

## INTRODUCTION TO THREE-NEUTRINO MIXING PARAMETERS LISTINGS

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**Introduction and Notation:** With the exception of the LSND anomaly, current accelerator, reactor, solar and atmospheric neutrino data can be described within the framework of a  $3 \times 3$  mixing matrix between the flavor eigenstates  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  and mass eigenstates  $\nu_1$ ,  $\nu_2$  and  $\nu_3$ . (See equation 13.79 of the review “Neutrino Mass, Mixing and Oscillations” by K. Nakamura and S.T. Petcov.) Whether or not this is the ultimately correct framework, it is currently widely used to parametrize neutrino mixing data and to plan new experiments.

The mass differences are called  $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$  and  $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$ . In these listings, we assume

$$\Delta m_{32}^2 \sim \Delta m_{31}^2 \quad (1)$$

although in the future, experiments may be precise enough to measure these separately. The angle are labeled  $\theta_{12}$ ,  $\theta_{23}$  and  $\theta_{13}$ . The CP violating phase is called  $\delta$ , but that does not yet appear in the listings. The familiar two neutrino form for oscillations is

$$P(\nu_a \rightarrow \nu_b) = \sin^2(2\theta) \sin^2(\Delta m^2 L/4E). \quad (2)$$

Despite the fact that the mixing angles have been measured to be much larger than in the quark sector, the two neutrino form is often a very good approximation and is used in many situations.

The angles appear in the equations below in many forms. They most often appear as  $\sin^2(2\theta)$ . The listings currently use this convention.

**Accelerator neutrino experiments:** Ignoring the small  $\Delta m_{21}^2$  scale,  $CP$  violation, and matter effects, the equations for the probability of appearance in an accelerator oscillation experiment are:

$$P(\nu_\mu \rightarrow \nu_\tau) = \sin^2(2\theta_{23}) \cos^4(\theta_{13}) \sin^2(\Delta m_{32}^2 L/4E) \quad (3)$$

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2(\Delta m_{32}^2 L/4E) \quad (4)$$

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2(\Delta m_{32}^2 L/4E) \quad (5)$$

$$P(\nu_e \rightarrow \nu_\tau) = \sin^2(2\theta_{13}) \cos^2(\theta_{23}) \sin^2(\Delta m_{32}^2 L/4E) . \quad (6)$$

For the case of negligible  $\theta_{13}$ , these probabilities vanish except for  $P(\nu_\mu \rightarrow \nu_\tau)$ , which then takes the familiar two-neutrino form.

Current and future long-baseline accelerator experiments are studying non-zero  $\theta_{13}$  through  $P(\nu_\mu \rightarrow \nu_e)$ . Including the CP terms and low mass scale, the equation for neutrino oscillation in vacuum is:

$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) &= P1 + P2 + P3 + P4 \\
 P1 &= \sin^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2(\Delta m_{32}^2 L/4E) \\
 P2 &= \cos^2(\theta_{23}) \sin^2(2\theta_{13}) \sin^2(\Delta m_{21}^2 L/4E) \\
 P3 &= -/+ J \sin(\delta) \sin(\Delta m_{32}^2 L/4E) \\
 P4 &= J \cos(\delta) \cos(\Delta m_{32}^2 L/4E) \quad (7)
 \end{aligned}$$

where

$$\begin{aligned}
 J &= \cos(\theta_{13}) \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \times \\
 &\quad \sin(\Delta m_{32}^2 L/4E) \sin(\Delta m_{21}^2 L/4E) \quad (8)
 \end{aligned}$$

and the sign in P3 is negative for neutrinos and positive for anti-neutrinos respectively. For most new long-baseline accelerator experiments, P2 can safely be neglected but the other three terms could be comparable. Also, depending on the distance and the mass hierarchy, matter effects will need to be included.

**Reactor neutrino experiments:** Nuclear reactors are prolific sources of  $\bar{\nu}_e$  with an energy near 4 MeV. The oscillation probability can be expressed

$$\begin{aligned}
 P(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - \cos^4(\theta_{13}) \sin^2(2\theta_{12}) \sin^2(\Delta m_{21}^2 L/4E) \\
 &\quad - \cos^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta m_{31}^2 L/4E) \\
 &\quad - \sin^2(\theta_{12}) \sin^2(2\theta_{13}) \sin^2(\Delta m_{32}^2 L/4E) \quad (9)
 \end{aligned}$$

not using the approximation in Eq. (1). For short distances ( $L < 5$  km) we can ignore the second term on the right and can reimpose approximation Eq. (1). This takes the familiar two neutrino form with  $\theta_{13}$  and  $\Delta m_{32}^2$ :

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2(2\theta_{13}) \sin^2(\Delta m_{32}^2 L/4E). \quad (10)$$

For long distances and small  $\theta_{13}$ , the last two terms in Eq. (9) oscillate rapidly and average to zero for an experiment with finite energy resolution, leading to the familiar two neutrino form but with  $\theta_{12}$  and  $\Delta m_{21}^2$ .

***Solar and Atmospheric neutrino experiments:*** Solar neutrino experiments are sensitive to  $\nu_e$  disappearance and have allowed the measurement of  $\theta_{12}$  and  $\Delta m_{21}^2$ . They are also sensitive to  $\theta_{13}$ . We identify  $\Delta m_{\odot}^2 = \Delta m_{21}^2$  and  $\theta_{\odot} = \theta_{12}$ .

Atmospheric neutrino experiments are primarily sensitive to  $\nu_{\mu}$  disappearance through  $\nu_{\mu} \rightarrow \nu_{\tau}$  oscillations, and have allowed the measurement of  $\theta_{23}$  and  $\Delta m_{32}^2$ . We identify  $\Delta m_A^2 = \Delta m_{32}^2$  and  $\theta_A = \theta_{23}$ . Despite the large  $\nu_e$  component of the atmospheric neutrino flux, it is difficult to measure  $\Delta m_{21}^2$  effects. This is because of a cancellation between  $\nu_{\mu} \rightarrow \nu_e$  and  $\nu_e \rightarrow \nu_{\mu}$  together with the fact that the ratio of  $\nu_{\mu}$  and  $\nu_e$  atmospheric fluxes, which arise from sequential  $\pi$  and  $\mu$  decay, is near 2.

***Oscillation Parameter Listings:*** In Section (B) we encode the three mixing angles  $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$  and two mass squared differences  $\Delta m_{21}^2$  and  $\Delta m_{32}^2$ . Our knowledge of  $\theta_{12}$  and  $\Delta m_{21}^2$  comes from the KamLAND reactor neutrino experiment together with solar neutrino experiments. Our knowledge of  $\theta_{23}$  and  $\Delta m_{32}^2$  comes from atmospheric neutrino experiments and long-baseline accelerator experiments. Results on  $\theta_{13}$  come from reactor antineutrino disappearance experiments. There are also results from long-baseline accelerator experiments looking for  $\nu_e$  appearance. The interpretation of both kinds of results depends on  $\Delta m_{32}^2$ , and the accelerator results also depend on the mass hierarchy,  $\theta_{23}$  and the CP violating phase  $\delta$ .