τ BRANCHING FRACTIONS

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There are now 42 measurements and 23 upper limits from Belle and BaBar on branching fractions of conventional τ-decay modes, up from 1 measurement and 3 upper limits in the 2006 edition of this Review. Eighteen of these measurements are used in the constrained fit to τ branching fractions, and 22 are for τ-decay modes for which older non-B-factory measurements exist. For those 22 measurements, the new B-factory measurements have on average about sixty times the number of events as the most precise earlier measurements, and the statistical uncertainties on the B-factory measurements are on average about eight times smaller. However, the systematic uncertainties now greatly exceed the statistical uncertainties of all B-factory branching fraction measurements of major τ-decay modes. For example, the average ratio of systematic to statistical uncertainty of the B-factory measurements of τ branching fractions larger than 10^{-3} is 17.6, while the average ratio for branching fractions smaller than 10^{-4} is 0.9. Thus, the total uncertainty on the branching fraction measurements from B-factories is on average only about 3.8 times smaller than the previous most precise non-B-factory measurements.

The constrained fit to τ branching fractions: The Lepton Summary Table and the List of τ-Decay Modes contain branching fractions for 131 conventional τ-decay modes and upper limits on the branching fractions for 39 other conventional τ-decay modes. Of the 131 modes with branching fractions, 82 are derived from a constrained fit to τ branching fraction data. The goal of the constrained fit is to make optimal use of the experimental data to determine τ branching fractions. For example, the branching fractions for the decay mode $\tau^- \rightarrow \pi^-\pi^+\pi^-\pi^0\nu_\tau$ is determined mostly from experimental measurements of the branching fraction for $\tau^- \rightarrow h^-h^-h^+\pi^0\nu_\tau$ and measurements of exclusive branching fractions for 3-prong modes containing charged kaons and 1 π^0.

Branching fractions from the constrained fit are derived from a set of basis modes. The basis modes form an exclusive set whose branching fractions are constrained to sum exactly to one. The set of selected basis modes expands as branching fraction measurements for new τ-decay modes are published. The number of basis modes has expanded from 12 in the year 1994 fit to 31 in the 2002 through 2013 fits. The 31 basis modes selected for the 2013 fit are listed in Table 1. See the 1996 edition of this *Review* [1] for a complete description of our notation for naming τ-decay modes and the selection of the basis modes. For each edition since the 1996 edition, the changes in the selected basis modes from the previous edition are described in the τ Branching Fractions Review. Figure 1 illustrates the basis mode branching fractions from the 2013 fit.

In selecting the basis modes, assumptions and choices must be made. For example, we assume the decays $\tau^- \to \pi^- K^+ \pi^- \geq 0\pi^0 \nu_\tau$ and $\tau^- \to \pi^+ K^- K^- \geq 0\pi^0 \nu \tau$ have negligible branching fractions. This is consistent with standard model predictions for τ decay, although the experimental limits for these branching fractions are not very stringent. The 95% confidence level upper limits for these branching fractions in the current Listings are $\mathrm{B}(\tau^- \to \pi^- K^+ \pi^- \geq 0\pi^0 \nu_\tau) < 0.25\%$ and $\mathrm{B}(\tau^- \to \pi^+ K^- K^- \geq 0\pi^0 \nu_\tau) < 0.09\%$, values not so different from measured branching fractions for allowed 3-prong modes containing charged kaons. Although our usual goal is to impose as few theoretical constraints as possible so that the world averages and fit results can be used to test the theoretical constraints (*i.e.*, we do not make use of the theoretical constraint from lepton universality on the ratio of the τ-leptonic branching fractions $\mathrm{B}(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)/\ \mathrm{B}(\tau^- \to e^- \overline{\nu}_e \nu_\tau) = 0.9726$), the experimental challenge to identify charged prongs in 3-prong τ decays is sufficiently difficult that experimenters have been forced to make these assumptions when measuring the branching fractions of the allowed decays. We are constrained by the assumptions made by the experimenters.

There are several τ-decay modes with small but well-measured (> 2.5 sigma from zero) branching fractions [2] which cannot be expressed in terms of the selected basis modes and are therefore left out of the fit:

Table 1: Basis modes and fit values(%) for the 2013 fit to τ branching fraction data.

$e^-\overline{\nu}_e\nu_\tau$	17.83 ± 0.04
$\mu^-\overline{\nu}_\mu\nu_\tau$	17.41 ± 0.04
$\pi^-\nu_\tau$	10.83 ± 0.06
$\pi^-\pi^0\nu_\tau$	25.52 ± 0.09
$\pi^-2\pi^0\nu_\tau$ (ex. K^0)	9.30 ± 0.11
$\pi^-3\pi^0\nu_\tau$ (ex. K^0)	1.05 ± 0.07
$h^-4\pi^0\nu_\tau$ (ex. K^0, η)	0.11 ± 0.04
$K^-\nu_\tau$	0.700 ± 0.010
$K^-\pi^0\nu_\tau$	0.429 ± 0.015
$K^-2\pi^0\nu_\tau$ (ex. K^0)	0.065 ± 0.023
$K^-3\pi^0\nu_\tau$ (ex. K^0, η)	0.048 ± 0.022
$\pi^-\overline{K}^0\nu_\tau$	0.84 ± 0.04
$\pi^-\overline{K}^0\pi^0\nu_\tau$	0.40 ± 0.04
$\pi^-K^0_SK^0_S\nu_\tau$	0.023 ± 0.002
$\pi^-K^0_SK^0_L\nu_\tau$	0.12 ± 0.04
$K^-K^0\nu_\tau$	0.159 ± 0.016
$K^-K^0\pi^0\nu_\tau$	0.159 ± 0.020
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0, ω)	8.99 ± 0.06
$\pi^-\pi^+\pi^-\pi^0\nu_\tau$ (ex. K^0, ω)	2.70 ± 0.08
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	0.294 ± 0.015
$K^-\pi^+\pi^-\pi^0\nu_\tau$ (ex. K^0, η)	0.078 ± 0.012
$K^-K^+\pi^-\nu_\tau$	0.144 ± 0.005
$K^-K^+\pi^-\pi^0\nu_\tau$	0.0061 ± 0.0025
$h^-h^-h^+2\pi^0\nu_\tau$ (ex. K^0, ω, η)	0.10 ± 0.04
$h^-h^-h^+3\pi^0\nu_\tau$	0.023 ± 0.006
$3h^-2h^+\nu_\tau$ (ex. K^0)	0.0839 ± 0.0035
$3h^-2h^+\pi^0\nu_\tau$ (ex. K^0)	0.0178 ± 0.0027
$h^-\omega\nu_\tau$	2.00 ± 0.08
$h^-\omega\pi^0\nu_\tau$	0.41 ± 0.04
$\eta\pi^-\pi^0\nu_\tau$	0.139 ± 0.010
$\eta K^-\nu_\tau$	0.0152 ± 0.0008

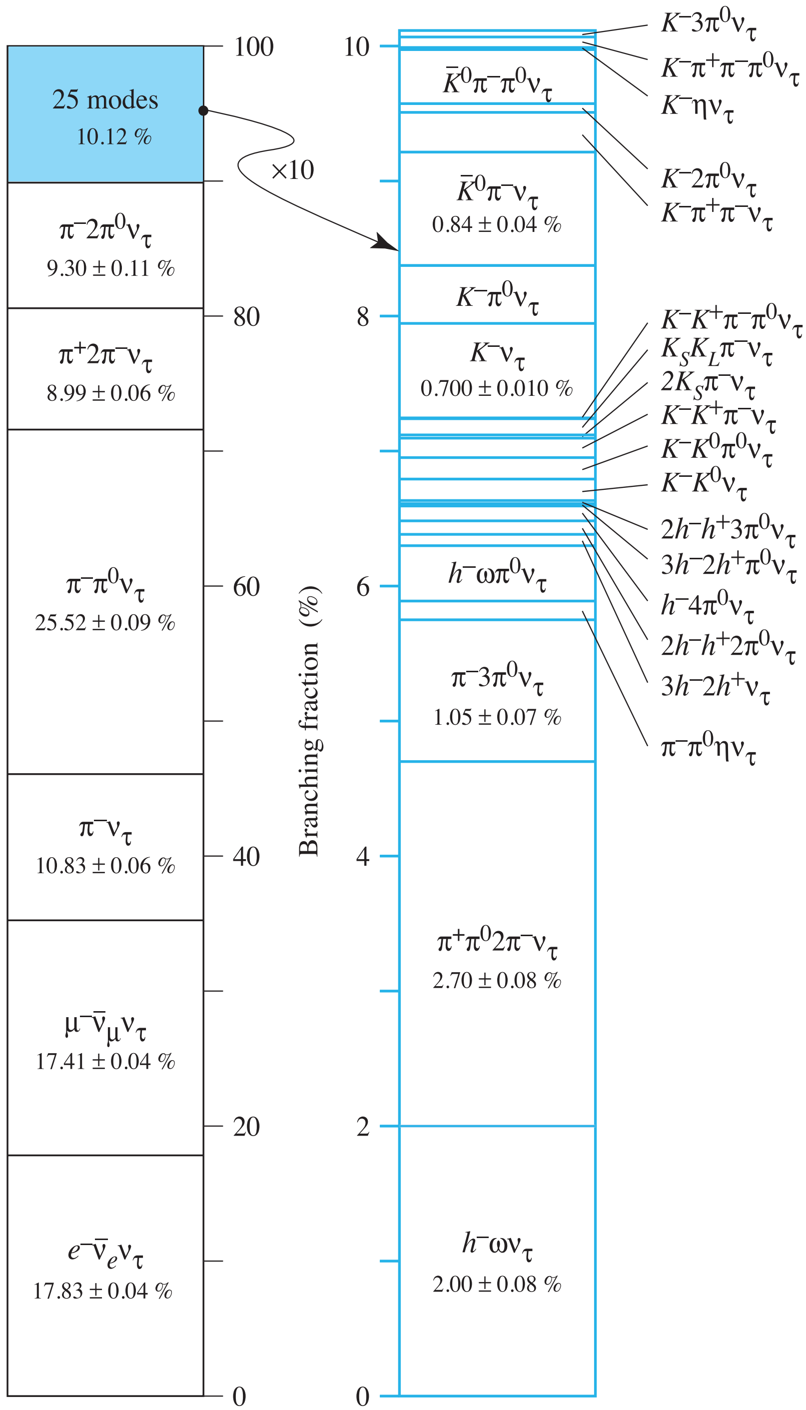

Figure 1: Basis mode branching fractions of the τ. Six modes account for 90% of the decays, 25 modes account for the last 10%. The list of excluded intermediate states for each basis mode has been suppressed.

$$\begin{aligned}
\mathrm{B}(\tau^- \to \pi^- K^0_S K^0_L \pi^0 \nu_\tau) &= (3.1 \pm 1.2) \times 10^{-4} \\
\mathrm{B}(\tau^- \to 2K^- K^+ \nu_\tau) &= (0.21 \pm 0.08) \times 10^{-4} \\
\mathrm{B}(\tau^- \to \eta K^- \pi^0 \nu_\tau) &= (0.48 \pm 0.12) \times 10^{-4} \\
\mathrm{B}(\tau^- \to \eta \overline{K}^0 \pi^- \nu_\tau) &= (0.93 \pm 0.15) \times 10^{-4}.
\end{aligned}$$

Certain components of other small but well-measured τ-decay modes cannot be expressed in terms of the selected basis modes and therefore are also left out of the fit:

$$\begin{aligned}
&\mathrm{B}(\tau^- \to \eta \pi^- \pi^0 \pi^0 \nu_\tau)\times \\
&\quad \mathrm{B}(\eta \to \gamma\gamma \text{ or } \eta \to \pi^+\pi^-\gamma \text{ or } \eta \to 3\pi^0) = (1.4 \pm 0.2) \times 10^{-4}, \\
&\mathrm{B}(\tau^- \to \eta \pi^- \pi^+ \pi^- \nu_\tau)\times \\
&\quad \mathrm{B}(\eta \to \gamma\gamma \text{ or } \eta \to \pi^+\pi^-\gamma) = (0.99 \pm 0.06) \times 10^{-4}, \\
&\mathrm{B}(\tau^- \to \phi K^- \nu_\tau)\times \\
&\quad \mathrm{B}(\phi \to K^0_S K^0_L \text{ or } \phi \to \eta\gamma) = (0.13 \pm 0.01) \times 10^{-4}, \\
&\mathrm{B}(\tau^- \to f_1(1285) \pi^- \nu_\tau)\mathrm{B}(f_1(1285) \to \rho^0\gamma) = (0.22 \pm 0.06) \times 10^{-4}, \\
&\mathrm{B}(\tau^- \to h^- \omega \pi^0 \pi^0 \nu_\tau)\mathrm{B}(\omega \to \pi^0\gamma) = (0.12 \pm 0.04) \times 10^{-4}, \\
&\mathrm{B}(\tau^- \to 2h^- h^+ \omega \nu_\tau)\mathrm{B}(\omega \to \pi^0\gamma) = (0.10 \pm 0.02) \times 10^{-4}.
\end{aligned}$$

The sum of these excluded branching fractions is $(0.08 \pm 0.01)\%$. This is near our goal of 0.1% for the internal consistency of the τ Listings for this edition, and thus for simplicity we do not include these small branching fraction decay modes in the basis set.

Beginning with the 2002 edition, the fit algorithm has been improved to allow for correlations between branching fraction measurements used in the fit. If only a few measurements are correlated, the correlation coefficients are listed in the footnote for each measurement. If a large number of measurements are correlated, then the full correlation matrix is listed in the footnote to the measurement that first appears in the τ Listings. Footnotes to the other measurements refer to the first measurement. For example, the large correlation matrices for the branching fraction measurements contained in Refs. [3,4] are listed in Footnotes to the $\Gamma(e^-\overline{\nu}_e\nu_\tau)/\Gamma_{\text{total}}$ and $\Gamma(h^-\nu_\tau)/\Gamma_{\text{total}}$ measurements respectively. Sometimes experimental papers contain correlation coefficients between measurements using only statistical errors without including systematic errors. We usually cannot make use of these correlation coefficients.

The 2013 constrained fit has a χ^2 of 128.9 for 109 degrees of freedom, and is essentially unchanged from the 2012 fit.

Inconsistencies in the τ lepton Branching Fraction Data: Several inconsistencies are known to exist in the branching fraction measurements that are used to determine the τ-lepton branching fractions. The sources of the inconsistencies are unknown. The treatment of discrepant data used for fits and averages is described in the introduction of this *Review*. Of the 82 branching fractions that are derived from the constrained fit, 12 (15%) have scale factors that are 1.5 or larger, and the largest is 2.7. Of the 49 branching fractions that are not derived from the constrained fit, 32 make use of only one measurement. Of the 17 averages that make use of more than one measurement, 4 (24%) have scale factors that are 1.5 or larger, and the largest is 5.4. Ideograms for 8 branching fractions are currently displayed in the τ Listings.

The τ branching fraction measurements by BaBar and Belle tend to be smaller than the non-B-factory measurements. There are 22 B-factory branching fraction measurements of τ-decay modes for which older non-B-factory measurements exist. Comparing the B-factory branching fraction measurements to the earlier non-B-factory measurements reveals a systematic discrepancy between the two sets of measurements. Figure 2 shows a histogram of the normalized difference ((B-factory value minus non-B-factory value)/estimated uncertainty in the difference) for the 22 measurements. The value used for the non-B-factory measurement is the value listed in the latest edition of this *Review* prior to the first B-factory measurement for that decay mode. Nineteen of the 22 B-factory branching fraction measurements are smaller than the non-B-factory values. The average normalized difference between the two sets of measurements is -1.08 (-1.41 for the 11 Belle measurements and -0.75 for the 11 BaBar measurements). The Heavy Flavor Averaging Group (HFAG) analysis of τ branching fractions includes a similar comparison of the B-factory and non-B-factory measurements [6].

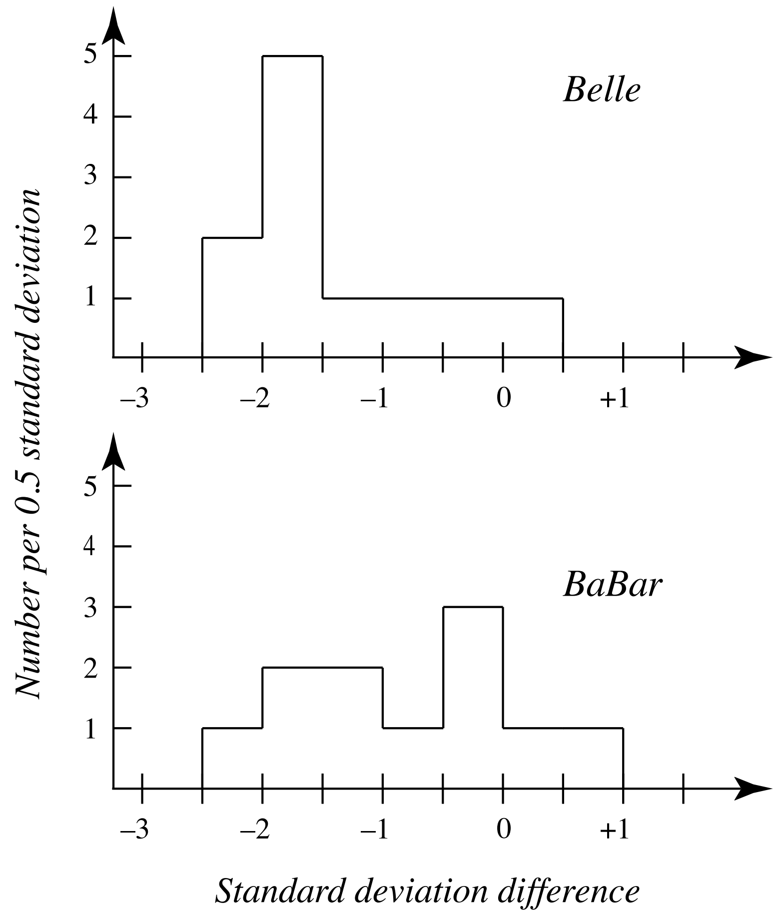

Figure 2: Distribution of the normalized difference between the 22 B-factory measurements of conventional τ-decay branching fractions and non-B-factory measurements. The Belle and BaBar collaborations have each published 11 measurements of τ-decay branching fractions for which older non-B-factory measurements exist.

Belle and BaBar have each published branching fraction measurements for the six τ-decay modes listed in Table 2. The normalized difference between the two measured values is calculated by subtracting the Belle value from the BaBar value and dividing this difference by the quadratic sum of the statistical and systematic errors for each measurement. When a measurement has asymmetric errors, the larger of the two values is used in the quadratic sum. It is apparent from the values in Table 2 that the Belle and BaBar values differ significantly for several of the τ-decay modes.

Table 2: Comparison of the Belle and Babar branching fraction measurements for the six τ-decay modes that both experiments have measured. The normalized difference is the difference between the Belle and BaBar branching fraction values divided by the quadratic sum of the statistical and systematic errors for both measurements.

Mode	BaBar − Belle Normalized Difference (#σ)
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	$+1.4$
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	-2.9
$K^-K^+\pi^-\nu_\tau$	-2.9
$K^-K^+K^-\nu_\tau$	-5.4
$\eta\ K^-\nu_\tau$	-1.0
$\phi\ K^-\nu_\tau$	-1.3

Overconsistency of Leptonic Branching Fraction Measurements: To minimize the effects of older experiments which often have larger systematic errors and sometimes make assumptions that have later been shown to be invalid, we exclude old measurements in decay modes which contain at least several newer data of much higher precision. As a rule, we exclude those experiments with large errors which together would contribute no more than 5% of the weight in the average. This procedure leaves five measurements for $\mathrm{B}_e \equiv \mathrm{B}(\tau^- \to e^-\overline{\nu}_e\nu_\tau)$ and five measurements for $\mathrm{B}_\mu \equiv \mathrm{B}(\tau^- \to \mu^-\overline{\nu}_\mu\nu_\tau)$. For both B_e and B_μ, the selected measurements are considerably more consistent with each other than should be expected from the quoted errors on the individual measurements. The χ^2 from the calculation of the average of the selected measurements is 0.34 for B_e and 0.08 for B_μ. Assuming normal errors, the probability of a smaller χ^2 is 1.3% for B_e and 0.08% for B_μ.

References

1. R.M. Barnett *et al.* (Particle Data Group), *Review of Particle Physics*, Phys. Rev. **D54**, 1 (1996).

2. See the τ Listings for references.
3. S. Schael *et al.* (ALEPH Collab.), Phys. Rep. **421**, 191 (2005).
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5. B. Aubert *et al.* (BaBar Collab.), Phys. Rev. Lett. **105**, 051602 (2010).
6. S. Banerjee *et al.* (HFAG), `http://arxiv.org/pdf/1101.5138v1.pdf`.