are in very good agreement with the theoretical predictions of Ref. 136. In both cases, supersymmetric corrections can contribute an observable shift from the Standard Model prediction in some regions of the MSSM parameter space [137,138].

The rare decay $B_s \to \mu^+\mu^-$ is especially sensitive to supersymmetric loop effects, with some loop contributions that scale as $\tan^6\beta$ when $\tan\beta \gg 1$ [139]. Current experimental limits [140] are within about a factor of five of the predicted Standard Model rate. The absence of a *significant* deviation in these and other $B$-physics observables from their Standard Model predictions places interesting constraints on the low-energy supersymmetry parameters [141].

***I.8. Massive neutrinos in low-energy supersymmetry:*** In the minimal Standard Model and its supersymmetric extension, there are no right-handed neutrinos, and Majorana mass terms for the left-handed neutrinos are absent. However, given the overwhelming evidence for neutrino masses and mixing [142,143], any viable model of fundamental particles must provide a mechanism for generating neutrino masses [144]. In extended supersymmetric models, various mechanisms exist for producing massive neutrinos [145]. Although one can devise models for generating massive Dirac neutrinos [146], the most common approaches for incorporating neutrino masses are based on $L$-violating supersymmetric extensions of the MSSM, which generate massive Majorana neutrinos. Two classes of $L$-violating supersymmetric models will now be considered.

***I.8.1. The supersymmetric seesaw:*** Neutrino masses can be incorporated into the Standard Model by introducing SU(3)×SU(2)×U(1) singlet right-handed neutrinos ($\nu_R$) and super-heavy Majorana masses (typically on the order of a grand unified mass) for the $\nu_R$. In addition, one must also include a standard Yukawa couplings between the lepton doublets, the Higgs doublet, and the $\nu_R$. The Higgs vacuum expectation value then induces an off-diagonal $\nu_L$–$\nu_R$ masses on the order of the electroweak scale. Diagonalizing the neutrino mass matrix (in the three-generation model) yields three superheavy neutrino states, and three very light neutrino states that are identified as the light neutrino states observed in nature. This is the seesaw mechanism [147].

The supersymmetric generalization of the seesaw model of neutrino masses is now easily constructed [148,149]. In the seesaw-extended Standard Model, lepton number is broken due to the presence of $\Delta L = 2$ terms in the Lagrangian (which include the Majorana mass terms for the light and superheavy neutrinos). Consequently, the seesaw-extended MSSM conserves R-parity. The supersymmetric analogue of the Majorana neutrino mass term in the sneutrino sector leads to sneutrino–antisneutrino mixing phenomena [149,150].

***I.8.2. R-parity-violating supersymmetry:*** A second approach to incorporating massive neutrinos in supersymmetric models is to retain the minimal particle content of the MSSM, while removing the assumption of R-parity invariance [152]. The most general R-parity-violating (RPV) model involving the MSSM spectrum introduces many new parameters to both the supersymmetry-conserving and the supersymmetry-breaking sectors. Each new interaction term violates either $B$ or $L$ conservation. For example, consider new scalar-fermion Yukawa couplings derived from the following interactions:

$$(\lambda_L)_{pmn}\widehat{L}_p\widehat{L}_m\widehat{E}^c_n+(\lambda'_L)_{pmn}\widehat{L}_p\widehat{Q}_m\widehat{D}^c_n+(\lambda_B)_{pmn}\widehat{U}^c_p\widehat{D}^c_m\widehat{D}^c_n\,, \quad (20)$$

where $p$, $m$, and $n$ are generation indices, and gauge group indices are suppressed. In the notation above, $\widehat{Q}$, $\widehat{U}^c$, $\widehat{D}^c$, $\widehat{L}$, and $\widehat{E}^c$ respectively represent $(u,d)_L$, $u^c_L$, $d^c_L$, $(\nu,\, e^-)_L$, and $e^c_L$ and the corresponding superpartners.

The Yukawa interactions are obtained from Eq. (20) by taking all possible combinations involving two fermions and one scalar superpartner. Note that the term in Eq. (20) proportional to $\lambda_B$ violates $B$, while the other two terms violate $L$. Even if all the terms of Eq. (20) are absent, there is one more possible supersymmetric source of R-parity violation. In the notation of Eq. (20), one can add a term of the form $(\mu_L)_p\widehat{H}_u\widehat{L}_p$, where $\widehat{H}_u$ represents the $Y=1$ Higgs doublet and its higgsino superpartner. This term is the RPV generalization of the supersymmetry-conserving Higgs mass parameter $\mu$ of the MSSM, in which the $Y=-1$ Higgs/higgsino super-multiplet $\widehat{H}_d$ is replaced by the slepton/lepton super-multiplet $\widehat{L}_p$. The RPV-parameters $(\mu_L)_p$ also violate $L$.

Phenomenological constraints derived from data on various low-energy $B$- and $L$-violating processes can be used to establish limits on each of the coefficients $(\lambda_L)_{pmn}$, $(\lambda'_L)_{pmn}$, and $(\lambda_B)_{pmn}$ taken one at a time [152,153]. If more than one coefficient is simultaneously non-zero, then the limits are, in general, more complicated [154]. All possible RPV terms cannot be simultaneously present and unsuppressed; otherwise the proton decay rate would be many orders of magnitude larger than the present experimental bound. One way to avoid proton decay is to impose $B$ or $L$ invariance (either one alone would suffice).

Otherwise, one must accept the requirement that certain RPV coefficients must be extremely suppressed.

One particularly interesting class of RPV models is one in which $B$ is conserved, but $L$ is violated. It is possible to enforce baryon number conservation, while allowing for lepton-number-violating interactions by imposing a discrete $\mathbf{Z_3}$ baryon *triality* symmetry on the low-energy theory [155], in place of the standard $\mathbf{Z_2}$ R-parity. Since the distinction between the Higgs and matter super-multiplets is lost in RPV models, R-parity violation permits the mixing of sleptons and Higgs bosons, the mixing of neutrinos and neutralinos, and the mixing of charged leptons and charginos, leading to more complicated mass matrices and mass eigenstates than in the MSSM. Recent attempts to fit neutrino masses and mixing in this framework can be found in Ref. 151.

The supersymmetric phenomenology of the RPV models exhibits features that are quite distinct from that of the MSSM [152]. The LSP is no longer stable, which implies that not all supersymmetric decay chains must yield missing-energy events at colliders. Nevertheless, the loss of the missing-energy signature is often compensated by other striking signals (which depend on which R-parity-violating parameters are dominant). For example, supersymmetric particles in RPV models can be singly produced (in contrast to R-parity-conserving models where supersymmetric particles must be produced in pairs). The phenomenology of pair-produced supersymmetric particles is also modified in RPV models due to new decay chains not present in R-parity-conserving supersymmetry [152].

In RPV models with lepton number violation (these include low-energy supersymmetry models with baryon triality mentioned above), both $\Delta L = 1$ and $\Delta L = 2$ phenomena are allowed, leading to neutrino masses and mixing [156], neutrinoless double-beta decay [157], sneutrino-antisneutrino mixing [158], $s$-channel resonant production of sneutrinos in $e^+e^-$ collisions [159] and charged sleptons in $p\bar{p}$ and $pp$ collisions [160].

***I.9. Extensions beyond the MSSM:*** Extensions of the MSSM have been proposed to solve a variety of theoretical problems. One such problem involves the $\mu$ parameter of the

MSSM. Although $\mu$ is a supersymmetric-*preserving* parameter, it must be of order the supersymmetry-breaking scale to yield a consistent supersymmetric phenomenology. In the MSSM, one must devise a theoretical mechanism to guarantee that the magnitude of $\mu$ is not larger than the TeV-scale (*e.g.*, in gravity-mediated supersymmetry, the Giudice-Masiero mechanism of Ref. 161 is the most cited explanation).

In extensions of the MSSM, new compelling solutions to the so-called $\mu$-problem are possible. For example, one can replace $\mu$ by the vacuum expectation value of a new SU(3)×SU(2)×U(1) singlet scalar field. In such a model, the Higgs sector of the MSSM is enlarged and the corresponding fermionic higgsino superpartner is added. This is the so-called NMSSM (here, NM stands for non-minimal) [162]. There are some advantages to extending the model further by adding an additional U(1) broken gauge symmetry [163] (which yields the USSM [72]) .

Non-minimal extensions of the MSSM involving additional matter and/or Higgs super-multiplets can also yield a less restrictive bound on the mass of the lightest Higgs boson (as compared to the bound quoted in Section I.5.2). For example, MSSM-extended models consistent with gauge coupling unification can be constructed in which the upper limit on the lightest Higgs boson mass can be as high as 200—300 GeV [164] (a similar relaxation of the Higgs mass bound occurs in split supersymmetry [165] and extra-dimensional scenarios [166]) .

Other MSSM extensions considered in the literature include an enlarged electroweak gauge group beyond SU(2)×U(1) [167]; and/or the addition of new, possibly exotic, matter supermultiplets (*e.g.*, new U(1) gauge groups and a vector-like color triplet with electric charge $\frac{1}{3}e$ that appear as low-energy remnants in $E_6$ grand unification models [168]) . A possible theoretical motivation for such new structures arises from the study of phenomenologically viable string theory ground states [169].

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