21. EXPERIMENTAL TESTS OF GRAVITATIONAL THEORY

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Einstein's General Relativity, the current "standard" theory of gravitation, describes gravity as a universal deformation of the Minkowski metric:

$$g_{\mu\nu}(x^\lambda) = \eta_{\mu\nu} + h_{\mu\nu}(x^\lambda)\ ,\ \text{where}\ \eta_{\mu\nu} = \mathrm{diag}(-1,+1,+1,+1)\ . \tag{21.1}$$

General Relativity is classically defined by two postulates. One postulate states that the Lagrangian density describing the propagation and self-interaction of the gravitational field is

$$\mathcal{L}_{\mathrm{Ein}}[g_{\mu\nu}] = \frac{c^4}{16\pi G_N}\sqrt{g}g^{\mu\nu}R_{\mu\nu}(g)\ , \tag{21.2}$$

$$R_{\mu\nu}(g) = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\alpha} + \Gamma^\beta_{\alpha\beta}\Gamma^\alpha_{\mu\nu} - \Gamma^\beta_{\alpha\nu}\Gamma^\alpha_{\mu\beta}\ , \tag{21.3}$$

$$\Gamma^\lambda_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})\ , \tag{21.4}$$

where G_N is Newton's constant, $g = -\det(g_{\mu\nu})$, and $g^{\mu\nu}$ is the matrix inverse of $g_{\mu\nu}$. A second postulate states that $g_{\mu\nu}$ couples universally, and minimally, to all the fields of the Standard Model by replacing everywhere the Minkowski metric $\eta_{\mu\nu}$. Schematically (suppressing matrix indices and labels for the various gauge fields and fermions and for the Higgs doublet),

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}}[\psi, A_\mu, H, g_{\mu\nu}] = &-\tfrac{1}{4}\sum \sqrt{g}g^{\mu\alpha}g^{\nu\beta}F^a_{\mu\nu}F^a_{\alpha\beta} - \sum \sqrt{g}\,\overline{\psi}\,\gamma^\mu D_\mu \psi \\ &- \tfrac{1}{2}\sqrt{g}g^{\mu\nu}\overline{D_\mu H}D_\nu H - \sqrt{g}\,V(H) - \sum \lambda\sqrt{g}\,\overline{\psi}\,H\psi\ ,\end{aligned} \tag{21.5}$$

where $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$, and where the covariant derivative D_μ contains, besides the usual gauge field terms, a spin-dependent gravitational contribution. From the total action follow Einstein's field equations,

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu}\ . \tag{21.6}$$

Here $R = g^{\mu\nu}R_{\mu\nu}$, $T_{\mu\nu} = g_{\mu\alpha}g_{\nu\beta}T^{\alpha\beta}$, and $T^{\mu\nu} = (2/\sqrt{g})\delta\mathcal{L}_{\mathrm{SM}}/\delta g_{\mu\nu}$ is the (symmetric) energy-momentum tensor of the Standard Model matter. The theory is invariant under arbitrary coordinate transformations: $x'^\mu = f^\mu(x^\nu)$. To solve the field equations Eq. (21.6), one needs to fix this coordinate gauge freedom. *E.g.*, the "harmonic gauge" (which is the analogue of the Lorenz gauge, $\partial_\mu A^\mu = 0$, in electromagnetism) corresponds to imposing the condition $\partial_\nu(\sqrt{g}g^{\mu\nu}) = 0$.

In this *Review*, we only consider the classical limit of gravitation (*i.e.* classical matter and classical gravity). Considering quantum matter in a classical gravitational background already poses interesting challenges, notably the possibility that the zero-point fluctuations of the matter fields generate a nonvanishing vacuum energy density $\rho_{\rm vac}$, corresponding to a term $-\sqrt{g}\,\rho_{\rm vac}$ in $\mathcal{L}_{\rm SM}$ [1]. This is equivalent to adding a "cosmological constant" term $+\Lambda\, g_{\mu\nu}$ on the left-hand side of Einstein's equations Eq. (21.6), with $\Lambda = 8\pi G_N\, \rho_{\rm vac}/c^4$. Recent cosmological observations (see the following *Reviews*) suggest a positive value of Λ corresponding to $\rho_{\rm vac} \approx (2.3 \times 10^{-3}{\rm eV})^4$. Such a small value has a negligible effect on the (non cosmological) tests discussed below.

21.1. Experimental tests of the coupling between matter and gravity

The universality of the coupling between $g_{\mu\nu}$ and the Standard Model matter postulated in Eq. (21.5) ("Equivalence Principle") has many observable consequences [2]. First, it predicts that the outcome of a local non-gravitational experiment, referred to local standards, does not depend on where, when, and in which locally inertial frame, the experiment is performed. This means, for instance, that local experiments should neither feel the cosmological evolution of the universe (constancy of the "constants"), nor exhibit preferred directions in spacetime (isotropy of space, local Lorentz invariance). These predictions are consistent with many experiments and observations. Stringent limits on a possible time variation of the basic coupling constants have been obtained by analyzing a natural fission reactor phenomenon which took place at Oklo, Gabon, two billion years ago [3,4]. These limits are at the 1×10^{-7} level for the fractional variation of the fine-structure constant $\alpha_{\rm em}$ [4], and at the 4×10^{-9} level for the fractional variation of the ratio m_q/Λ_{QCD} between the light quark masses and Λ_{QCD} [5]. The determination of the lifetime of Rhenium 187 from isotopic measurements of some meteorites dating back to the formation of the solar system (about 4.6 Gyr ago) yields comparably strong limits [6]. Measurements of absorption lines in astronomical spectra also give stringent limits on the variability of both $\alpha_{\rm em}$ (at the 10^{-5} level [7]) , and $\mu = m_p/m_e$, e.g.

$$|\Delta\mu/\mu| < 1.8 \times 10^{-6} (95\%\ C.L.)\ , \tag{21.7}$$

at a redshift $z = 0.68466$ [8], and $\Delta\mu/\mu = (0.3 \pm 3.2_{\rm stat} \pm 1.9_{\rm sys}) \times 10^{-6}$ at the large redshift $z = 2.811$ [9]. Direct laboratory limits (based on monitoring the frequency ratio of several different atomic clocks) on the present time variation of $\alpha_{\rm em}$, $\mu = m_p/m_e$, and m_q/Λ_{QCD} have reached the levels [10]:

$$\begin{aligned} d\ln(\alpha_{\rm em})/dt &= (-2.5 \pm 2.6) \times 10^{-17}{\rm yr}^{-1}, \\ d\ln(\mu)/dt &= (-1.5 \pm 3.0) \times 10^{-16}{\rm yr}^{-1}, \\ d\ln(m_q/\Lambda_{QCD})/dt &= (7.1 \pm 4.4) \times 10^{-15}{\rm yr}^{-1}. \end{aligned} \tag{21.8}$$

There are also experimental limits on a possible dependence of coupling constants on the gravitational potential [10,11]. See Ref. 12 for a review of the issue of "variable constants."

The highest precision tests of the isotropy of space have been performed by looking for possible quadrupolar shifts of nuclear energy levels [13]. The (null) results can be interpreted as testing the fact that the various pieces in the matter Lagrangian Eq. (21.5) are indeed coupled to one and the same external metric $g_{\mu\nu}$ to the 10^{-29} level. For astrophysical constraints on possible Planck-scale violations of Lorentz invariance, see Ref. 14.

The universal coupling to $g_{\mu\nu}$ postulated in Eq. (21.5) implies that two (electrically neutral) test bodies dropped at the same location and with the same velocity in an external gravitational field fall in the same way, independently of their masses and compositions. The universality of the acceleration of free fall has been verified at the 10^{-13} level for laboratory bodies, notably Beryllium-Titanium, and Beryllium-Aluminum test bodies [15,16],

$$(\Delta a/a)_{\rm BeTi} = (0.3 \pm 1.8) \times 10^{-13} \ ,$$

$$(\Delta a/a)_{\rm BeAl} = (-0.7 \pm 1.3) \times 10^{-13} \ , \tag{21.9}$$

as well as for the gravitational accelerations of the Earth and the Moon toward the Sun [17],

$$(\Delta a/a)_{\rm EarthMoon} = (-0.8 \pm 1.3) \times 10^{-13} \ . \tag{21.10}$$

The latter result constrains not only how $g_{\mu\nu}$ couples to matter, but also how it couples to itself [18]("strong equivalence principle"; see Eq. (21.16) below, and the end of the section on binary pulsar tests). See also Ref. 19 for a review of torsion balance experiments.

Finally, Eq. (21.5) also implies that two identically constructed clocks located at two different positions in a static external Newtonian potential $U(\boldsymbol{x}) = \sum G_N m/r$ exhibit, when intercompared by means of electromagnetic signals, the (apparent) difference in clock rate, $\tau_1/\tau_2 = \nu_2/\nu_1 = 1 + [U(\boldsymbol{x}_1) - U(\boldsymbol{x}_2)]/c^2 + O(1/c^4)$, independently of their nature and constitution. This universal gravitational redshift of clock rates has been verified at the 10^{-4} level by comparing a hydrogen-maser clock flying on a rocket up to an altitude $\sim 10,000$ km to a similar clock on the ground [20]. The redshift due to a height change of only 33 cm has been detected by comparing two optical clocks based on $^{27}Al^+$ ions [21].

21.2. Tests of the dynamics of the gravitational field in the weak field regime

The effect on matter of one-graviton exchange, *i.e.*, the interaction Lagrangian obtained when solving Einstein's field equations Eq. (21.6) written in, say, the harmonic gauge at first order in $h_{\mu\nu}$,

$$\Box h_{\mu\nu} = -\frac{16\pi G_N}{c^4}(T_{\mu\nu} - \tfrac{1}{2}T\eta_{\mu\nu}) + O(h^2) + O(hT) \ , \tag{21.11}$$

reads $-(8\pi G_N/c^4)T^{\mu\nu}\Box^{-1}(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu})$. For a system of N moving point masses, with free Lagrangian $L^{(1)} = \sum_{A=1}^{N} - m_A c^2 \sqrt{1 - \boldsymbol{v}_A^2/c^2}$, this interaction, expanded to order v^2/c^2, reads (with $r_{AB} \equiv |\boldsymbol{x}_A - \boldsymbol{x}_B|$, $\boldsymbol{n}_{AB} \equiv (\boldsymbol{x}_A - \boldsymbol{x}_B)/r_{AB}$)

$$L^{(2)} = \frac{1}{2} \sum_{A \neq B} \frac{G_N\, m_A\, m_B}{r_{AB}} \left[1 + \frac{3}{2c^2}(\boldsymbol{v}_A^2 + \boldsymbol{v}_B^2) - \frac{7}{2c^2}(\boldsymbol{v}_A \cdot \boldsymbol{v}_B) - \frac{1}{2c^2}(\boldsymbol{n}_{AB} \cdot \boldsymbol{v}_A)(\boldsymbol{n}_{AB} \cdot \boldsymbol{v}_B) + O\left(\frac{1}{c^4}\right) \right] . \quad (21.12)$$

The two-body interactions, Eq. (21.12), exhibit v^2/c^2 corrections to Newton's $1/r$ potential induced by spin-2 exchange ("gravito-magnetism"). Consistency at the "post-Newtonian" level $v^2/c^2 \sim G_N\, m/rc^2$ requires that one also considers the three-body interactions induced by some of the three-graviton vertices and other nonlinearities (terms $O(h^2)$ and $O(hT)$ in Eq. (21.11)),

$$L^{(3)} = -\frac{1}{2} \sum_{B \neq A \neq C} \frac{G_N^2\, m_A\, m_B\, m_C}{r_{AB}\, r_{AC}\, c^2} + O\left(\frac{1}{c^4}\right) . \quad (21.13)$$

All currently performed gravitational experiments in the solar system, including perihelion advances of planetary orbits, the bending and delay of electromagnetic signals passing near the Sun, and very accurate ranging data to the Moon obtained by laser echoes, are compatible with the post-Newtonian results Eqs. (21.11)–(21.13). The "gravito-magnetic" interactions $\propto v_A v_B$ contained in Eq. (21.12) are involved in many of these experimental tests. They have been particularly tested in lunar laser ranging data [17], in the LAGEOS satellite observations [22,23], and in the dedicated Gravity Probe B mission [24]. The recently launched LARES satellite promises to improve the accuracy of such tests [23].

Similar to what is done in discussions of precision electroweak experiments, it is useful to quantify the significance of precision gravitational experiments by parameterizing plausible deviations from General Relativity. The addition of a mass-term in Einstein's field equations leads to a score of theoretical difficulties which have not yet received any consensual solution. We shall, therefore, not consider here the ill-defined "mass of the graviton" as a possible deviation parameter from General Relativity (see, however, Ref. 25). Deviations from Einstein's pure spin-2 theory are then defined by adding new, bosonic light or massless, macroscopically coupled fields. The possibility of new gravitational-strength couplings leading (on small, and possibly large, scales) to deviations from Einsteinian (and Newtonian) gravity is suggested by String Theory [26], and by Brane World ideas [27]. For reviews of experimental constraints on Yukawa-type additional interactions, see Refs. [19,28,16]. Experiments have set limits on non-Newtonian forces down to 0.056 mm [29].

Here, we shall focus on the parametrization of long-range deviations from relativistic gravity obtained by adding a strictly massless (*i.e.* without self-interaction $V(\varphi) = 0$) scalar field φ coupled to the trace of the energy-momentum tensor $T = g_{\mu\nu} T^{\mu\nu}$ [30]. The most general such theory contains an arbitrary function $a(\varphi)$ of the scalar field, and can be defined by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{tot}}[g_{\mu\nu}, \varphi, \psi, A_\mu, H] =& \frac{c^4}{16\pi G} \sqrt{g}(R(g_{\mu\nu}) - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) \\ &+ \mathcal{L}_{\text{SM}}[\psi, A_\mu, H, \widetilde{g}_{\mu\nu}] \ ,\end{aligned} \tag{21.14}$$

where G is a "bare" Newton constant, and where the Standard Model matter is coupled not to the "Einstein" (pure spin-2) metric $g_{\mu\nu}$, but to the conformally related ("Jordan-Fierz") metric $\widetilde{g}_{\mu\nu} = \exp(2a(\varphi))g_{\mu\nu}$. The scalar field equation $\Box_g \varphi = -(4\pi G/c^4)\alpha(\varphi)T$ displays $\alpha(\varphi) \equiv \partial a(\varphi)/\partial\varphi$ as the basic (field-dependent) coupling between φ and matter [31]. The one-parameter (ω) Jordan-Fierz-Brans-Dicke theory [30] is the special case $a(\varphi) = \alpha_0\varphi$ leading to a field-independent coupling $\alpha(\varphi) = \alpha_0$ (with ${\alpha_0}^2 = 1/(2\omega + 3)$). The addition of a self-interaction term $V(\varphi)$ in Eq. (21.14) introduces new phenomenological possibilities; notably the "chameleon mechanism" [32].

In the weak-field slow-motion limit appropriate to describing gravitational experiments in the solar system, the addition of φ modifies Einstein's predictions only through the appearance of two "post-Einstein" dimensionless parameters: $\overline{\gamma} = -2\alpha_0^2/(1 + \alpha_0^2)$ and $\overline{\beta} = +\frac{1}{2}\beta_0\alpha_0^2/(1 + \alpha_0^2)^2$, where $\alpha_0 \equiv \alpha(\varphi_0)$, $\beta_0 \equiv \partial\alpha(\varphi_0)/\partial\varphi_0$, φ_0 denoting the vacuum expectation value of φ. These parameters show up also naturally (in the form $\gamma_{\text{PPN}} = 1 + \overline{\gamma}$, $\beta_{\text{PPN}} = 1 + \overline{\beta}$) in phenomenological discussions of possible deviations from General Relativity [2]. The parameter $\overline{\gamma}$ measures the admixture of spin 0 to Einstein's graviton, and contributes an extra term $+\overline{\gamma}(\boldsymbol{v}_A - \boldsymbol{v}_B)^2/c^2$ in the square brackets of the two-body Lagrangian Eq. (21.12). The parameter $\overline{\beta}$ modifies the three-body interaction Eq. (21.13) by an overall multiplicative factor $1 + 2\overline{\beta}$. Moreover, the combination $\eta \equiv 4\overline{\beta} - \overline{\gamma}$ parameterizes the lowest order effect of the self-gravity of orbiting masses by modifying the Newtonian interaction energy terms in Eq. (21.12) into $G_{AB} m_A m_B / r_{AB}$, with a body-dependent gravitational "constant" $G_{AB} = G_N[1 + \eta(E_A^{\text{grav}}/m_A c^2 + E_B^{\text{grav}}/m_B c^2) + O(1/c^4)]$, where $G_N = G \exp[2a(\varphi_0)](1 + \alpha_0^2)$ and where E_A^{grav} denotes the gravitational binding energy of body A.

The best current limits on the post-Einstein parameters $\overline{\gamma}$ and $\overline{\beta}$ are (at the 68% confidence level):

$$\overline{\gamma} = (2.1 \pm 2.3) \times 10^{-5} \ , \tag{21.15}$$

deduced from the additional Doppler shift experienced by radio-wave beams connecting the Earth to the Cassini spacecraft when they passed near the Sun [33], and

$$4\overline{\beta} - \overline{\gamma} = (1.8 \pm 2.9) \times 10^{-4} \ , \tag{21.16}$$

from Lunar Laser Ranging measurements [17] of a possible polarization of the Moon toward the Sun [18]. More stringent limits on $\overline{\gamma}$ are obtained in models (*e.g.*, string-inspired ones [26]) where scalar couplings violate the Equivalence Principle.

21.3. Tests of the dynamics of the gravitational field in the radiative and/or strong field regimes

The discovery of pulsars (*i.e.*, rotating neutron stars emitting a beam of radio noise) in gravitationally bound orbits [34,35] has opened up an entirely new testing ground for relativistic gravity, giving us an experimental handle on the regime of radiative and/or strong gravitational fields. In these systems, the finite velocity of propagation of the gravitational interaction between the pulsar and its companion generates damping-like terms at order $(v/c)^5$ in the equations of motion [36]. These damping forces are the local counterparts of the gravitational radiation emitted at infinity by the system ("gravitational radiation reaction"). They cause the binary orbit to shrink and its orbital period P_b to decrease. The remarkable stability of pulsar clocks has allowed one to measure the corresponding very small orbital period decay $\dot{P}_b \equiv dP_b/dt \sim -(v/c)^5 \sim -10^{-12}$ in several binary systems, thereby giving us a direct experimental confirmation of the propagation properties of the gravitational field, and, in particular, an experimental confirmation that the speed of propagation of gravity is equal to the velocity of light to better than a part in a thousand. In addition, the surface gravitational potential of a neutron star $h_{00}(R) \simeq 2Gm/c^2R \simeq 0.4$ being a factor $\sim 10^8$ higher than the surface potential of the Earth, and a mere factor 2.5 below the black hole limit ($h_{00}(R) = 1$), pulsar data have allowed one to obtain several accurate tests of the strong-gravitational-field regime, as we discuss next.

Binary pulsar timing data record the times of arrival of successive electromagnetic pulses emitted by a pulsar orbiting around the center of mass of a binary system. After correcting for the Earth motion around the Sun and for the dispersion due to propagation in the interstellar plasma, the time of arrival of the Nth pulse t_N can be described by a generic, parameterized "timing formula" [37] whose functional form is common to the whole class of tensor-scalar gravitation theories:

$$t_N - t_0 = F[T_N(\nu_p, \dot{\nu}_p, \ddot{\nu}_p) \, ; \, \{p^K\} \, ; \, \{p^{PK}\}] \; . \tag{21.17}$$

Here, T_N is the pulsar proper time corresponding to the Nth turn given by $N/2\pi = \nu_p T_N + \frac{1}{2}\dot{\nu}_p T_N^2 + \frac{1}{6}\ddot{\nu}_p T_N^3$ (with $\nu_p \equiv 1/P_p$ the spin frequency of the pulsar, *etc.*), $\{p^K\} = \{P_b, T_0, e, \omega_0, x\}$ is the set of "Keplerian" parameters (notably, orbital period P_b, eccentricity e, periastron longitude ω_0 and projected semi-major axis $x = a \sin i/c$), and $\{p^{PK}\} = \{k, \gamma_{\rm timing}, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\}$ denotes the set of (separately measurable) "post-Keplerian" parameters. Most important among these are: the fractional periastron advance per orbit $k \equiv \dot{\omega} P_b/2\pi$, a dimensionful time-dilation parameter $\gamma_{\rm timing}$, the orbital period derivative $\dot{P}_b$, and the "range" and "shape" parameters of the gravitational time delay caused by the companion, r and s.

Without assuming any specific theory of gravity, one can phenomenologically analyze the data from any binary pulsar by least-squares fitting the observed sequence of pulse arrival times to the timing formula Eq. (21.17). This fit yields the "measured" values of the parameters $\{\nu_p, \dot{\nu}_p, \ddot{\nu}_p\}$, $\{p^K\}$, $\{p^{PK}\}$. Now, each specific relativistic theory of gravity predicts that, for instance, k, $\gamma_{\rm timing}$, $\dot{P}_b$, r and s (to quote parameters that have been successfully measured from some binary pulsar data) are some theory-dependent functions of the Keplerian parameters and of the (unknown) masses m_1, m_2 of the pulsar and its companion. For instance, in General Relativity, one finds (with $M \equiv m_1 + m_2$, $n \equiv 2\pi/P_b$)

$$\begin{aligned}
k^{\rm GR}(m_1, m_2) =& 3(1-e^2)^{-1}(G_N M n/c^3)^{2/3} \ , \\
\gamma^{\rm GR}_{\rm timing}(m_1, m_2) =& e n^{-1}(G_N M n/c^3)^{2/3} m_2(m_1 + 2m_2)/M^2 \ , \\
\dot{P}^{\rm GR}_b(m_1, m_2) =& -(192\pi/5)(1-e^2)^{-7/2}\left(1 + \tfrac{73}{24}e^2 + \tfrac{37}{96}e^4\right) \\
& \times (G_N M n/c^3)^{5/3} m_1 m_2/M^2 \ , \\
r(m_1, m_2) =& G_N m_2/c^3 \ , \\
s(m_1, m_2) =& n x (G_N M n/c^3)^{-1/3} M/m_2 \ .
\end{aligned} \tag{21.18}$$

In tensor-scalar theories, each of the functions $k^{\rm theory}(m_1, m_2)$, $\gamma^{\rm theory}_{\rm timing}(m_1, m_2)$, $\dot{P}^{\rm theory}_b(m_1, m_2)$, *etc.*, is modified by quasi-static strong field effects (associated with the self-gravities of the pulsar and its companion), while the particular function $\dot{P}^{\rm theory}_b(m_1, m_2)$ is further modified by radiative effects (associated with the spin 0 propagator) [31,38,39].

Let us give some highlights of the current experimental situation. In the first discovered binary pulsar PSR 1913+16 [34,35], it has been possible to measure with accuracy *three* post-Keplerian parameters: k, $\gamma_{\rm timing}$ and $\dot{P}_b$. The three equations $k^{\rm measured} = k^{\rm theory}(m_1, m_2)$, $\gamma^{\rm measured}_{\rm timing} = \gamma^{\rm theory}_{\rm timing}(m_1, m_2)$, $\dot{P}^{\rm measured}_b = \dot{P}^{\rm theory}_b(m_1, m_2)$ determine, for each given theory, three curves in the two-dimensional mass plane. This yields *one* (combined radiative/strong-field) test of the specified theory, according to whether the three curves meet at one point, as they should. After subtracting a small ($\sim 10^{-14}$ level in $\dot{P}^{\rm obs}_b = (-2.423 \pm 0.001) \times 10^{-12}$), but significant, "galactic" perturbing effect (linked to galactic accelerations and to the pulsar proper motion) [40], one finds that General Relativity passes this $(k - \gamma_{\rm timing} - \dot{P}_b)_{1913+16}$ test with complete success at the 10^{-3} level [35,41,42]

$$\left[\frac{\dot{P}^{\rm obs}_b - \dot{P}^{\rm gal}_b}{\dot{P}^{\rm GR}_b[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]}\right]_{1913+16} = 0.997 \pm 0.002 \ . \tag{21.19}$$

Here $\dot{P}^{\rm GR}_b[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]$ is the result of inserting in $\dot{P}^{\rm GR}_b(m_1, m_2)$ the values of the masses predicted by the two equations $k^{\rm obs} = k^{\rm GR}(m_1, m_2)$, $\gamma^{\rm obs}_{\rm timing} = \gamma^{\rm GR}_{\rm timing}(m_1, m_2)$. This yields experimental evidence for the reality of gravitational radiation damping forces at the $(-3 \pm 2) \times 10^{-3}$ level.

The discovery of the binary pulsar PSR 1534+12 [43] has allowed one to measure *five* post-Keplerian parameters: k, $\gamma_{\rm timing}$, r, s, and (with less accuracy) $\dot{P}_b$ [44,45]. This allows one to obtain *three* (five observables minus two masses) tests of relativistic gravity. Two among these tests probe strong field gravity, without mixing of radiative effects [44]. General Relativity passes all these tests within the measurement accuracy. The most precise of the new, pure strong-field tests is the one obtained by combining the measurements of k, γ, and s. Using the most recent data [45], one finds agreement at the 1% level:

$$\left[\frac{s^{\rm obs}}{s^{\rm GR}[k^{\rm obs}, \gamma^{\rm obs}_{\rm timing}]}\right]_{1534+12} = 1.000 \pm 0.007 \; . \tag{21.20}$$

The discovery of the binary pulsar PSR J1141−6545 [46](whose companion is probably a white dwarf) has allowed one to measure *four* observable parameters: $k, \gamma_{\rm timing}, \dot{P}_b$ [47,48], and the parameter s [49,48]. The latter parameter (which is equal to the sine of the inclination angle, $s = \sin i$) was consistently measured in two ways: from a scintillation analysis [49], and from timing measurements [48]. General Relativity passes all the corresponding tests within measurement accuracy. See Fig. 21.1 which uses the (more precise) scintillation measurement of $s = \sin i$.

The discovery of the remarkable *double* binary pulsar PSR J0737−3039 A and B [50,51] has led to the measurement of *seven* independent parameters [52,53]: five of them are the post-Keplerian parameters k, $\gamma_{\rm timing}$, r, s and $\dot{P}_b$ entering the relativistic timing formula of the fast-spinning pulsar PSR J0737−3039 A, a sixth is the ratio $R = x_B/x_A$ between the projected semi-major axis of the more slowly spinning companion pulsar PSR J0737−3039 B, and that of PSR J0737−3039 A. [The theoretical prediction for the ratio $R = x_B/x_A$, considered as a function of the (inertial) masses $m_1 = m_A$ and $m_2 = m_B$, is $R^{\rm theory} = m_1/m_2 + O((v/c)^4)$ [37], independently of the gravitational theory considered.] Finally, the seventh parameter $\Omega_{\rm SO,B}$ is the angular rate of (spin-orbit) precession of PSR J0737−3039 B around the total angular momentum [53]. These seven measurements give us *five* tests of relativistic gravity [52,54,55]. General Relativity passes all those tests with flying colors (see Fig. 21.1). Let us highlight here two of them (from [55]) .

One test is a new confirmation of the reality of gravitational radiation at the 10^{-3} level

$$\left[\frac{\dot{P}_b^{\rm obs}}{\dot{P}_b^{\rm GR}[k^{\rm obs}, R^{\rm obs}]}\right]_{0737-3039} = 1.000 \pm 0.001 \; . \tag{21.21}$$

Another one is a new, 5×10^{-4} level, strong-field confirmation of General Relativity:

$$\left[\frac{s^{\rm obs}}{s^{\rm GR}[k^{\rm obs}, R^{\rm obs}]}\right]_{0737-3039} = 1.0000 \pm 0.0005 \; . \tag{21.22}$$

Fig. 21.1 illustrates all the tests of strong-field and radiative gravity derived from the above-mentioned binary pulsars: $(3-2=)$ one test from PSR1913+16, $(5-2=)$ 3 tests from PSR1534+12, $(4-2=)$ 2 tests from PSR J1141−6545, and $(7-2=)$ 5 tests from PSR J0737−3039.

Data from several nearly circular binary systems (made of a neutron star and a white dwarf) have also led to strong-field confirmations (at the 4.6×10^{-3} level) of the 'strong

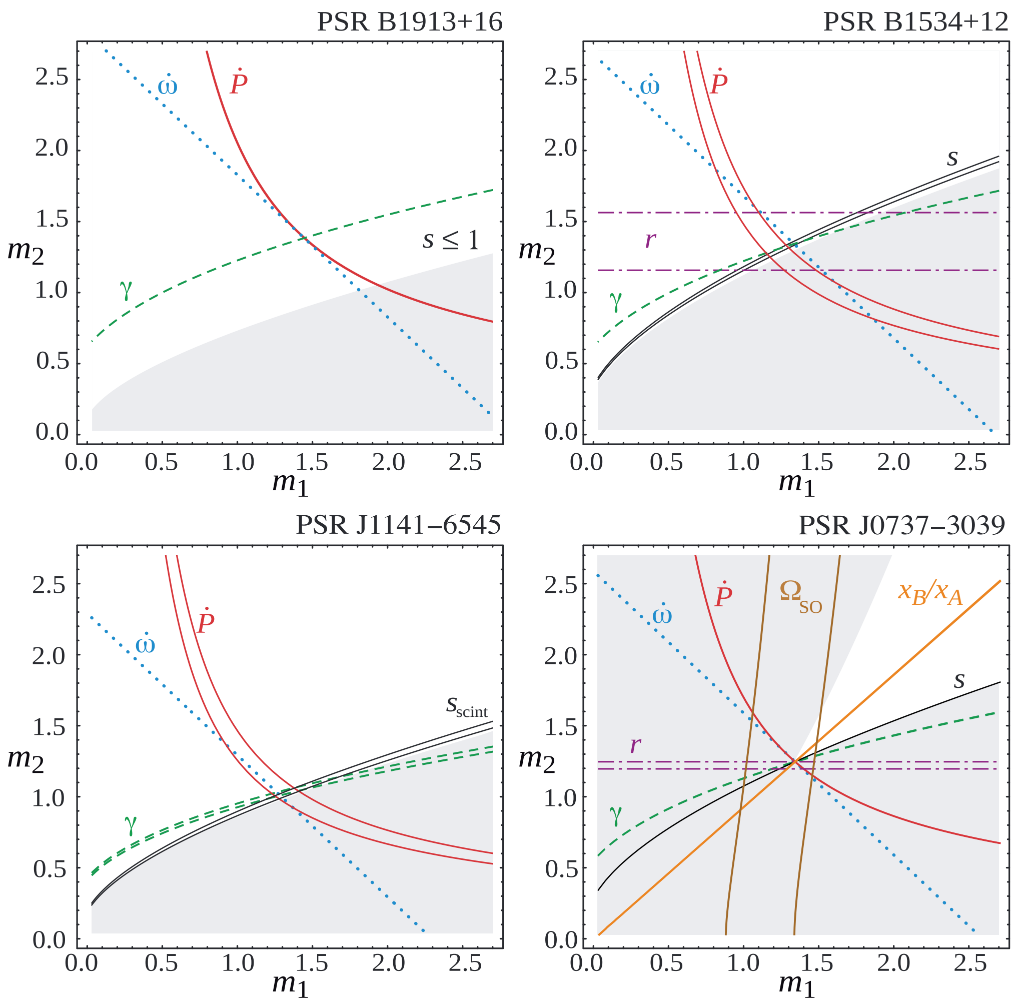

Figure 21.1: Illustration of the *eleven* tests of relativistic gravity obtained in the four different binary pulsar systems PSR1913+16 (one test), PSR1534+12 (3 tests), PSR J1141−6545 (2 tests), and PSR J0737−3039 A,B (5 tests). Each curve (or strip) in the mass plane corresponds to the interpretation, within General Relativity, of some observable parameter among: $\dot{P}_b$, $k \equiv \dot{\omega} P_b / 2\pi$, $\gamma_{\rm timing}$, r, $s = \sin i$, $\Omega_{\rm SO,B}$ and R. (Figure updated from [66]; courtesy of G. Esposito-Farèse.)

equivalence principle,' *i.e.*, the fact that neutron stars and white dwarfs fall with the same acceleration in the gravitational field of the Galaxy [56,57,58]. The measurements of $\dot{P}_b$ in some pulsar-white dwarf systems lead to strong constraints on the variation of Newton's G_N, and on the existence of gravitational dipole radiation [59,60,61,62]. In addition, arrays of millisecond pulsars are sensitive detectors of ultra low frequency gravitational waves ($f \sim 10^{-9} - 10^{-8}$ Hz) [63,64]. Such waves might be generated by supermassive black-hole binary systems, by cosmic strings and/or during the inflationary era. The sensitivity of pulsar timing arrays is comparable to predicted gravitational wave signal levels [65].

The constraints on tensor-scalar theories provided by the various binary-pulsar "experiments" have been analyzed in [39,66,67,61] and shown to exclude a large portion of the parameter space allowed by solar-system tests. The most stringent tests follow from the measurement of the orbital period decay $\dot{P}_b$ of the low-eccentricity 8.5-hour pulsar-white dwarf system PSR J1738+0333 with [61]

$$\left[\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm gal} - \dot{P}_b^{\rm GR}\right]_{1738+0333} = (2.0 \pm 3.7) \times 10^{-15}. \tag{21.23}$$

Dissymetric binary systems are strong emitters of dipolar gravitational radiation in tensor-scalar theories, with $\dot{P}_b$ scaling (modulo matter-scalar couplings) like $m_1 m_2/(m_1+m_2)^2(v/c)^3$ ($\sim 10^{-9}$ for PSR J1738+0333), instead of the smaller quadrupolar radiation $\dot{P}_b \sim (v/c)^5$ [2,31]. Thereby, the result Eq. (21.23) constrains the basic matter-scalar coupling α_0^2 more strongly, over most of the parameter space, than the best current solar-system limits Eq. (21.15), Eq. (21.16) (namely below the 10^{-5} level) [61]. In the particular case of the Jordan-Fierz-Brans-Dicke theory, the pulsar bound on α_0^2 is (when choosing an equation of state of medium stiffness) $\alpha_0^2 < 2 \times 10^{-5}$, wich is within a factor two of the Cassini bound Eq. (21.15) (where $\overline{\gamma} = -2\alpha_0^2/(1+\alpha_0^2)$).

Finally, measurements over several years of the pulse profiles of various pulsars have detected secular profile changes compatible with the prediction [68] that the general relativistic spin-orbit coupling should cause a secular change in the orientation of the pulsar beam with respect to the line of sight ("geodetic precession"). Such confirmations of general-relativistic spin-orbit effects were obtained in PSR 1913+16 [69], PSR B1534+12 [70], PSR J1141−6545 [71], and PSR J0737−3039 [53].

The tests considered above have examined the gravitational interaction on scales between a fraction of a millimeter and a few astronomical units. The general relativistic action on light and matter of an external gravitational field have been verified on much larger scales in many gravitational lensing systems [72]. Some tests on cosmological scales are also available [73].

21.4. Conclusions

All present experimental tests are compatible with the predictions of the current "standard" theory of gravitation: Einstein's General Relativity. The universality of the coupling between matter and gravity (Equivalence Principle) has been verified around the 10^{-13} level. Solar system experiments have tested the weak-field predictions of Einstein's theory at the 10^{-4} level (and down to the 2×10^{-5} level for the post-Einstein parameter $\overline{\gamma}$). The propagation properties of relativistic gravity, as well as several of its strong-field aspects, have been verified at the 10^{-3} level (or better) in several binary pulsar experiments. Recent laboratory experiments have set strong constraints on sub-millimeter modifications of Newtonian gravity. Quantitative confirmations of General Relativity have also been obtained on astrophysical and cosmological scales (assuming dark matter and a cosmological constant).

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