D_S^+ BRANCHING FRACTIONS

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More than a dozen papers on the D_s^+, most of them from the CLEO experiment, have been published since the 2008 Review. We now know enough to attempt an overview of the branching fractions. Figure 1 shows a partial breakdown of the fractions. The rest of this note is about how the figure was constructed. The values shown make heavy use of CLEO measurements of inclusive branching fractions [1] For other data and references cited in the following, see the Listings.

Modes with leptons: The bottom $(20.0 \pm 0.9)\%$ of Fig. 1 shows the fractions for the exclusive modes that include leptons. Measured $e^+\nu_e$ fractions have been doubled to get the semileptonic $\ell^+\nu$ fractions. The sum of the exclusive $e^+\nu_e$ fractions is $(6.9 \pm 0.4)\%$, consistent with an inclusive semileptonic $e^+\nu_e$ measurement of $(6.5 \pm 0.4)\%$. There seems to be little missing here.

Inclusive hadronic $K\overline{K}$ fractions: The Cabibbo-favored $c \to s$ decay in D_s^+ decay produces a final state with both an s and an $\bar{s}$; and thus decay modes with a $K\overline{K}$ pair or with an η, ω, η', or ϕ predominate (see, for example, in Fig. 1 the fractions with leptons). We consider the $K\overline{K}$ modes first. A complete picture of the exclusive $K\overline{K}$ charge modes is not yet possible, because branching fractions for more than half of those modes have yet to be measured. However, CLEO has measured the inclusive K^+, K^-, K_S^0, K^+K^-, $K^+K_S^0$, $K^-K_S^0$, and $2K_S^0$ fractions (which include modes with leptons) [1]. And each of these inclusive fractions f with a K_S^0 is equal to the corresponding fraction with a K_L^0: $f(K^+K_L^0) = f(K^+K_S^0)$, $f(2K_L^0) = f(2K_S^0)$, *etc.* Therefore, of all inclusive fractions pairing a K^+, K_S^0, or K_L^0 with a K^-, K_S^0, or K_L^0, we know all but $f(K_S^0K_L^0)$.

We can get that fraction. The total K_S^0 fraction is

$$f(K_S^0) = f(K^+K_S^0) + f(K^-K_S^0) + 2f(2K_S^0) + f(K_S^0K_L^0) + f(\text{single } K_S^0)\ ,$$

where $f(\text{single } K_S^0)$ is the sum of the branching fractions for modes such as $K_S^0\pi^+2\pi^0$ with a K_S^0 and no second K. The $K_S^0\pi^+2\pi^0$ mode is in fact the only unmeasured single-K_S^0 mode (throughout, we shall assume that fractions for modes with a K or $K\overline{K}$ and more than three pions are negligible), and we shall take its fraction to be the same as for the $K_S^0 2\pi^+\pi^-$ mode, $(0.29 \pm 0.11)\%$. Any reasonable deviation from this value would be too small to matter much in the following. Adding the several small single-K_S^0 branching fractions, including those from semileptonic modes, we get $f(\text{single } K_S^0) = (1.67 \pm 0.26)\%$.

Using this, we have:

$$\begin{aligned} f(K_S^0K_L^0) &= f(K_S^0) - f(K^+K_S^0) - f(K^-K_S^0) - 2f(2K_S^0) \\ &\quad - f(\text{single } K_S^0) \\ &= (19.0 \pm 1.1) - (5.8 \pm 0.5) - (1.9 \pm 0.4) \\ &\quad - 2 \times (1.70 \pm 0.32) - (1.67 \pm 0.26) \\ &= (6.2 \pm 1.4)\%\ . \end{aligned}$$

Here and below we treat the errors as uncorrelated, although often they are not. However, our main aim is to get numbers for Fig. 1; errors will be secondary.

There is a check on our result: The ϕ inclusive branching fraction is $(15.7 \pm 1.0)\%$, of which 34%, or $(5.34 \pm 0.34)\%$ of D_s^+ decays, produces a $K_S^0K_L^0$. Our $f(K_S^0K_L^0) = (6.2 \pm 1.4)\%$ has to be at least this large—and it is.

We now make a table. The first column gives the various particle pairings; here we use $f(K^+\overline{K}^0) = 2\,f(K^+K_S^0)$, and likewise for $f(K^-K^0)$. The second column gives the inclusive branching fractions; the third column gives the fractions for K^+K^- and $K_S^0K_L^0$ from $\phi\ell^+\nu$ decay; the last column subtracts these off to get the purely hadronic $K\overline{K}$ inclusive fractions.

K^+K^-	15.8 (0.7)%	2.44 (0.14)%	13.4 (0.7)%
$K^+\overline{K}^0$	11.6 (1.0)		11.6 (1.0)
K^-K^0	3.8 (0.8)		3.8 (0.8)
$K_S^0K_S^0 + K_L^0K_L^0$	3.4 (0.64)		3.4 (0.64)
$K_S^0K_L^0$	6.2 (1.4)	1.69 (0.10)	4.5 (1.4) .

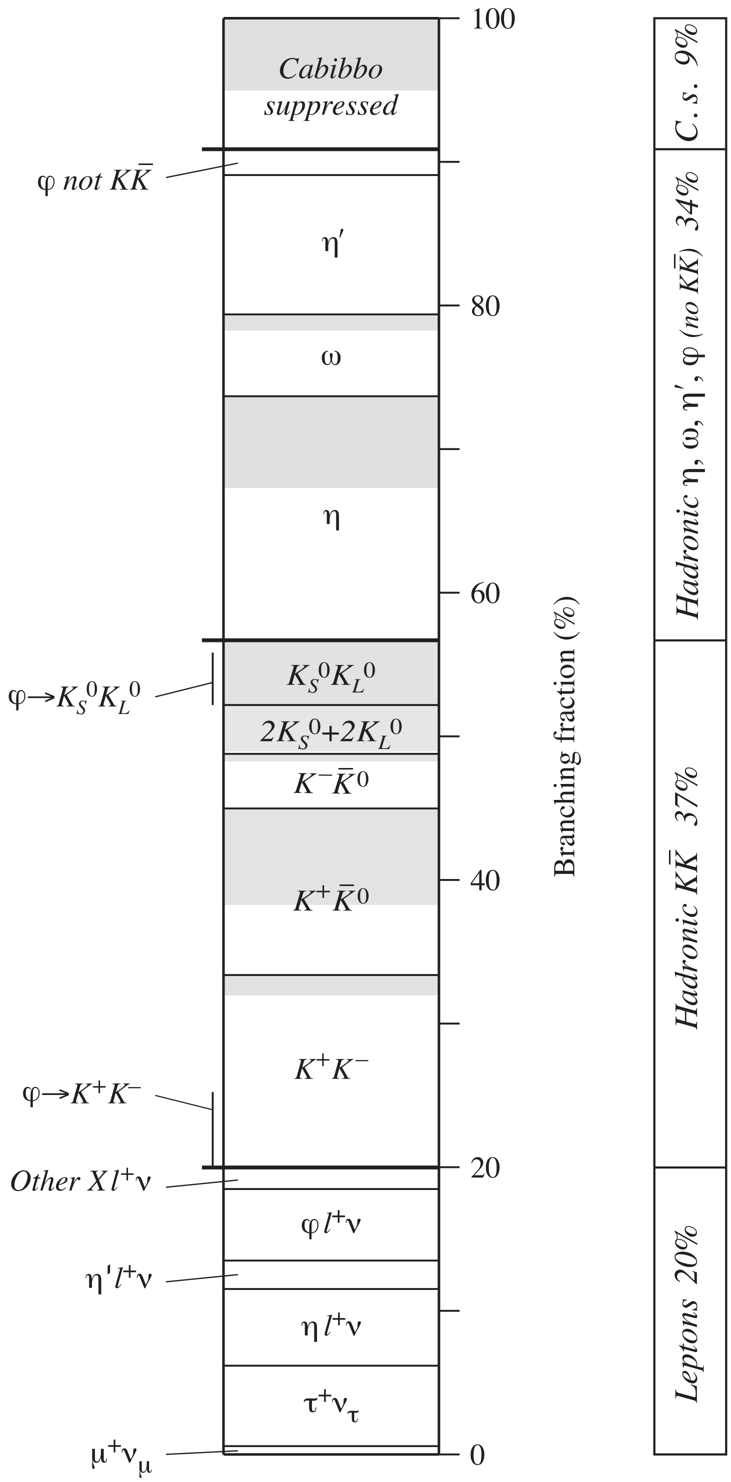

Figure 1: A partial breakdown of D_s^+ branching fractions. Shading indicates parts of bins allotted to as-yet unmeasured exclusive modes. The inclusive hadronic ϕ fraction is spread over three bins. See the text for further explanations.

The values in the last column are shown in Fig. 1. Their sum is $(36.7 \pm 2.1)\%$.

We can add more information to the figure by summing up measured branching fractions for exclusive modes within each bin:

K^+K^- *modes*—The sum of measured $K^+K^-\pi^+$, $K^+K^-\pi^+\pi^0$, and $K^+K^-2\pi^+\pi^-$ branching fractions is $(12.0 \pm 0.6)\%$. That leaves $(1.4 \pm 0.9)\%$ for the $K^+K^-\pi^+2\pi^0$ mode, which is the only other K^+K^- mode with three or fewer pions. In Fig. 1, this unmeasured part of the K^+K^- bin is shaded.

$K^+\overline{K}^0$ *modes*—Twice the sum of measured $K^+K^0_S$ and $K^+K^0_S\pi^+\pi^-$ branching fractions is $(4.9 \pm 0.3)\%$. This leaves $(6.7 \pm 1.0)\%$ for the unmeasured $K^+\overline{K}^0$ modes (there are four such modes with three or fewer pions). This is shaded in the figure.

K^-K^0 *modes*—Twice the $K^-K^0_S2\pi^+$ fraction is $(3.28 \pm 0.24)\%$, which leaves about $(0.5 \pm 0.8)\%$ for $K^-K^02\pi^+\pi^0$, the only other K^-K^0 mode with three or fewer pions.

$K^0\overline{K}^0$ *modes*—The only measurement of $K^0\overline{K}^0$ decays is of the $2K^0_S2\pi^+\pi^-$ fraction, $(0.084 \pm 0.035)\%$; so nearly everything is shaded here. However, most of the $K^0_SK^0_L$ fraction is accounted for by ϕ decays (see below).

Inclusive hadronic η, ω, η', and ϕ fractions: These are easier. We start with the inclusive branching fractions, and then, to avoid double counting, subtract: (1) fractions for modes with leptons; (2) η mesons that are included in the inclusive η' fraction; and (3) K^+K^- and $K^0_SK^0_L$ from ϕ decays:

$$\begin{aligned}
f(\eta \text{ hadronic}) &= f(\eta \text{ inclusive}) - 0.65\, f(\eta' \text{ inclusive}) \\
&\quad - f(\eta\ell^+\nu) = (17.0 \pm 3.1)\% \\
f(\omega \text{ hadronic}) &= f(\omega \text{ inclusive}) - 0.03\, f(\eta' \text{ inclusive}) \\
&= (5.7 \pm 1.4)\% \\
f(\eta' \text{ hadronic}) &= f(\eta' \text{ inclusive}) - f(\eta'\ell^+\nu) \\
&= (9.7 \pm 1.9)\% \\
f(\phi \text{ hadronic}, \not\to K\overline{K}) &= 0.17\, \big[f(\phi \text{ inclusive}) \\
&\quad - f(\phi\ell^+\nu)\big] = (1.8 \pm 0.2)\% \, .
\end{aligned}$$

The factors 0.65, 0.03, and 0.17 are the $\eta' \to \eta$, $\eta' \to \omega$, and $\phi \not\to K\overline{K}$ branching fractions. Figure 1 shows the results; the sum is $(34.2 \pm 3.9)\%$, which is about equal to the hadronic $K\overline{K}$ total.

Note that the bin marked ϕ near the top of Fig. 1 includes neither the $\phi\ell^+\nu$ decays nor the 83% of other ϕ decays that produce a $K\overline{K}$ pair. Compared to the size of that ϕ bin, there is twice as much ϕ in the $K^0_S K^0_L$ bin, and nearly three times as much in the K^+K^- bin. These contributions are indicated in those bins.

Again, we can show how much of each bin is accounted for by measured exclusive branching fractions:

η modes—The sum of $\eta\pi^+$, $\eta\rho^+$, and ηK^+ branching fractions is $(10.6 \pm 0.8)\%$, which leaves a good part of the inclusive hadronic η fraction, $(17.0 \pm 3.1)\%$, to be accounted for. This is shaded in the figure.

ω modes—The sum of $\omega\pi^+$, $\omega\pi^+\pi^0$, and $\omega 2\pi^+\pi^-$ fractions is $(4.6 \pm 0.9)\%$, which is nearly as large as the inclusive hadronic ω fraction, $(5.7 \pm 1.4)\%$.

η' modes—The sum of $\eta'\pi^+$, $\eta'\rho^+$, and $\eta' K^+$ fractions is $(16.5 \pm 2.2)\%$, which is much larger than the inclusive hadronic η' fraction, $(9.7 \pm 1.9)\%$. If an exclusive measurement is at fault, it almost has to be the $\eta'\rho^+$ fraction, which is $(12.5 \pm 2.2)\%$. It has been suggested that some of this signal might instead be misidentified kinematic reflections of other modes [2].

Cabibbo-suppressed modes: Remaining is $(9.1 \pm 4.5)\%$ for hadronic Cabibbo-suppressed modes having no η, ω, η', or ϕ. The contributions are:

K^0 + pions—Above, we found that $f(\text{single } K^0_S) = (1.67 \pm 0.26)\%$; subtracting leptonic contributions leaves $(1.20 \pm 0.24)\%$. The hadronic single-K^0 fraction is twice this, $(2.40 \pm 0.48)\%$.

K^+ + pions—The $K^+\pi^0$ and $K^+\pi^+\pi^-$ fractions sum to $(0.77 \pm 0.05)\%$. Much of the $K^+ n\pi$ modes, where $n \geq 3$, is already in the η, ω, and η' bins, and the rest is not measured. The total K^+ fraction wanted here is probably in the 1-to-2% range.

Multi-pions—The $2\pi^+\pi^-$, $\pi^+2\pi^0$, and $3\pi^+2\pi^-$ fractions total $(2.6 \pm 0.2)\%$. Modes not measured might double this.

The sum of the three contributions is certainly not inconsistent with the Cabibbo-suppressed total of $(9.1 \pm 4.5)\%$. The sum of actually measured fractions is $(4.2 \pm 0.2)\%$.

A model: With CLEO about to publish inclusive branching fractions [1], Gronau and Rosner predicted those fractions using a "statistical isospin" model [2]. Consider, say, the $D_s^+ \to K\overline{K}\pi$ charge modes: the $K^+K^-\pi^+$ branching fraction is measured, the $K^+\overline{K}^0\pi^0$ and $K^0\overline{K}^0\pi^+$ fractions are not. The statistical isospin model assumes that all the independent isospin amplitudes for $D_s^+ \to K\overline{K}\pi$ decay are equal in magnitude and incoherent in phase—in which case, the ratio of the three fractions here is 3:3:2. (Actually, use was also made of the fact that $D_s^+ \to K\overline{K}\pi$ decay is dominated by $\phi\pi^+$, $K^+\overline{K}^{*0}$, and $K^{*+}\overline{K}^0$ submodes; but the estimated charge-mode ratios were not far from 3:3:2.) A different, quark-antiquark pair-production model was used to estimate systematic uncertainties.

In this way, unmeasured exclusive fractions were calculated from measured exclusive fractions (the latter were taken from the 2008 Review, and so did not benefit from recent results). In the hadronic sector, the measured total of 59.4% of D_s^+ decays led to an estimated total of 24.2% for unmeasured modes. Weighted counts of π^+, K_S^0, *etc.*, were then made to get the inclusive fractions.

Of interest here is that the sum of all the exclusive fractions—a way-stop in getting the inclusive values—was a nearly correct 103%. In the absence of complete measurements, the model is a way to, in effect, average over ignorance. It probably works better summed over a number of charge-mode sets than in detail. It is known to sometimes give incorrect results when there are sufficient measurements to test it.

References

1. S. Dobbs *et al.*, Phys. Rev. **D79**, 112008 (2009).
2. M. Gronau and J.L. Rosner, Phys. Rev. **D79**, 074022 (2009).