V_{ud}, V_{us}, THE CABIBBO ANGLE, AND CKM UNITARITY

Updated November 2009 by E. Blucher (Univ. of Chicago) and W.J. Marciano (BNL)

The Cabibbo-Kobayashi-Maskawa (CKM) [1,2] three-generation quark mixing matrix written in terms of the Wolfenstein parameters (λ, A, ρ, η) [3] nicely illustrates the orthonormality constraint of unitarity and central role played by λ.

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\ . \quad (1)$$

That cornerstone is a carryover from the two-generation Cabibbo angle, $\lambda = \sin(\theta_{\rm Cabibbo}) = V_{us}$. Its value is a critical ingredient in determinations of the other parameters and in tests of CKM unitarity.

Unfortunately, the precise value of λ has been somewhat controversial in the past, with kaon decays suggesting [4] $\lambda \simeq 0.220$, while hyperon decays [5] and indirect determinations via nuclear β-decays imply a somewhat larger $\lambda \simeq 0.225 - 0.230$. That discrepancy is often discussed in terms of a deviation from the unitarity requirement

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \quad (2)$$

For many years, using a value of V_{us} derived from $K \to \pi e \nu$ (K_{e3}) decays, that sum was consistently 2–2.5 sigma below unity, a potential signal [6] for new physics effects. Below, we discuss the current status of V_{ud}, V_{us}, and their associated unitarity test in Eq. (2). (Since $|V_{ub}|^2 \simeq 1 \times 10^{-5}$ is negligibly small, it is ignored in this discussion.)

V_{ud}

The value of V_{ud} has been obtained from superallowed nuclear, neutron, and pion decays. Currently, the most precise determination of V_{ud} comes from superallowed nuclear beta-decays [6] ($0^+ \to 0^+$ transitions). Measuring their half-lives, t,

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and Q values which give the decay rate factor, f, leads to a precise determination of V_{ud} via the master formula [7–9]

$$|V_{ud}|^2 = \frac{2984.48(5)\,\mathrm{sec}}{ft(1+\mathrm{RC})} \tag{3}$$

where RC denotes the entire effect of electroweak radiative corrections, nuclear structure, and isospin violating nuclear effects. RC is nucleus-dependent, ranging from about +3.0% to +3.6% for the best measured superallowed decays. The most recent analysis of Hardy and Towner [10, 11] gives a weighted average (with errors combined in quadrature) of

$$V_{ud} = 0.97425(22)\ \text{(superallowed)}\ , \tag{4}$$

which, assuming unitarity, corresponds to $\lambda = 0.2255(10)$. The new average value of V_{ud} is shifted upward compared to our 2007 value of 0.97418(27) primarily because of improvements in the experimental ft values and nuclear isospin breaking corrections employed. We note, however, that the possibility of additional nuclear coulombic corrections has been raised recently [12].

Combined measurements of the neutron lifetime, τ_n, and the ratio of axial-vector/vector couplings, $g_A \equiv G_A/G_V$, via neutron decay asymmetries can also be used to determine V_{ud}:

$$|V_{ud}|^2 = \frac{4908.7(1.9)\ \mathrm{sec}}{\tau_n(1+3g_A^2)}, \tag{5}$$

where the error stems from uncertainties in the electroweak radiative corrections [8] due to hadronic loop effects. Those effects have been recently updated and their error was reduced by about a factor of 2 [9], leading to a ±0.0002 theoretical uncertainty in V_{ud} (common to all V_{ud} extractions). Using the world averages from this *Review*

$$\begin{aligned} \tau_n^{\mathrm{ave}} &= 885.7(8)\ \mathrm{sec} \\ g_A^{\mathrm{ave}} &= 1.2695(29) \end{aligned} \tag{6}$$

leads to

$$V_{ud} = 0.9746(4)_{\tau_n}(18)_{g_A}(2)_{\mathrm{RC}} \tag{7}$$

with the error dominated by g_A uncertainties (which have been expanded due to experimental inconsistencies). We note that a recent precise measurement [13] of $\tau_n = 878.5(7)(3)$ sec is also inconsistent with the world average from this *Review* and would lead to a considerably larger $V_{ud} = 0.9786(4)(18)(2)$. Alternatively, accepting the recent shorter lifetime measurement as correct, and employing it along with the value of V_{ud} in Eq. (4), leads to $g_A = 1.2763(7)$, which is outside of the range of Eq. (6) but in good accord with the most recent direct measurements of g_A [14]. Future neutron studies are expected to resolve these inconsistencies and significantly reduce the uncertainties in g_A and τ_n, potentially making them the best way to determine V_{ud}.

The recently completed PIBETA experiment at PSI measured the very small ($\mathcal{O}(10^{-8})$) branching ratio for $\pi^+ \to \pi^o e^+ \nu_e$ with about ±1/2% precision. Their result gives [15]

$$V_{ud} = 0.9749(26) \left[\frac{BR(\pi^+ \to e^+ \nu_e(\gamma))}{1.2352 \times 10^{-4}} \right]^{\frac{1}{2}} \tag{8}$$

which is normalized using the very precisely determined theoretical prediction for $BR(\pi^+ \to e^+\nu_e(\gamma)) = 1.2352(5) \times 10^{-4}$ [7], rather than the experimental branching ratio from this *Review* of $1.230(4)\times 10^{-4}$ which would lower the value to $V_{ud} = 0.9728(30)$. Theoretical uncertainties in that determination are very small; however, much higher statistics would be required to make this approach competitive with others.

V_{us}

$|V_{us}|$ may be determined from kaon decays, hyperon decays, and tau decays. Previous determinations have most often used $K\ell 3$ decays:

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta_K^\ell + \delta_{SU2}) C^2 \left| V_{us} \right|^2 f_+^2(0) I_K^\ell . \tag{9}$$

Here, ℓ refers to either e or μ, G_F is the Fermi constant, M_K is the kaon mass, S_{EW} is the short-distance radiative correction, δ_K^ℓ is the mode-dependent long-distance radiative correction, $f_+(0)$ is the calculated form factor at zero momentum transfer for the $\ell\nu$ system, and I_K^ℓ is the phase-space integral, which depends on measured semileptonic form factors. For charged kaon decays, δ_{SU2} is the deviation from one of the ratio of

$f_+(0)$ for the charged to neutral kaon decay; it is zero for the neutral kaon. C^2 is 1 (1/2) for neutral (charged) kaon decays. Most determinations of $|V_{us}|$ have been based only on $K \to \pi e \nu$ decays; $K \to \pi \mu \nu$ decays have not been used because of large uncertainties in I_K^μ. The experimental measurements are the semileptonic decay widths (based on the semileptonic branching fractions and lifetime) and form factors (allowing calculation of the phase space integrals). Theory is needed for S_{EW}, δ_K^ℓ, δ_{SU2}, and $f_+(0)$.

Many new measurements during the last few years have resulted in a significant shift in V_{us}. Most importantly, recent measurements of the $K \to \pi e \nu$ branching fractions are significantly different than earlier PDG averages, probably as a result of inadequate treatment of radiation in older experiments. This effect was first observed by BNL E865 [16] in the charged kaon system and then by KTeV [17,18] in the neutral kaon system; subsequent measurements were made by KLOE [19–22], NA48 [23–25], and ISTRA+ [26]. Current averages (*e.g.*, by the PDG [27] or Flavianet [28]) of the semileptonic branching fractions are based only on recent, high-statistics experiments where the treatment of radiation is clear. In addition to measurements of branching fractions, new measurements of lifetimes [29] and form factors [30–34], have resulted in improved precision for all of the experimental inputs to V_{us}. Precise measurements of form factors for $K_{\mu 3}$ decay now make it possible to use both semileptonic decay modes to extract V_{us}.

Following the analysis of the Flavianet group [28], one finds the values of $|V_{us}|f_+(0)$ in Table 1. The average of these measurements gives

$$f_+(0)|V_{us}| = 0.21664(48). \tag{10}$$

Figure 1 shows a comparison of these results with the PDG evaluation from 2002 [35], as well as $f_+(0)(1-|V_{ud}|^2-|V_{ub}|^2)^{1/2}$, the expectation for $f_+(0)|V_{us}|$ assuming unitarity, based on $|V_{ud}| = 0.9742 \pm 0.0003$, $|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$, and the lattice calculation of $f_+(0) = 0.9644 \pm 0.0049$ [36](Lattice calculations of $f_+(0)$ have improved significantly in recent years, and therefore replace the classic calculation of Leutwyler and Roos [37].) Combining the result in Eq. (10) with the above value of $f_+(0)$ gives

$$|V_{us}| = \lambda = 0.2246 \pm 0.0012. \tag{11}$$

Table 1: $|V_{us}|f_+(0)$ from $K_{\ell 3}$.

Decay Mode	$\lvert V_{us}\rvert f_+(0)$
$K^{\pm}e3$	0.2173 ± 0.0008
$K^{\pm}\mu3$	0.2176 ± 0.0011
K_Le3	0.2163 ± 0.0006
$K_L\mu3$	0.2168 ± 0.0007
K_Se3	0.2154 ± 0.0013
Average	0.2166 ± 0.0005

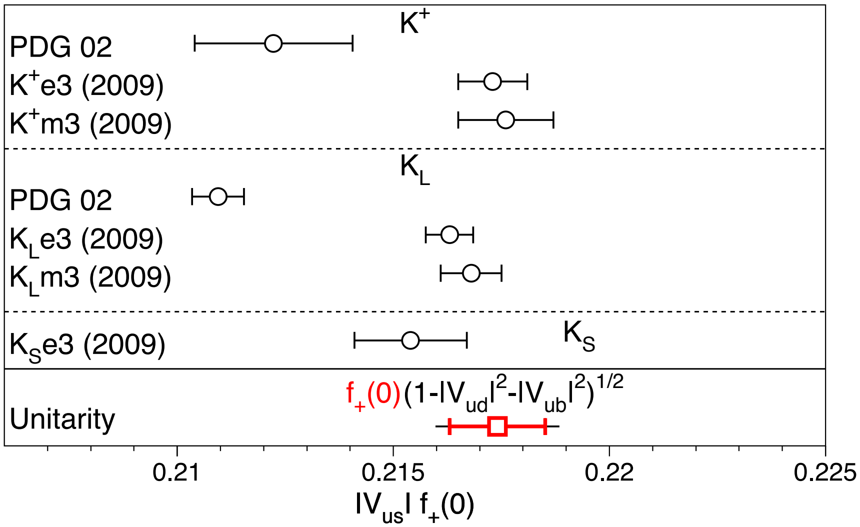

Figure 1: Comparison of determinations of $|V_{us}|f_+(0)$ from this review (labeled 2009), from the PDG 2002, and with the prediction from unitarity using $|V_{ud}|$ and the lattice calculation of $f_+(0)$ [36]. For $f_+(0)(1 - |V_{ud}|^2 - |V_{ub}|^2)^{1/2}$, the inner error bars are from the quoted uncertainty in $f_+(0)$; the total uncertainties include the $|V_{ud}|$ and $|V_{ub}|$ errors.

A value of V_{us} can also be obtained from a comparison of the radiative inclusive decay rates for $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$ combined with a lattice gauge theory calculation of f_K/f_π via [42]

$$\frac{|V_{us}|f_K}{|V_{ud}|f_\pi} = 0.2387(4) \left[\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}\right]^{\frac{1}{2}} \tag{12}$$

with the small error coming from electroweak radiative corrections. Employing

$$\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))} = 1.3337(46), \tag{13}$$

which averages in the KLOE result [43], $B(K \to \mu\nu(\gamma)) = 63.66(9)(15)\%$ and [44]

$$f_K/f_\pi = 1.189(7) \tag{14}$$

along with the value of V_{ud} in Eq. (4) leads to

$$|V_{us}| = 0.2259(5)(13). \tag{15}$$

It should be mentioned that hyperon decay fits suggest [5]

$$|V_{us}| = 0.2250(27) \text{ Hyperon Decays} \tag{16}$$

modulo SU(3) breaking effects that could shift that value up or down. We note that a recent representative effort [45] that incorporates SU(3) breaking found $V_{us} = 0.226(5)$. Similarly, inclusive strangeness changing tau decays give [46]

$$|V_{us}| = 0.2208(34) \text{ Tau Decays} \tag{17}$$

where the central value depends on the strange quark mass. However, a recent BaBar study [47] of $\tau \to K\nu/\tau \to \pi\nu$ using the lattice value of f_K/f_π from Eq. (14) finds $V_{us} = 0.2255(24)$, in good agreement with other determinations.

Employing the value of V_{ud} in Eq. (4) and $V_{us} = 0.2252(9)$, the average of the $K\ell 3$ (Eq. (11)) and $K\mu 2$ (Eq. (15) determinations of V_{us}, leads to the unitarity consistency check

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(4)(4). \tag{18}$$

where the first error is the uncertainty from $|V_{ud}|^2$ and the second error is the uncertainty from $|V_{us}|^2$.

CKM Unitarity Constraints

The current good experimental agreement with unitarity, $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 0.9999(6)$, provides strong confirmation of Standard Model radiative corrections (which range between 3-4% depending on the nucleus used) at better than the 50 sigma level [48]. In addition, it implies constraints on "New Physics" effects at both the tree and quantum loop levels. Those effects could be in the form of contributions to nuclear beta decays, K decays and/or muon decays, with the last of these providing normalization via the muon lifetime [49], which is used to obtain the Fermi constant, $G_\mu = 1.166371(6) \times 10^{-5}\text{GeV}^{-2}$.

In the following sections, we illustrate the implications of CKM unitarity for (1) exotic muon decays [50](beyond ordinary muon decay $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$) and (2) new heavy quark mixing V_{uD} [51]. Other examples in the literature [52,53] include Z_χ boson quantum loop effects, supersymmetry, leptoquarks, compositeness etc.

Exotic Muon Decays

If additional lepton flavor violating decays such as $\mu^+ \to e^+\bar{\nu}_e\nu_\mu$ (wrong neutrinos) occur, they would cause confusion in searches for neutrino oscillations at, for example, muon storage rings/neutrino factories or other neutrino sources from muon decays. Calling the rate for all such decays $\Gamma(\text{exotic } \mu \text{ decays})$, they should be subtracted before the extraction of G_μ and normalization of the CKM matrix. Since that is not done and unitarity works, one has (at one-sided 95% CL)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - BR(\text{exotic } \mu \text{ decays}) \geq 0.9989 \tag{19}$$

or

$$BR(\text{exotic } \mu \text{ decays}) < 0.001 \ . \tag{20}$$

This bound is a factor of 10 better than the direct experimental bound on $\mu^+ \to e^+\bar{\nu}_e\nu_\mu$.

New Heavy Quark Mixing

Heavy D quarks naturally occur in fourth quark generation models and some heavy quark "new physics" scenarios such as E_6 grand unification. Their mixing with ordinary quarks gives rise to V_{ud} which is constrained by unitarity (one sided 95% CL)

$$\begin{aligned} |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{uD}|^2 &> 0.9989 \\ |V_{uD}| &< 0.03 \ . \end{aligned} \tag{21}$$

A similar constraint applies to heavy neutrino mixing and the couplings $V_{\mu N}$ and V_{eN}.

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