N AND Δ RESONANCES

Written April 2012 by E. Klempt (University of Bonn) and R.L Workman, (George Washington University).

I. Introduction

The excited states of the nucleon have been studied in a large number of formation and production experiments. The Breit-Wigner masses and widths, the pole positions, and the elasticities of the N and Δ resonances in the Baryon Summary Table come largely from partial-wave analyses of πN total, elastic, and charge-exchange scattering data. The most comprehensive analyses were carried out by the Karlsruhe-Helsinki (KH80) [1], Carnegie Mellon-Berkeley (CMB80) [2], and George Washington U (GWU) [3] groups. Partial-wave analyses have also been performed on much smaller πN reaction data sets to get $N\eta$, ΛK, and ΣK branching fractions. Other branching fractions come from analyses of $\pi N \rightarrow N\pi\pi$ data. A number of groups have undertaken multichannel analyses of these and associated photo-induced reactions (see Sec. VI).

In recent years, a large amount of data on photoproduction of many final states has been accumulated, and these data are beginning to make a significant impact on the properties of baryon resonances. A survey of data on photoproduction can be found in the proceedings of recent conferences [4] and workshops [5], and in a recent review [6].

II. Naming scheme for baryon resonances

In the past, when nearly all resonance information came from elastic πN scattering, it was common to label resonances with the incoming partial wave $L_{2I,2J}$, as in $\Delta(1232)P_{33}$ and $N(1680)F_{15}$. However, most recent information has come from γN experiments. Therefore, we have replaced $L_{2I,2J}$ with the spin-parity J^P of the state, as in $\Delta(1232)$ $3/2^+$ and $N(1680)$ $5/2^+$. This applies to all baryons, including those such as the Ξ resonances and charm baryons that are not produced in formation experiments. Names of the stable baryons $(N, \Lambda, \Sigma, \Xi, \Omega, \Lambda_c, \cdots)$ have no spin, parity, or mass attached.

Table 1. The status of the N resonances. Only those with an overall status of *** or **** are included in the main Baryon Summary Table.

Particle	J^P	Status overall	πN	γN	Status as seen in — $N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$
N	$1/2^+$	****									
$N(1440)$	$1/2^+$	****	****	****		***				*	***
$N(1520)$	$3/2^-$	****	****	****	***					***	***
$N(1535)$	$1/2^-$	****	****	****	****					**	*
$N(1650)$	$1/2^-$	****	****	***	***			***	**	**	***
$N(1675)$	$5/2^-$	****	****	***	*			*		*	***
$N(1680)$	$5/2^+$	****	****	****	*	**				***	***
$N(1685)$	$?^?$	*									
$N(1700)$	$3/2^-$	***	***	**	*			*	*	*	***
$N(1710)$	$1/2^+$	***	***	***	***		**	***	**	*	**
$N(1720)$	$3/2^+$	****	****	***	***			**	**	**	*
$N(1860)$	$5/2^+$	**	**							*	*
$N(1875)$	$3/2^-$	***	*	***			**	***	**		***
$N(1880)$	$1/2^+$	**	*	*		**		*			
$N(1895)$	$1/2^-$	**	*	**	**			**	*		
$N(1900)$	$3/2^+$	***	**	***	**		**	***	**	*	**
$N(1990)$	$7/2^+$	**	**	**					*		
$N(2000)$	$5/2^+$	**	*	**	**			**	*	**	
$N(2040)$	$3/2^+$	*									
$N(2060)$	$5/2^-$	**	**	**	*				**		
$N(2100)$	$1/2^+$	*									
$N(2150)$	$3/2^-$	**	**	**				**			**
$N(2190)$	$7/2^-$	****	****	***			*	**		*	
$N(2220)$	$9/2^+$	****	****								
$N(2250)$	$9/2^-$	****	****								
$N(2600)$	$11/2^-$	***	***								
$N(2700)$	$13/2^+$	**	**								

**** Existence is certain, and properties are at least fairly well explored.
*** Existence is very likely but further confirmation of quantum numbers and branching fractions is required.
** Evidence of existence is only fair.
* Evidence of existence is poor.

Table 2. The status of the Δ resonances. Only those with an overall status of ∗∗∗ or ∗∗∗∗ are included in the main Baryon Summary Table.

	Status			Status as seen in —						
Particle J^P	overall	πN	γN	$N\eta$	$N\sigma$	$N\omega$	ΛK	ΣK	$N\rho$	$\Delta\pi$
Δ(1232) $3/2^+$	∗∗∗∗	∗∗∗∗	∗∗∗∗	F						
Δ(1600) $3/2^+$	∗∗∗	∗∗∗	∗∗∗	o					∗	∗∗∗
Δ(1620) $1/2^-$	∗∗∗∗	∗∗∗∗	∗∗∗		r				∗∗∗	∗∗∗
Δ(1700) $3/2^-$	∗∗∗∗	∗∗∗∗	∗∗∗∗		b				∗∗	∗∗∗
Δ(1750) $1/2^+$	∗	∗			i					
Δ(1900) $1/2^-$	∗∗	∗∗	∗∗			d		∗∗	∗∗	∗∗
Δ(1905) $5/2^+$	∗∗∗∗	∗∗∗∗	∗∗∗∗			d		∗∗∗	∗∗	∗∗
Δ(1910) $1/2^+$	∗∗∗∗	∗∗∗∗	∗∗			e		∗	∗	∗∗
Δ(1920) $3/2^+$	∗∗∗	∗∗∗	∗∗				n	∗∗∗		∗∗
Δ(1930) $5/2^-$	∗∗∗	∗∗∗								
Δ(1940) $3/2^-$	∗∗	∗	∗∗	F				(seen	in	$\Delta\eta$)
Δ(1950) $7/2^+$	∗∗∗∗	∗∗∗∗	∗∗∗∗	o				∗∗∗	∗	∗∗∗
Δ(2000) $5/2^+$	∗∗				r					∗∗
Δ(2150) $1/2^-$	∗	∗			b					
Δ(2200) $7/2^-$	∗	∗			i					
Δ(2300) $9/2^+$	∗∗	∗∗				d				
Δ(2350) $5/2^-$	∗	∗				d				
Δ(2390) $7/2^+$	∗	∗				e				
Δ(2400) $9/2^-$	∗∗	∗∗					n			
Δ(2420) $11/2^+$	∗∗∗∗	∗∗∗∗	∗							
Δ(2750) $13/2^-$	∗∗	∗∗								
Δ(2950) $15/2^+$	∗∗	∗∗								

∗∗∗∗	Existence is certain, and properties are at least fairly well explored.
∗∗∗	Existence is very likely but further confirmation of quantum numbers and branching fractions is required.
∗∗	Evidence of existence is only fair.
∗	Evidence of existence is poor.

III. Using the N and Δ listings

Tables 1 and 2 list all the N and Δ entries in the Baryon Listings and give our evaluation of the overall status, the status from $\pi N \to N\pi$ scattering data and from photoproduction experiments, and the status channel by channel. Only the established resonances (overall status 3 or 4 stars) are promoted to the Baryon Summary Table. We have omitted from the Listings information from old analyses, prior to KH80 and CMB80 which can be found in earlier editions. A rather complete survey of older results was given in our 1982 edition [7].

The star rating assigned to a resonance depends on the data base and the analysis. As a rule, we award an overall status *** or **** only to those resonances which are confirmed by independent analyses and which are derived from analyses based on complete information, *i.e.*, for analyses based on three observables in πN scattering or eight properly chosen observables in photoproduction. Use of dispersion relations (as in the KH80, CMB80, and GWU analyses) may lift these requirements. Three and four-star resonances should be observed in one of their strongest decay modes. Weak signals or signals emerging in analyses with incomplete experimental information are given ** or * status. We do not consider new results without proper error evaluation.

In the Data Listings, we give first the Breit-Wigner mass and width but warn the reader that Breit-Wigner parameters depend on the formalism used, such as for angular momentum barrier factors or cut-off parameters, and the assumed or modeled background. Then we give pole-related quantities, such as the position of the pole and its elastic residue. For the first time, we give residues and phases of hadronic transition amplitudes and helicity amplitudes. Branching ratios and photoproduction amplitudes follow.

IV. Properties of resonances

Resonances are defined by poles of the scattering amplitude in the complex energy $w = \sqrt{s}$ plane [8]. In contrast to other quantities related to resonance phenomena, such as the Breit-Wigner mass or the K-matrix pole, a pole of the scattering amplitude does not depend on the chosen field parameterization, and production and decay properties factorize. It is the pole position which should be compared to eigenvalues of the Hamiltonian of full QCD.

Examining the Listings, one finds a much larger spread in Breit-Wigner parameters compared to pole parameters. In his *pole-emic* against Breit-Wigner parameters, Höhler [9] concluded: *"In contrast to the conventional* (Breit-Wigner) *parameters, the pole positions and speed plots have a well-defined relation to S-matrix theory. They also give more information on the resonances and thresholds and can be used for the prediction on other reactions that couple to the excited states* [italics in original]."

In scattering theory, the amplitude for the scattering process leading from the initial state a to the final state b is given by the S matrix, which can be decomposed as follows:

$$S_{ab} = I_{ab} + 2i\sqrt{\rho_a}T_{ab}\sqrt{\rho_b}\,. \tag{1}$$

Here I_{ab} is the identity operator, and T_{ab} describes the transition from the initial state to the final state (e.g. πN to ΣK). T_{ab} contains coupling constants, the decay momenta k to the power L to yield the correct threshold behavior when angular momenta are involved, and a correction $F(L, r^2, k^2)$, e.g. in Blatt-Weisskopf form, with a range parameter r. The two-body phase-space ρ is given (see Eq. 39.17 in Sec. 39) by

$$\rho(s) = \frac{1}{16\pi}\frac{2|\vec{k}|}{\sqrt{s}}\,. \tag{2}$$

The transition amplitude T contains poles due to resonances and background terms. Above the threshold for inelastic reactions, a resonance is associated with a cluster of poles in different Riemann sheets. The pole closest to the real axis has the strongest impact on the data. It is situated on the second Riemann sheet, starting at the highest threshold below the pole position. If the threshold is close to the pole position, poles in other sheets may have an important impact as well.

Other complications may occur: Broad resonances are difficult to disentangle from background amplitudes, *e.g.*, due to left-hand cuts originating from meson and baryon exchange forces. A two-particle subsystem generates a square-root singularity at its threshold; poles in a two-body subsystem, *e.g.*, the ρ meson in the $\pi\pi$ system, lead to branch points in the complex energy plane. Neglecting some of these aspects leads to a model dependence of the pole position. These uncertainties increase with the particle width.

Several particle properties are related to poles. First, poles exist on multiple Riemann sheets. In the Listings, we give for each resonance the position of the most relevant pole. The poles of the scattering amplitude can be found by analytic continuation of the amplitude. The real part of the pole position in the complex energy plane defines the particle mass, the imaginary part its half width: $w_{\rm pole} = m_{\rm pole} - i\Gamma_{\rm pole}/2$. Residues of transition amplitudes are the first term in a Laurent expansion and can be calculated through a contour integral of the amplitude T_{ab} around the pole position in the energy plane:

$$\begin{aligned} Res(a \to b) &= \oint \frac{d\sqrt{s}}{2\pi i} \sqrt{\rho_a}\; T_{ab}(s) \sqrt{\rho_b} \\ &= \frac{1}{2w_{\rm pole}} \sqrt{\rho_a(s_{\rm pole})}\; g_a\, g_b \sqrt{\rho_b(s_{\rm pole})}\ , \end{aligned} \tag{4}$$

where g_a and g_b are coupling constants. In the Listings, we give normalized residues, $2\,Res(a \to b)/\Gamma_{\rm pole}$. For elastic scattering, *e.g.*, for $\pi N \to N\pi$, this gives the elastic residue:

$$Res(a \to a) = \frac{1}{2w_{\rm pole}} \rho_a(s_{\rm pole}) g_a^2. \tag{5}$$

Branching ratios of a pole can be defined by

$$BR_{\rm pole}(\text{channel } b) = \frac{|Res(\pi N \to b)|^2}{|Res(\pi N \to N\pi)| \cdot \left(\Gamma_{\rm pole}/2\right)}. \tag{6}$$

This information is, however, not given in the literature.

Within models, background amplitudes can be parameterized using an effective Lagrangian approach (as in dynamical coupled-channel approaches), or by low-order polynomial functions. In the latter case, resonances are then added, sometimes in the form of Breit-Wigner amplitudes. In the Listings, particle properties related to fits to data using Breit-Wigner amplitudes are given as well. These are the Breit-Wigner mass and width, the partial decay widths, and the branching ratios. It should be noted that Breit-Wigner parameters depend on the background parameterization.

The multichannel relativistic Breit-Wigner amplitude is given by

$$A_{ab} = \sqrt{\rho_a} T_{ab} \sqrt{\rho_b} = \frac{-g_a g_b \sqrt{\rho_a \rho_b}}{s - m_{\mathrm{BW}}^2 + i \sum\limits_a g_a^2 \rho_a} \ , \tag{7}$$

where m_{BW} is called the Breit-Wigner mass. In the case of two channels, Eq. (7) is known as the Flatté formula. The inclusion of angular momenta leads to additional factors. The energy-dependent partial decay widths, defined by $\sqrt{s}\,\Gamma_a(s) = g_a^2 \rho_a(s)$, can be used to bring Eq. (7) into the form of Eq. (39.57). Evaluated at the Breit-Wigner mass, it gives the partial decay width Γ_a at the resonance position

$$m_{\mathrm{BW}} \Gamma_a = g_a^2 \rho_a (m_{BW}^2). \tag{8}$$

The branching ratio for the decay of a resonance into channel a,

$$BR_a = \Gamma_a / \Gamma_{\mathrm{BW}} \, , \tag{9}$$

vanishes by definition for decay modes with thresholds above the Breit-Wigner mass. That the sum $\sum_a BR_a$ equals one follows from the definition. Unobserved decay modes lead to the inequality $\sum_a BR_a \leq 1$. In the case of broad resonances, definitions (8) and (9) may be counter-intuitive. Branching ratios can also be defined as

$$BR' = \int\limits_{\text{threshold}}^{\infty} \frac{ds}{\pi} \frac{g_a^2 \rho(s)}{(m_{\mathrm{BW}}^2 - s)^2 + (\sum\limits_a g_a^2 \rho_a(s))^2}. \tag{10}$$

Here $\rho(s)$ should not be continued below threshold. These branching ratios include decays of resonances into channels with thresholds above their nominal masses. The relation $\sum_a BR'_a = 1$ is needed for normalization.

V. Electromagnetic interactions

A new approach to the nucleon excitation spectrum is provided by dedicated facilities at the Universities of Bonn and

Mainz, and at the national laboratories Jefferson Lab in the US and SPring-8 in Japan. High-precision cross sections and polarization observables in photoproduction of pseudoscalar mesons provide a data set that is nearly a "complete experiment," one that fully constrains the four complex amplitudes describing the spin-structure of the reaction. A large number of photoproduction reactions has been studied.

In photoproduction, the spins of the photon and nucleon can be parallel or anti-parallel, and there are spin-flip and non-flip transitions. Four independent amplitudes can be defined using the photon polarization and the hadronic current [10]. The amplitudes can be expanded in a series of electric and magnetic multipoles. In general, two amplitudes, one electric and one magnetic, contribute to one J^P combination. For a given resonance, these two amplitudes are related to the helicity amplitudes $A_{1/2}$ and $A_{3/2}$. The final state may have isospin $I = 1/2$ or $I = 3/2$.

If a Breit-Wigner parametrization is used, the $N\gamma$ partial width, Γ_γ, is given in terms of the helicity amplitudes $A_{1/2}$ and $A_{3/2}$ by

$$\Gamma_\gamma = \frac{k_{\rm BW}^2}{\pi}\frac{2m_N}{(2J+1)m_{\rm BW}}\Big(|A_{1/2}|^2 + |A_{3/2}|^2\Big) \ . \qquad (11)$$

Here m_N and $m_{\rm BW}$ are the nucleon and resonance masses, J is the resonance spin, and $k_{\rm BW}$ is the photon c.m. decay momentum. Most earlier analyses have quoted the real quantities $A_{1/2}$ and $A_{3/2}$.

Other more recent studies have quoted related complex quantities, evaluated at the T-matrix pole. The complex helicity amplitudes for photoproduction of the final state b, $\tilde{A}_{1/2}$ and $\tilde{A}_{3/2}$, are given by

$$Res\Big((\gamma N)_h \to b\Big) = \frac{\tilde{A}^h g_b}{2w_{\rm pole}}|\,k_{\rm pole}|\sqrt{\frac{2m_N}{(2J+1)\pi}\cdot\rho_b(w_{\rm pole}^2)}. \qquad (12)$$

$\tilde{A}_{1/2}$ and $\tilde{A}_{3/2}$ are defined at the pole position, and are normalized to reproduce Eq. (11) when the pole position is replaced by the Breit-Wigner mass.

The amplitudes $\tilde{A}_{1/2}$ and $\tilde{A}_{3/2}$, the elastic residues, and the residues of the transition amplitudes are complex numbers. Eq. (8) defines $g_{N\pi}$ up to a sign. (Here, $g_{N\pi}$ is the $N\pi$ decay constant of a resonance, not the πN coupling constant!) Due to Eq. (12), the phase of the helicity amplitude depends on this definition. We define the phase of $g_{N\pi}$ clockwise.

The determination of eight real numbers from four complex amplitudes (with one overall phase undetermined) requires at least seven independent data points. At least one further measurement is required to resolve discrete ambiguities that result from the fact that data are proportional to squared amplitudes. Photon beams and nucleon targets can be polarized (with linear or circular polarization $P_\perp$, $P_\odot$, and $\vec{T}$, respectively); and the recoil polarization of the outgoing nucleon $\vec{R}$ can be measured. Experiments can be divided into three classes: those with polarized photons and a polarized target (BT); and those measuring the baryon recoil polarization and using either a polarized photon (BR) or a polarized target (TR). Different sign conventions are used in the literature, as summarized in Ref. 12.

A large number of polarization observables has been determined that constrain energy-dependent partial-wave solutions. One of the best studied reactions is $\gamma p \to \Lambda K^+$. Published data include differential cross sections, the beam asymmetry Σ, the target asymmetry T, the recoil polarization P, and the BR double-polarization variables C_x, C_z, O_x, and O_z. For $\gamma p \to p\pi^0$, $\gamma p \to n\pi^+$, and $\gamma p \to p\eta$, differential cross sections and beam asymmetries have been published; BT data for E, F, G, and H have been presented at conferences [13].

Electroproduction of mesons provides information on the internal structure of resonances. The helicity amplitudes become functions of the momentum transfer, and a third amplitude, $S_{1/2}$, contributes to the process. Recent experimental results and their interpretation are reviewed by I.G. Aznauryan and V.D. Burkert [14] and by L. Tiator, D. Drechsel, S.S. Kamalov, and M. Vanderhaeghen [15].

VI. Partial wave analyses

Several PWA groups are now actively involved in the analysis of the new data. Of the three "classical" analysis groups at

KH, CMB, and GWU, only the GWU group is still active. This group maintains a nearly complete database, covering reactions from πN and KN elastic scattering to $\gamma N \to N\pi$, $N\eta$, and $N\eta'$. It is presently the only group determining energy-independent πN elastic amplitudes from scattering data. Given the high-precision of photoproduction data already collected and to be taken in the near future, we estimate that an improved spectrum of N and Δ resonances should become available in the forthcoming years.

Energy-dependent fits are performed by various groups with the aim to understand the reaction dynamics and to identify N and Δ resonances. Ideally, the Bethe-Salpeter equation should be solved to describe the data. For practical reasons, approximations have to be made. We mention here: (1) The Mainz unitary isobar model [16] focusses on the correct treatment of the low-energy domain; resonances are added to the unitary amplitude as a sum of Breit-Wigner amplitudes. (2) Multichannel analyses using K-matrix parameterizations derive background terms from a chiral Lagrangian—providing a microscopical description of the background—(Giessen [17,18]), or from phenomenology (Bonn-Gatchina [19]). (3) Several groups (Argonne-Osaka [20], Bonn-Jülich [22,23], Dubna-Mainz-Taipeh [21], EBAC-Jlab [24], Valencia [25]) use dynamical reaction models, driven by chiral Lagrangians, which take dispersive parts of intermediate states into account. The Giessen group pioneered multichannel analyses of large data sets on pion- and photo-induced reactions [17,18]. The Bonn-Gatchina group included recent high-statistics data and reported systematic searches for new baryon resonances in all relevant partial waves. A summary of their results can be found in Ref. 19.

References

1. G. Höhler, *Pion-Nucleon Scattering,*Landolt-Börnstein, Vol. I/9b2 (1983), ed. H. Schopper, Springer Verlag.
2. R.E. Cutkosky *et al.*, Baryon 1980, *IV International Conference on Baryon Resonances*, Toronto, ed. N. Isgur, p. 19.
3. R.A. Arndt *et al.*, Phys. Rev. **C74**, 045205 (2006).

4. *Hadron 2011: 14th International Conference on Hadron Spectroscopy,* München, Germany, June 13-17, 2011, published in eConf.
5. *NSTAR 2011: 8th International Workshop on the Physics of Excited Nucleons,"* May 17-20, 2011, Newport News, Virginia, USA.
6. E. Klempt and J.M. Richard, Rev. Mod. Phys. **82**, 1095 (2010).
7. M. Roos *et al.*, Phys. Lett. **B111**, 1 (1982).
8. R.H. Dalitz and R.G. Moorhouse, Proc. Roy. Soc. Lond. **A318**, 279 (1970).
9. C. Caso *et al.*, Eur. Phys. J. **C3**, 1 (1998).
10. G.F. Chew *et al.*, Phys. Rev. **106**, 1345 (1957).
11. C.G. Fasano *et al.*, Phys. Rev. **C46**, 2430 (1992).
12. A.M. Sandorfi *et al.*, "A Rosetta Stone relating conventions in photo-meson partial wave analyses," `arXiv:1108.5411 [nucl-th]`.
13. R. Beck, "Light baryon spectroscopy: recent results," presented at *Hadron2011,* op.cit.
14. I.G. Aznauryan and V.D. Burkert, Prog. Part. Nucl. Phys. **67**, 1 (2012).
15. L. Tiator *et al.*, Eur. Phys. J. ST **198**, 141 (2011).
16. D. Drechsel, S.S. Kamalov, and L. Tiator, Eur. Phys. J. A **34**, 69 (2007).
17. G. Penner and U. Mosel, Phys. Rev. C **66**, 055211 (2002).
18. G. Penner and U. Mosel, Phys. Rev. C **66** 055212 (2002).
19. A.V. Anisovich *et al.*, EPJ **A48**, 15 (2012).
20. T. Sato and T. -S. H. Lee, J. Phys. G **36**, 073001 (2009).
21. G.Y. Chen *et al.*, Phys. Rev. C **76**, 035206 (2007).
22. M. Döring *et al.*, Phys. Lett. B **681**, 26 (2009).
23. M. Döring *et al.*, Nucl. Phys. A **829**, 170 (2009).
24. A. Matsuyama, T. Sato, and T.-S.H. Lee, Phys. Rept. **439**, 193 (2007).
25. S. Sarkar, E. Oset, and M.J. Vicente Vacas, Nucl. Phys. A **750**, 294 (2005) [Erratum-ibid. A **780**, 78 (2006)].