HYPERONS AND HEAVY MESONS (SYSTEMATICS AND DECAY[1])

By Murray Gell-Mann
Department of Physics, California Institute of Technology, Pasadena, California

AND

Arthur H. Rosenfeld
Department of Physics and Radiation Laboratory, University of California Berkeley, California

1. Introduction

We attempt, in this article, to summarize the information now available, both experimental and theoretical, on the classification and decays of hyperons and heavy mesons. Our principal emphasis is on the "weak interactions" responsible for the slow decays of these particles. The "strong interactions" involved in production and scattering phenomena form a separate topic, which we do not discuss at length. We do, however, mention the hyperfragments, the study of which bears on both kinds of interactions.

In Section 2 we take up some general features of elementary particle phenomena—the families of particles, the types of interactions, and some symmetry principles. The systematics of hyperons and heavy mesons is treated in Section 3 according to the "strangeness" theory. Section 4 is devoted to the weak interactions, not only of hyperons and heavy mesons, but also of the more familiar particles; we emphasize especially the recent work on nonconservation of parity and the classification of all weak processes. The detailed phenomenology of hyperon and heavy meson decays is discussed in Sections 5 and 6 respectively.

A great deal of the theoretical material is presented in a rather dogmatic way. We take for granted, for example, the correctness of charge independence, the strangeness theory, and the two-component neutrino, because we feel that an adequate discussion of the present experimental evidence on these questions would lead us too far afield. The reader must bear in mind that some of these theoretical principles may ultimately be proved wrong, although there is no doubt that they have been very useful so far.

In the phenomenological work we make extensive use of further hypotheses such as CP invariance, spin 0 for the heavy mesons, and spin $\frac{1}{2}$ for the hyperons. In the case of these assumptions, however, we give some discussion of the relevant experimental evidence, either in the text or in an appendix.

Most of the experimental data in the article are presented in the form of tables. In many cases, the numbers represent weighted averages of the results of many groups, and references to the individual experimental papers

[1] The survey of literature pertaining to this review was completed in July, 1957.

are to be found in the footnotes to the tables. We restrict ourselves, however, principally to the most recent work, and references to earlier research, including many pioneering experiments, must be looked for in the later papers. In particular, we apologize for having slighted the cosmic ray experiments, in which most of the salient features of the new particle field were first revealed.

Many references to theoretical papers are lacking also. An extensive list of these is given by Dalitz (R3). We present, at the beginning of the bibliography, six references to useful review articles and compilations of information on strange particles (R1 to R6).

2. Elementary Particles

2.1. *Field theory.*—We are at present very far from having a satisfactory theory of the elementary particles. The enumeration of the particles, with their spins, their masses, and the nature and strength of their couplings must be taken wholly or largely from experiment. If we are given this information, we can attempt a detailed description of particle phenomena by means of the quantum theory of fields, which is, in fact, the only apparatus we have for such a description. It is not known, however, whether field theory, with its strict requirements of microscopic causality and relativistic invariance, is applicable to phenomena at very small distances, say $\sim 10^{-14}$ cm.[2]

At larger distances, field theory has scored some success. Quantum electrodynamics is in excellent accord with experiment; the Yukawa theory, especially in the simplified form studied by Chew and collaborators, gives a semi-quantitative description of the nucleon-pion system; and the Fermi theory of β-decay, later extended to other weak processes, has been very useful. It may be worthwhile to apply the methods of field theory to the new particles as well, but we do not attempt anything of the kind in this section. Let us refer, however, to one or two very general results of field theory.

2.2. *Particle and antiparticle;* CPT *invariance.*—The connection between spin and statistics is an example of a general principle that can be proved from the basic structure of field theory. Particles of integral spin must obey Bose-Einstein statistics and are called "bosons"; those of odd half-integral spin obey Fermi-Dirac statistics and are called "fermions."

Recently (1) attention has been called to another general property of present-day field theory:

For every particle there exists an antiparticle (which may or may not be identical with the particle itself). For every possible state Ψ of a system of particles, there is a possible state Ψ' of the corresponding system of antiparticles which looks just like the state Ψ with space and time inverted.

We may restate the result in terms of invariance. We define an operation

[2] We use the symbol $\sim$ for "is of the order of magnitude of," $\approx$ for "is approximately equal to," and $\propto$ for "is proportional to."

C (called "charge conjugation") that carries each particle into its antiparticle without disturbing space-time, an operation P that reflects space coordinates, and an operation T that reverses time. Then what we are discussing is the automatic invariance of field theories under the product of these operations, CPT.[3]

The behavior of the electromagnetic field is such that particle and antiparticle have equal and opposite electric charge. Thus charged particles like π^+ and p have antiparticles distinct from themselves (π^- and $\bar{p}$, respectively). A neutral particle may be identical with its antiparticle (e.g., π^0 or the photon γ) or distinct from its antiparticle (e.g. n and $\bar{n}$).

From CPT invariance it follows that the spins, for example, of particle and antiparticle are the same and that their masses are exactly equal; if they are unstable, their lifetimes are equal as well (2).

In field theory, the destruction of a particle and the creation of its antiparticle are described by the same field operator and are closely related phenomena. In fact, we may say that scattering of a particle and the production and annihilation of particle-antiparticle pairs are all different forms of the same process. (For example, they are all described by the same Feynman diagram.)

2.3. *Types of interaction.*—All the known interactions of the elementary particles appear to fall into three categories: (*a*) The strong interactions, typified by the virtual Yukawa process $N \Leftrightarrow N+\pi$ principally responsible for nuclear forces; here, N represents a nucleon (n or p). The strength of this process may be measured by the dimensionless coupling constant $g^2/4\pi\hbar c \approx 15$, or alternatively by the constant

$$f^2/4\pi\hbar c = \left(\frac{m_\pi}{2m_N}\right)^2 g^2/4\pi\hbar c \approx .08.$$

In general, the strong couplings are characterized by coupling constants of the order of unity. (*b*) The electromagnetic interaction, through which a photon may be virtually emitted or absorbed by any charged particle, real or virtual. Here the universal parameter of strength is the fine structure constant $e^2/4\pi\hbar c \approx 1/137$. (*c*) The weak interactions, of which the classic example is the β-decay coupling that induces the decay $n \rightarrow p+e^-+\bar{\nu}$.[4] The Fermi constant C that measures the strength of β-decay[5] is $\sim 10^{-49}$ erg cm.[3] and can be written in dimensionless form only if a length is specified. For our purposes a convenient length is the Compton wave-length of the

[3] The CPT invariance of field theory arises essentially from the Hermiticity of the Hamiltonian, which is necessary to insure positive probabilities. Roughly speaking, the operation CPT for a conventional field theory Hamiltonian is equivalent to Hermitian conjugation.

[4] We use heavy arrows to denote strong or electromagnetic processes, whether real or virtual, for example, $n \Leftrightarrow p+\pi^-$, $\pi^0 \Leftrightarrow 2\gamma$. Light arrows indicate weak processes: $n \rightarrow p+e^-+\bar{\nu}$, etc.

[5] There are really several such constants C; they are defined in 4.1.

charged pion $\hbar/m_\pi c$; in terms of this we have $C{\sim}10^{-7}\,\hbar c\,(\hbar/m_\pi c)^2$. Actually the probability of β-decay is proportional to C^2 and we may therefore take as a reasonable dimensionless measure of the strength of the weak couplings a value like 10^{-14}. That this estimate works not only for β-decay but for all the known weak interactions is a remarkable law of nature, the "universality of strength" of the weak couplings (see 4.1).

2.4. *Families of particles.*—The known elementary particles can be classified in the following way (see Table I): (*a*) The photon, which is coupled to all charged particles by the electromagnetic interaction. No other coupling of the photon is known except this familiar interaction with charges and currents; the apparent absence of other couplings is sometimes referred to as the principle (3) of "minimal electromagnetic interaction."

Thus the "anomalous moments" of proton and neutron, for example, are attributed not to a special magnetic coupling of these particles but to the ordinary interaction of the electromagnetic field with the charges and currents of the meson cloud. (*b*) Baryons and antibaryons, which are fermions possessing strong couplings and satisfying a rigorous conservation law, the "conservation of baryons." This is the law responsible for the stability of nuclei; it states that baryons (such as the proton) cannot be created or destroyed except in baryon-antibaryon pair production and annihilation. However, one baryon may be transformed into another, as when a neutron turns into a proton in β-decay. The word "hyperon" means an elementary particle heavier than the nucleon. In fact, all the known hyperons are baryons: they are fermions and they are made from nucleons and ultimately decay into nucleons. The antihyperons have not yet been observed, but the recent discovery of the antiproton and antineutron makes it virtually certain that antihyperons exist also, as required by CPT invariance. (*c*) Mesons, which in our terminology are bosons possessing strong couplings. (We do not call the muon a meson, but a lepton.) Unlike baryons, mesons can be created or destroyed; they are radiated or absorbed in the course of baryon transformations much as photons are radiated or absorbed by charged particles. The lightest known meson is the pion. The term "K particle" is applied in principle to any particle intermediate in mass between pion and nucleon. In fact, the known K particles are "mesons," in our sense of the word. (*d*) Leptons, which are fermions possessing no strong couplings. (Strictly speaking, we should distinguish, as in Table I, between leptons and antileptons, but the nature of this distinction is only now becoming clear. See 4.4.)

The known leptons, all with spin $\frac{1}{2}$, are the electron and positron e^- and e^+, the negative and positive muon μ^- and μ^+, and the neutrino and antineutrino. The last two we shall denote by ν and $\bar{\nu}$ respectively or else, for convenience, by $+\nu$ and $-\nu$ respectively. Thus instead of writing "$\pi^+\to\mu^++\nu$ and $\pi^-\to\mu^-+\bar{\nu}$" we shall write "$\pi^\pm\to\mu^\pm\pm\nu^\pm$."

2.5. *Nonconservation of parity; hypothesis of* CP *invariance.*—We have mentioned in 2.2 the automatic invariance under CPT of field theories, for which one need assume only invariance under proper Lorentz transforma-

TABLE I

Masses and Lifetimes of Elementary Particles

	Particle	Spin	Mass (Errors represent standard deviation) (Mev)	Mass difference (Mev)	Mean life (sec)	Decay rate (number per second)
tons	γ	1	0		stable	0.0
tons and ntileptons	$\nu, \bar{\nu}$	$\frac{1}{2}$	0		stable	0.0
	e^-, e^+	$\frac{1}{2}$	0.510976*		stable	0.0
	μ^-, μ^+	$\frac{1}{2}$	105.70 ±0.06*		$(2.22\pm0.02)\times10^{-6}$*	0.45×10^{6}
ons	$\pi^\pm$	0	139.63 ±0.06*	4.6*	$(2.56\pm0.05)\times10^{-8}$*	0.39×10^{8}
	π^0	0	135.04 ±0.16*		$(0.0<\tau<0.4)\times10^{-15}$ (O)	$>2.5\times10^{15}$
	$K^\pm$	0	494.0 ±0.20 (a)		$(1.224\pm.013)\times10^{-8}$ (b)	0.815×10^{8}
				1±5	K_1: $(0.95\pm.08)\times10^{-10}$ (P)	1.05×10^{10}
	K^0	0	493 ±5 (Th)		K_2: $(3<\tau<100)\times10^{-8}$ (L)(P)	$(>0.01<0.3)\times10^{8}$
ons†	p	$\frac{1}{2}$	938.213±0.01*		stable	0.0
	n	$\frac{1}{2}$	939.506±0.01*		$(1.04\pm0.13)\times10^{+3}$ *	0.96×10^{-3}
	Λ	$\frac{1}{2}$?	1115.2 ±0.13 (B)		$(2.77\pm0.15)\times10^{-10}$ (d)	0.36×10^{10}
	Σ^+	$\frac{1}{2}$?	1189.3 ±0.35 (B)	7.1±0.4	$(0.78\pm0.074)\times10^{-10}$ (e)	1.28×10^{10}
	Σ^-	$\frac{1}{2}$?	1196.4 ±0.5 (B)	7.6_{-2}^{+3}	$(1.58\pm0.17)\times10^{-10}$ (f)	0.64×10^{10}
	Σ^0	$\frac{1}{2}$?	1188.8_{-1}^{+2} (g)		$(<0.1)\times10^{-10}$ (A)	$>10\times10^{10}$
					theoretically $\sim10^{-19}$	theoretically $\sim10^{19}$
	Ξ^-	?	1321 ±3.5*		$(4.6<\tau<200)\times10^{-10}$ (Tr)	$(>0.005, <0.2)\times10^{10}$
	Ξ^0	?	?		?	

From compilations by Cohen, Crowe, and DuMond, *Nuovo cimento*, **5**, 541 (1957), and "Fundamental Constants of Phys- to be published by Interscience, New York, 1957. They include all data available before January 1, 1957.

Antibaryons have the same spin, mass, and mean life as baryons.

) Alvarez, Bradner, Falk-Vairant, Gow, Rosenfeld, Solmitz, and Tripp, K^- Interactions in Hydrogen, UCRL-3775, May 1957.

) Barkas and Rosenfeld UCRL-8030, (1957).

) Lande, Booth, Impeduglia, Lederman, and Chinowsky, *Phys. Rev.*, **103**, 1901 (1956).

) Orear, Harris, and Taylor, *Bull. Am. Phys. Soc.*, **2**, 26.

) Plano, Samios, Schwartz, and Steinberger, *Phys. Rev.* (to be published) 1957.

h) Thompson, Burwell, and Huggett, Supplemento 2 *Nuovo cimento*, **4**, 286 (1956).

r) G. H. Trilling and G. Neugebauer, *Phys. Rev.*, **104**, 1688 (1956).

) R. S. White, compilation of all emulsion data available from all laboratories, prepared for 7th Rochester Conference (private communication).

$m_{K^\pm}=3m_{\pi^\pm}+Q_\tau$, where Q_τ is the weighted average from Heckman, Smith, and Barkas, *Nuovo cimento*, **4**, 51 (56); from Roy Haddock, *Nuovo cimento*, **4**, 240 (56); and from Bacchella, Berthelot, *et al.*, *Nuovo cimento*, **4**, 1529 (56). We have assumed that the K^- is the antiparticle of the K^+ and shares the same mass and lifetime. The present experimental mass of the K^- is consistent with this assumption, namely 493.4±0.5 Mev (W).

Weighted average of

1.227±0.015 (Alvarez, Crawford, Good, and Stevenson, *Phys. Rev.* (to be published)).

1.211±0.026 (V. Fitch and R. Motley, *Phys. Rev.*, **101**, 496 (1956); *Phys. Rev.*, **105**, 265 (1957); and private communication). The quoted errors are statistical only. We have assumed that the K^- is the antiparticle of the K^+ and shares the same mean life. The present experimental mean life is consistent with this assumption, namely $\tau_{K^-}=1.25\pm0.11$ (W. H. Barkas, Seventh Rochester Conference).

Weighted average of

1.9±0.4 (Graves, Brown, Glaser, and Perl, *Bull. Am. Phys. Soc.*, **2**, 221 (1957)).

2.77±0.2 (Eisler, Plano, Samios, Steinberger, and Schwartz, *Bull. Am. Phys. Soc.*, **2**, 221 (1957)).

3.1±0.5 (A)

3.25±0.33*

Weighted average of

0.95±0.30 (Graves, Brown, Glaser, and Perl, *Bull. Am. Phys. Soc.*, **2**, 221 (1957)).

0.69±0.1 (A)

0.89±0.12 (compilation of all emulsion data available from all laboratories, prepared for 7th Rochester Conference by G. Snow (private communication)).

Weighted average of

1.5±0.35 (Eisler, Plano, Samios, Steinberger, and Schwartz, *Bull. Am. Phys. Soc.*, **2**, 221 (1957)).

1.6±0.2 (A)

Combined result from Alvarez *et al.*, K^- Interactions in Hydrogen, UCRL-3583, Nov. 1956, and a private communication from M. Schwartz and R. Plano giving $Q=73.5\pm3.5$ for $\Sigma^0\rightarrow\Lambda+\gamma+Q$.

tions (those not involving space or time inversion). Until recently it was thought that the laws of physics were also invariant under C, P, and T separately. (For the strong and electromagnetic interactions, this still appears to be true. See 3.10.)

Let us discuss in particular invariance under P, which is equivalent to the conservation of the quantum number called parity (also denoted by P). If parity is exactly conserved, then physical laws do not distinguish between right and left: the mirror reflection of any state of a system of particles is also a possible state of the same system of particles. The conservation of parity is now known to be violated by the weak interactions. This was first suspected in K particle decay (see Section 6). It was then suggested by Lee and Yang (4) that nonconservation of P be looked for in nuclear β-decay and in the decays of $\pi^{\pm}$ and $\mu^{\pm}$. Their conjectures were brilliantly confirmed by a series of experiments early in 1957 (see Section 4). As an example, we may take the β^--decay of oriented Co^{60}. The spinning nucleus emits electrons preferentially "down," where "up" is defined by the nuclear spin using the right hand rule. The angular distribution of electrons is of the form $1+a \cos\theta$ with a around -1 for the fastest electrons (5); the "up-down" ratio is then $(1+a)/(1-a)$. Clearly parity is not conserved; the mirror image of decaying Co^{60} is not a possible state of Co^{60}.

Does this mean that the microscopic laws of nature do define a right hand? Not necessarily. There is still the possibility that physical laws are exactly invariant under CP, i.e., that the mirror reflection of a state of a system of particles is always a possible state of the corresponding system of antiparticles. For example, the β^+-decay of anti-Co^{60} would then have an angular distribution exactly the opposite of that of Co^{60}, namely $1-a\cos\theta$. We could then not define a right hand by means of β-decay unless we specified that we were talking about matter and not antimatter. But there would be no intrinsic way to tell matter from antimatter except the very handedness we are trying to define.

Exact invariance under CP was predicted by Landau (6). So far all the experiments on nonconservation of parity are consistent with this hypothesis, but further tests of its validity are still required.[6] In Appendix A we discuss the possibility of interpreting present information without CP invariance, but in the text we shall make extensive use of the hypothesis.

Let us emphasize here that if CP invariance should fail in β-decay, then for a nucleus like Ag^{111}, if it could be oriented, the β^- angular distribution would be, say, $1+b\cos\theta$ and yet the β^+ angular distribution of oriented

[6] Given nonconservation of P the conservation of CP implies, of course, nonconservation of C. The nonconservation of C has been verified spectacularly in all of the parity experiments involving neutrinos, which suggest that the neutrino is always right circularly polarized, while its charge conjugate, the antineutrino, is always left circularly polarized. (See Section 4.) This is evidently a clear-cut violation of C invariance.

anti-Ag^{111} would be $1-b' \cos\theta$ with b' different from b.[7] This difference would permit an absolute definition of matter and antimatter and of right and left.

2.6. *Consequences of* CP *invariance;* T *invariance.*—If CP is an exact symmetry of nature, then isolated matter and antimatter differ only in handedness. The decay schemes of particle and antiparticle are always exactly the same and their decays mirror images of each other. (We assume here that particle and antiparticle always decay into different final states, as π^+ and π^- do because of conservation of charge or as n and $\bar{n}$ do because of conservation of baryons. When particle and antiparticle can decay into the same final states, we have a quite different situation like the one described in 3.9.)

Just as P invariance would be equivalent to the conservation of the quantum number P, so CP invariance is equivalent to the conservation of a quantum number CP. However, CP is less directly useful as a quantum number than P would be, since a system of particles can be in an eigenstate of CP only when it is neutral, has equal numbers of baryons and antibaryons. etc. For example, a deuteron cannot be in an eigenstate of CP, since charge conjugation would turn it into an antideuteron.

However, a neutral pair of pions $\pi^+ + \pi^-$ or $2\pi^0$ in their center of mass system *are* in an eigenstate[8] of CP with eigenvalue $+1$. In 3.9 we shall use this result, together with the assumption of exact CP invariance, to prove that a particle (K_2^0) with $CP = -1$ cannot decay into two pions, even through the weak interactions.

If CP is exactly conserved, then the CPT invariance of field theory guarantees exact symmetry under time reversal T. Now T invariance does not correspond directly to a conservation law. Instead, it determines the *phases* of transition matrix elements (7a, 7b). The contrast between the behavior of T and that of other symmetry operators, such as CP or angular momentum, can be appreciated if we introduce the famous unitary operator

[7] We have chosen Ag^{111} rather than Co^{60} as an example here because the difference between b' and b probably vanishes for Co^{60}, while it must be non-zero for Ag^{111} if CP invariance is violated in β-decay and if the two-component theory of the neutrino is correct. This can be shown, using scalar and tensor (or vector and axial vector) interactions (and assuming that nuclear matrix elements do not accidentally vanish), from the fact that the transition is of the form $\frac{1}{2}^+ \rightarrow \frac{1}{2}^-$. The difference $b'-b$ arises from a Coulomb effect.

[8] For two identical bosons such as $2\pi^0$, the wave function must be symmetric under the operator X that exchanges the two particles, i.e., $X = +1$. For π^+ and π^-, which are not identical, we can define a generalized wave function by including a charge coordinate that distinguishes π^+ and π^-. This generalized wave function must again be symmetric under the exchange operator X, since boson field operators commute.

Now for either case, $2\pi^0$ or $\pi^+ + \pi^-$, the operator X is identical with CP in the center of mass system, because CP interchanges both charge and position for the two particles. Thus $CP = X = +1$.

$\mathcal{S}$. Often called the $\mathcal{S}$ matrix, it transforms initial into final states in a collision problem. The law of conservation of CP may be written in the form $CP\mathcal{S}=\mathcal{S}CP$ and this implies that the eigenvalue of CP is the same before and after the collision. In the case of time reversal, however, the initial and final states must be interchanged, so that T invariance implies $T\mathcal{S}=\mathcal{S}^{-1}T$. This relation does not give selection rules; but when it is combined with the unitarity of $\mathcal{S}$, it gives conditions on the phases of $\mathcal{S}$ matrix elements. For example, in the photopion effect $\gamma+N\Rightarrow\pi+N$ there is the following familiar result (7b). The phase of the matrix element leading to a final state of given energy, angular momentum, parity, and isotopic spin is given by the phase shift for the scattering $\pi\Leftrightarrow N+\pi+N$ in the same state. (Strictly speaking, this particular statement is true only to lowest order in the fine structure constant.)

Landau (6) has pointed out that CP or T invariance forbids the existence of static electric dipole moments for elementary particles, even when P invariance is violated. Consider the neutron, for example. A tiny fraction of the time it is virtually dissociated into proton, electron, and antineutrino. The wave function ψ of this system contains both even and odd parity terms ψ_e and ψ_0, respectively, since P is not conserved in β-decay. We might therefore expect that the electric dipole moment operator $\boldsymbol{D}$ could have a small expectation value $(\psi_e, \boldsymbol{D}\psi_0)+(\psi_0, \boldsymbol{D}\psi_e)$ for the neutron. However, it can be shown that T invariance requires ψ_e and ψ_0 to be 90° out of phase, so that the expectation value of the real operator $\boldsymbol{D}$ is zero.

Of course if two elementary particles with spin were degenerate with each other and electromagnetic transitions were allowed between them, then a static electric dipole moment might arise (approximately as it does for molecules.) No such situation is known, however, among the particles.

3. The New Particles; Strangeness Theory

3.1. *Charge independence.*—A striking property of the strong interactions and the particles that possess them (baryons, antibaryons, and mesons) is the principle of *charge independence* or *conservation of isotopic spin* (8).

Each strongly coupled particle belongs to a charge multiplet with an isotopic spin quantum number I and multiplicity $2I+1$. The components of the multiplet are characterized by values of I_z ranging from $-I$ to I; corresponding to the variation of I_z there is a variation in the electric charge which increases in steps of e as I_z increases in steps of one. These properties are illustrated in Figure 1 and by Eq. 1 below. Each multiplet carries an isotopic spin vector $\boldsymbol{I}$ and the charge independence of the strong interactions means that they conserve the total $\boldsymbol{I}$ in any process. Similarly, the strong interactions leave all charge multiplets rigorously degenerate.

Electromagnetic interactions are manifestly charge dependent. They violate the conservation of isotopic spin and remove the degeneracy of the charge multiplets. For example, the n-n and p-p forces are identical as far as the strong interactions are concerned, but the Coulomb force obviously destroys the equality. Similarly, electromagnetic effects presumably give rise to the mass difference between neutron and proton. (See 3.5.)

3.2. *Conservation of strangeness.*—Although it violates conservation of I, the electromagnetic interaction does not affect the conservation of I_z. We can understand this on the basis of "minimal electromagnetic interaction." The photon is coupled only through the charge; since the charge is a function of I_z, we see that the electromagnetic coupling transforms in isotopic spin space like a function of I_z and commutes with the total I_z.

The conservation of I_z is usually restated in a more convenient form. We write the relation between charge and I_z for the members of a multiplet as follows:

$$Q/e = I_z + \tfrac{1}{2}Y \qquad 1.$$

Here $\frac{1}{2}Y$ is the center of charge, or average charge, of the multiplet; it is indicated by a fulcrum in Figure 1. Since Q/e is always integral, we see that Y is an even integer when I is integral and an odd integer when I is half-integral. Since Q is rigorously conserved, the conservation of I_z is the same as the conservation of Y.

A still more convenient notation can be used if we introduce the quantity

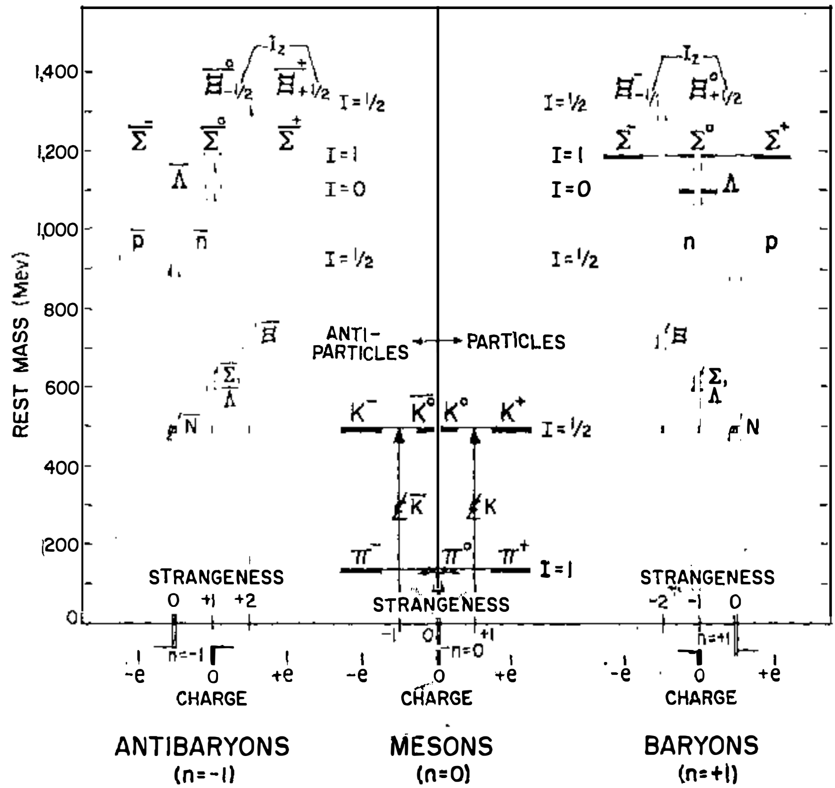

FIG. 1. Strongly interacting particles (SIPs). Particles unstable against electromagnetic decay are represented with a dotted bar; the rest are drawn solid.

n, the number of baryons minus the number of antibaryons. The law of conservation of baryons then states that the "baryon number" n is rigorously conserved. For the nucleon, antinucleon, and pion the center of charge $\frac{1}{2}Y$ is just equal to $\frac{1}{2}n$; that is, it is $+\frac{1}{2}$ for the nucleon, and $-\frac{1}{2}$ for the antinucleon, and 0 for the pion. We can measure the displacement of the center of charge from its familiar values, therefore, by writing $Y=n+S$ or

$$Q/e = I_z + \tfrac{1}{2}n + \tfrac{1}{2}S \qquad 2.$$

S is then an integer, conserved whenever I_z is, and zero in the case of nucleon (N), antinucleon ($\overline{N}$), and π. It is called "strangeness" and the strongly coupled particles for which $S\neq 0$ are "strange" particles. The quantum number S is conserved by both strong and electromagnetic interactions; since these conserve I_z; only the weak interactions can violate *conservation of strangeness* (3, 9, 9a).

3.3. *The strange particles.*—The known hyperons are all strange baryons and the known K particles are strange mesons (right-hand column of Fig. 1.)

Let us consider the baryons first. The nucleon N, of course, is a doublet with $S=0$, consisting of the proton p and the neutron n. The strange baryons are, in order of increasing mass: Λ, a singlet with $S=-1$ (Λ). Σ, a triplet with $S=-1$ (Σ^+, Σ^0, Σ^-). Ξ, a doublet with $S=-2$. (Ξ^0, Ξ^-. The Ξ^0 is so far hypothetical.)

The antibaryons (left-hand column of Fig. 1) follow the same pattern, but with the signs of charge, strangeness, and I_z all reversed.[9] So far only the antinucleons have been detected experimentally.

Finally we have the mesons (middle column of Fig. 1). The pion is a triplet with $S=0$ (π^+, π^0, π^-). The strange mesons are a pair of doublets: K^+ and K^0, with $S=+1$, and $\overline{K^0}$ and K^- with $S=-1$. We shall speak of the first doublet collectively as K and the second as $\overline{K}$; this notation emphasizes that K and $\overline{K}$ are each other's antiparticles.

In principle the symbol K should be reserved for the class of all particles with masses between m_π and m_N. However, this causes no difficulty at present since the specific particles we call K and $\overline{K}$ are the only members of the class now known.

3.4. *Conservation of S and I in particle reactions.*—The rule that $\Delta S=0$ in strong and electromagnetic processes gives not only particle stability, as discussed in 3.2, but also severe restrictions on particle reactions. For example, in all collisions of nucleons, pions, and antinucleons the initial strangeness is zero. If in such collisions a strange particle is produced ($S\neq 0$), it must be accompanied by at least one other strange particle; this is the famous law of "associated production." However, the conservation of strangeness is much more stringent than the requirement of associated production. For example, processes like $\pi+N\Rightarrow\overline{K}+\Sigma$ or $N+N\Rightarrow\Lambda+\Lambda$ are forbidden while $\pi+N\Rightarrow K+\Sigma$ and $\gamma+N\Rightarrow K+\Lambda$ are allowed.

[9] We can see that this arrangement is necessary since production and annihilation of particle-antiparticle pairs must be possible without violating conservation laws. (See 2.2.)

Besides the conservation of I_z or strangeness, we must take into account conservation of total I (or charge independence) in strong processes involving the new particles. Of course the latter law requires corrections from electromagnetic effects, which the former does not.

Charge independence tells us, for example, that the force between Λ and the proton is the same as between Λ and the neutron. It gives us also intensity rules like the following (3, 10):

$$d\sigma(K^- + d \Rightarrow \Lambda + n + \pi^0) = \tfrac{1}{2} d\sigma(K^- + d \Rightarrow \Lambda + p + \pi^-) \qquad 3.$$

and

$$d\sigma(K^- + d \Rightarrow \Sigma^0 + \pi^- + p) = d\sigma(K^- + d \Rightarrow \Sigma^- + \pi^0 + p). \qquad 4.$$

The electromagnetic corrections to such rules are small (perhaps of the order of the fine structure constant ~1 per cent in amplitude) and the rules are therefore quite useful. These rules have not yet been tested in experiments with strange particles, but they can easily be checked by stopping K^- in a deuterium bubble chamber.

3.5. *Electromagnetic mass differences.* Apart from the Coulomb force, the most striking effect of the electromagnetic violation of charge independence is the removal of degeneracy of charge multiplets. The neutron-proton mass difference of 1.3 Mev has, of course, been known for a long time. Its electromagnetic origin was long in doubt, since it was hard to understand why the neutron should be heavier. Recently Feynman & Speisman (11) have shown, by means of a crude model, that electromagnetic effects could give a heavier neutron. However, we have no quantitative theory of the mass differences, and we shall restrict ourselves to the experimental facts. (See Table I.)

The mass difference between charged and neutral K particles is very poorly known; the mass of K^+ has been accurately determined but we know about the mass of K^0 only that it is the same to within ~5 Mev. The mass difference of charged and neutral pions (4.6 Mev) is best measured by means of the Panofsky effect: $\pi^- + p \Rightarrow \pi^0 + n$ for captured π^-.

Next the baryons—we have mentioned the nucleon above; we have no information about Ξ since Ξ^0 has never been detected; Λ is a singlet; thus we have only the Σ triplet to discuss. The difference in mass of Σ^+ and Σ^- (7.2 ± 0.1 Mev with Σ^- heavier, see Table I) has been determined as follows by emulsion workers. About 1 per cent of the K^- which come to rest in emulsion are captured by free protons to give the reactions $K^- + p \Rightarrow \Sigma^\pm + \pi^\mp$. The difference in the Σ ranges $R_- - R_+$ leads to an accurate determination of the mass difference. The difference in mass between the charged and neutral Σ cannot be measured with such an accurate tool as emulsions, and at present we know only that Σ^0 and Σ^+ have the same mass within ± 3 Mev: Σ^- particles have been captured by protons in a hydrogen bubble chamber, yielding $\Sigma^0 + n$, followed by the decay sequence $\Sigma^0 \Rightarrow \gamma + \Lambda$, $\Lambda \rightarrow p + \pi^-$. However only three such complete sequences have been seen (11a) and a more accurate though less direct value for m_{Σ^0} has been obtained with a propane bubble chamber by observing the gamma ray in the sequence of processes $\pi^- + p \Rightarrow \Sigma^0 + K^0$, $\Sigma^0 \Rightarrow \gamma + \Lambda$ (12). The mass of Σ^0 can then be determined with respect to that of Λ, which is known to about $\frac{1}{5}$ Mev.

3.6. *Stability and electromagnetic decay.*—In the presence of strong and electromagnetic interactions only, the particles we have listed would, with three exceptions, all be stable. Although allowed by conservation of S, such decays as $N \Rightarrow P + \pi^-$ or $\Lambda \Rightarrow P + K^-$ or $\Sigma^+ \Rightarrow \Lambda + \pi^+$ are forbidden by conservation of energy. Energetically possible decays like $\Lambda \Rightarrow p + \pi^-$ or $K^+ \Rightarrow \pi^+ + \pi^0$ are forbidden by conservation of strangeness.

The exceptions are the particles π^0, Σ^0 and $\tilde{\Sigma}^0$. Each of these would be stable under strong interactions only, but they can all decay electromagnetically. The π^0 meson, like the vacuum, has zero charge and zero strangeness and thus the transition $\pi^0 \Rightarrow$vacuum can occur with the emission of two γ rays. The necessary change of I by one unit is easily accomplished by the electromagnetic coupling.[10] This process $\pi^0 \Rightarrow 2\gamma$ is known experimentally to have a mean life of $\lesssim 10^{-15}$ sec. (See Table I.)

In a similar way, the transition $\Sigma^0 \Rightarrow \Lambda + \gamma$ requires no change in charge or strangeness and a change of I by only one unit. A rough theoretical estimate of the lifetime can be obtained if we assume a magnetic dipole transition between particles of spin $\frac{1}{2}$, with a transition magnetic moment of the same order as the neutron magnetic moment. We then obtain a Σ^0 lifetime $\tau \sim 5 \times 10^{-20}$ sec. Such a mean life is rather inaccessible to experiment. It is not long enough to permit Σ^0 to go a measurable distance before decaying ($c\tau = 1.5 \times 10^{-9}$ cm.); nor is it short enough to produce a measurable width of the Σ^0 state ($\hbar/\tau = 10$ Kev). There are no direct measurements of this width (even the mass is unknown to ± 3 Mev). In fact, the present experimental limits on the mean life of Σ^0 are roughly analogous to a determination that the moon is closer than the sun and farther than the ceiling.

3.7. *Decay through the weak interactions.*—So far we have ignored the effects of the weak interactions. It is very difficult to observe their effects in collision processes, because of the extremely small cross-sections involved. To date only one such experiment has been successful: the search (12) for inverse β-decay $\bar{\nu} + p \rightarrow e^+ + n$, with a cross-section around 4×10^{-44} cm.^{2} (averaged over the spectrum of high-energy neutrinos from a pile).

In collisions involving strange particles, violations of the strangeness selection rules by the weak interactions should be of the order of one part in 10^{13} or 10^{14}, since this is the magnitude of the dimensionless strength parameter of Section 2.3.

Thus the weak interactions manifest themselves almost exclusively in

[10] The electromagnetic interaction is proportional to charge Q and therefore linear in I_z (see Eq. 1). In other words, it transforms in isotopic spin space like a scalar plus the z-component of a vector. Therefore, each time the electromagnetic coupling occurs (in the sense of perturbation theory) it can change the total isotopic spin either by zero or by one unit ($|\Delta I| = 0$ or 1). A first order electromagnetic process (first order in e in the amplitude) can change I by as much as one unit, a second order process by as much as two units, etc.

decay processes.[11] We have seen that most mesons and baryons are stable in the absence of the weak interactions; they "wait," so to speak, for the weak couplings to induce their decay. A typical lifetime for weak decay can be estimated by taking a "nuclear time" like $(\hbar/m_\pi c^2) \sim \frac{1}{2}\times 10^{-23}$ sec. and multiplying it by 10^{13} or 10^{14}. In fact the lifetimes do range mostly from 10^{-8} to 10^{-10} sec. Of course, severely limited phase space in the final state can vastly increase the lifetime, as in the case of the neutron, which has a mean life of about 1040 sec.

All the baryons, antibaryons, and mesons that are stable under strong and electromagnetic effects disintegrate by means of the weak interactions, with the exception of the proton and antiproton, which are absolutely stable since they are the lightest baryon and antibaryon, respectively.

Let us refer to baryons, antibaryons, and mesons as "strongly interacting particles" or *SIP*s. The disintegration products in the weak decay of a *SIP* are either: (*a*) *SIP*s alone, as in the case $\Lambda \rightarrow p+\pi^-$, or (*b*) a lepton-antilepton pair, with or without *SIP*s, as in the decays $n\rightarrow p+e^-+\bar{\nu}$, $K^+\rightarrow \mu^++\nu$, $K^+\rightarrow \mu^++\nu+\pi^0$, etc.

There are also decays involving both weak and electromagnetic interactions. These "weak electromagnetic decays" may be either inner bremsstrahlung processes or else decays in which a γ-ray is essential.

3.8. *Decay into strongly interacting particles; possible selection rules.*—The weak interactions are known to induce the following decays of *SIP*s into *SIP*s, in violation of conservation of strangeness: Baryons—$\Xi\rightarrow\Lambda+\pi$, $\Sigma^\pm\rightarrow N+\pi$, $\Lambda\rightarrow N+\pi$. Antibaryons—presumably the corresponding processes. Mesons—$K\rightarrow 2\pi$, $K\rightarrow 3\pi$, $\overline{K}\rightarrow 2\pi$, $\overline{K}\rightarrow 3\pi$.

These are all the energetically possible decays of this kind allowed by conservation of baryons, with one exception: the unobserved decay $\Xi\rightarrow N+\pi$. Since only about a dozen Ξ events have been observed, all in cosmic rays, it is by no means clear that this process is absent. However, if it is indeed forbidden, it is significant that it involves a change in strangeness of two units, while all the others involve a change of only one unit. It may be, then (9a), that there is a rule $\Delta S=\pm 1$ for weak decays of *SIP*s into *SIP*s or, what is the same thing, a rule $\Delta I_z=\pm\frac{1}{2}$. It is important that this rule be checked by a further search for decays of the form $\Xi^-\rightarrow\pi^-+n$ or $\Xi^0\rightarrow\pi^-+p$.

In the decays we are discussing, not only strangeness but isotopic spin too is defined in initial and final states. It has been suggested that there may be a further rule (9a) that $|\Delta I|=\frac{1}{2}$. This would, of course, be subject to electromagnetic corrections. In Sections 5 and 6 we discuss the evidence for and against the rule.

A convenient formal device for treating the rule $|\Delta I|=\frac{1}{2}$ was introduced

[11] The absorption of μ^- by nuclei is, of course, a weak process and well known experimentally, although not a decay. However, the absorption always takes place from the ground state of a muonic atom, which would be stable in the absence of the absorption. The situation is therefore just like that of a decaying particle.

by Wentzel (R1). In a weak decay of an *S1P* into *S1P*s, we imagine that an additional particle is emitted or absorbed that carries no energy or momentum or charge, but does carry an isotopic spin of $\frac{1}{2}$, with whichever z-component ($\pm\frac{1}{2}$) is needed to balance I_z in the reaction. This particle is aptly named the "spurion." With the spurion included, we then assume that I is conserved in the decay process. Such an assumption is clearly equivalent to the rule $|\Delta I|=\frac{1}{2}$. For applications of the spurion method, see for example 5.6 and 5.7.

3.9. *Decay of neutral K particles; K_1^0 and K_2^0.*—The weak decays of the strange particles K^0 and $\overline{K^0}$ are subject to some interesting special considerations. The reader will note that in Table I we do not list lifetimes for K^0 and $\overline{K^0}$. The reason is that neither of these particles has a unique lifetime. Instead, we must consider two linear combinations of the states representing K^0 and $\overline{K^0}$; these combinations are called K_1^0 and K_2^0 and it is they that are characterized by unique lifetimes τ_1 and τ_2 respectively. This situation was predicted theoretically (14) on the basis of the strangeness theory and has since been confirmed in several respects by experiment (15).

In our discussion we assume, as before, the exact conservation of CP, or, what is equivalent, exact symmetry under time reversal T. (In the original treatment (14), exact conservation of C was taken for granted even for the weak interactions; the old argument can, however, be modernized if we replace C by CP. The possibility of nonconservation of CP is taken up in Appendix A.)

In the production of a neutral K particle, the conservation of strangeness brings about a sharp distinction between K^0 and $\overline{K^0}$. Only K^0, for example, can be produced in the reaction $\pi^-+p\Rightarrow K^0+\Lambda^0$; only $\overline{K^0}$ in the reaction $K^-+p\Rightarrow\overline{K^0}+n$. However, in the subsequent decay of an isolated K^0 or $\overline{K^0}$, strangeness is not conserved and plays no important role. We must consider instead the quantum number CP, which is conserved in the decay.

For convenience, let us take K to be spinless. Denote by $|K^0\rangle$ the state of a K^0 meson at rest and by $|\overline{K^0}\rangle$ the state of a $\overline{K^0}$ meson at rest. These are eigenstates of strangeness with eigenvalues $+1$ and -1, respectively. In each of the reactions mentioned above, the produced meson, in its own rest frame, is in one of these states. Now under CP these two states evidently go into each other, since K^0 and $\overline{K^0}$ are antiparticles. We can adjust the relative phase of the states so that CP $|K^0\rangle=|\overline{K^0}\rangle$ and CP $|\overline{K^0}\rangle=|K^0\rangle$.

In treating the decay, we must form eigenstates of CP instead of strangeness: we define $|K_1^0\rangle\equiv 2^{-1/2}(|K^0\rangle+|\overline{K^0}\rangle)$ and $|K_2^0\rangle\equiv 2^{-1/2}(|K^0\rangle-|\overline{K^0}\rangle)$. Then $|K_1^0\rangle$ corresponds to the eigenvalue $+1$ and $|K_2^0\rangle$ to the eigenvalue -1 of CP. We may refer to these states as even and odd respectively under CP. The equations defining $|K_1^0\rangle$ and $|K_2^0\rangle$ may be inverted so that the original states $|K^0\rangle$ and $|K^0\rangle$ are expressed in terms of them: $|K^0\rangle=2^{-1/2}(|K_1^0\rangle+|K_2^0\rangle)$ and $|\overline{K^0}\rangle=2^{-1/2}(|K_1^0\rangle-|K_2^0\rangle)$.

We may say, then, that the production of a K^0 meson (or a $\overline{K^0}$ meson) corresponds to the production, with equal probability and prescribed relative phase, of a "K_1^0 meson" or a "K_2^0 meson." Each particle K_1^0 or K_2^0 is its own antiparticle, the former being even under CP and the latter odd under CP. Since CP is assumed conserved in the decay, some decay modes are available to K_1^0 that are forbidden to K_2^0 and vice versa. Thus these two particles must have different lifetimes.

For example, consider the familiar decay into two pions $\pi^+ + \pi^-$ or $\pi^0 + \pi^0$; this final state is even under CP , as we have mentioned in 2.6. Thus decay into two pions is allowed for K_1^0 and forbidden for K_2^0. Now in fact two neutral K particles have been observed. One of these decays nearly always into 2π and has a lifetime of about 10^{-10} sec.; it is to be identified with the theoretical K_1^0 meson. The other has a much longer lifetime (at least 300 times longer) and does not appear to give 2π; it must be the K_2^0 meson. Apparently the 2π decay has a much higher rate than other possible modes for either K_1^0 or K_2^0.

Suppose now we generate a "beam" of 1000 K^0 mesons, for example, by the reaction $\pi^- + p \Rightarrow \Lambda^0 + K^0$. In terms of K_1^0 and K_2^0, we have 500 of each. After around 10^{-9} sec. (in the rest system of the mesons), nearly all the K_1^0 have decayed, mostly into 2π; very few of the K_2^0 have decayed, however. Our "beam" now contains about 500 K_2^0. These are not in a pure state of strangeness. They have with equal probability $S = +1$ and $S = -1$. In striking matter, half of them (around 250) are capable of reactions like $K^0 + p \Rightarrow K^+ + n$ characteristic of $S = +1$ and the other half are capable of reactions like $\overline{K^0} + n \Rightarrow K^- + p$ or $\overline{K^0} + p \Rightarrow \Sigma^+ + \pi^0$ characteristic of $S = -1$ (15). This behavior of the "stale beam" is to be contrasted with that of the "fresh beam" of 1000 K^0 mesons, all of which had strangeness $+1$.

This thought-experiment illustrates the characteristic feature of the situation, that K^0 and $\overline{K^0}$ are the important entities in strong processes but K_1^0 and K_2^0 in decay.

We may note that since K_1^0 and K_2^0 have different lifetimes (and therefore different "level widths") they must also have different values of the tiny self-energy due to virtual weak decays. In other words, the weak interactions should give rise to a mass difference between K_1^0 and K_2^0; this difference cannot be calculated at the present time but we may crudely estimate it to be of the same order of magnitude as the level width of the K_1^0 state which is $\hbar/\tau_1 \approx 6 \times 10^{-12}$ Mev. Despite the smallness of this quantity, there is a chance that it can be measured (see 6.9).

3.10. *Invariance under* C; *parity conservation.*—In Section 3 up to this point we have emphasized one special property of the strong interactions, namely charge independence. We have seen that one aspect of charge independence, the conservation of S, applies also to electromagnetism, although not to the weak interactions.

We have mentioned in Section 2 the existence of another conservation law that seems to be valid for the strong and electromagnetic interactions and violated by the weak ones. We have referred to this law as the conservation of parity P. However, we have adopted in 2.5 the point of view of Landau that the symmetry of nature between right and left is manifested in the exact invariance of physical laws under CP, even for weak interactions. (Of course this must still be checked by further experiments.) From this viewpoint, conservation of parity P and of charge conjugation C imply each other.

Can we find any way of connecting the separate conservation of C and P with the conservation of S, which has the same domain of validity? Perhaps so, if we concentrate our attention on C invariance and regard P invariance merely as a consequence of it. The point is that C invariance may be thought of as related to symmetry under reflection in isotopic spin space,[12] while conservation of S is related to conservation of I_z or symmetry under rotations about the z-axis is isotopic spin space.

There is a difficulty in this formulation, however. The separate conservation of C and P applies not only to baryons, antibaryons, and mesons, but also to the electromagnetic coupling of electrons and muons, for which isotopic spin and strangeness have, so far as we know, no useful meaning.

The conservation of parity by electromagnetism is established experimentally down to about one part in 10^6 in intensity by the validity of selection rules in atomic transitions (4). In the case of strong interactions, the best evidence seems to be the experiment of Tanner (17), which indicates that parity is conserved by nuclear forces down to one part in 10^7 in intensity. (He has investigated the reaction $p+F^{19}\Rightarrow\alpha+O^{16}$, which would have a resonance corresponding to a known 1^+ state of Ne^{20} if parity were not conserved. The resonance is not observed.)

The consequences of the separate conservation of C and P are well known. It may be instructive, however, to give an example of the sort of interaction that is forbidden by C or P invariance, although allowed by CP invariance, charge independence, and all other known symmetries. Let us use the notation of field theory and denote by the same symbol a particle and the field operator that destroys it. Then consider a coupling term of the form[13]

[12] We may consider in place of C the operator G introduced by Yang and Lee (16) in discussing nucleon-antinucleon annihilation; G is the product of C by rotation in isotopic spin space of 180° about the y-axis. It commutes with **I** but carries baryons into antibaryons and K into $\overline{K}$; the pion field changes sign under G. The operator G behaves in every way like a reflection of all three coordinates in isotopic spin space, and it might be useful to interpret it in that way. We may then regard C invariance of the strong interactions as an extension of charge independence—the interactions are invariant under reflection G as well as under rotations in isotopic spin space. When we "turn on" the electromagnetic coupling of the photon with charged particles, conservation of C, like that of strangeness, remains exact.

[13] The reader may wonder why we have used nucleons, hyperons, and K particles for our counterexample rather than electrons and photons, or nucleons and pions. The reason is that in these latter systems there seems to be no simple coupling (for instance, bilinear in the fermion fields and not involving gradients) that violates separate C and P invariance while obeying CP conservation and the other known laws.

In a sense, therefore, hyperon phenomena will provide a more stringent test of parity conservation than the more familiar reactions. Still, we do not expect a spectacular violation of the law by the strong couplings of hyperons, since such a violation should probably have shown up, by means of virtual processes, in Tanner's experiment on nuclei (15).

$$\bar{p}(g_s + g_p\gamma_5)\Lambda K^+ + \bar{n}(g_s + g_p\gamma_5)\Lambda K^0 + \text{Herm. conj.},$$

where g_s and g_p are required to be real by CP invariance. (We assume Λ has spin $\frac{1}{2}$ and K spin 0.) If g_s and g_p were both $\neq 0$, then this coupling would violate C and P invariance. It would introduce a handedness into hyperon production phenomena; for example, in $\pi^- + p \Rightarrow \Lambda + K^0$ the Λ would be, in general, longitudinally polarized.

With C and P separately conserved by the strong interactions, either g_s or g_p must be zero; the K-particle must have a unique parity relative to N and Λ, as far as strong and electromagnetic processes are concerned.

4. The Weak Interactions

4.1. *Four-fermion couplings; the Puppi triangle.*—Let us begin our detailed discussion of the weak interactions by reviewing the general properties we have mentioned in the previous sections: (*a*) They exhibit an apparent universality of strength—around 10^{13} times weaker in intensity than the strong couplings (18). (*b*) They violate conservation of strangeness S and the separate conservation of parity P and charge conjugation C. (*c*) They manifest themselves mostly in the decay of systems that would be stable under strong and electromagnetic processes alone.

We must now attempt to list in some coherent fashion the known weak processes. Before the discovery of the strange particles the situation could be summarized by means of the Puppi triangle shown in Figure 2. At each vertex is a pair of spin $\frac{1}{2}$ particles, one charged and one neutral. The sides of the triangle represent interactions between one such pair and another. For simplicity only the positive pair is indicated at each vertex, for example $p\bar{n}$; of course $n\bar{p}$ is coupled, too.

The line C connecting $p\bar{n}$ and $e^+\nu$ stands for the β-decay interaction responsible for the following observed processes: $n \to p + e^- + \bar{\nu}$ (β^--decay of the free neutron or a neutron in a nucleus); $p \to n + e^+ + \nu$ (β^+-decay of a proton in a nucleus); $e^- + p \to n + \nu$ (K-capture by a proton in a nucleus); $\bar{\nu} + p \to n + e^+$ (detection of the antineutrino). These reactions can, of course, be derived from one another by two fundamental operations: reversing a

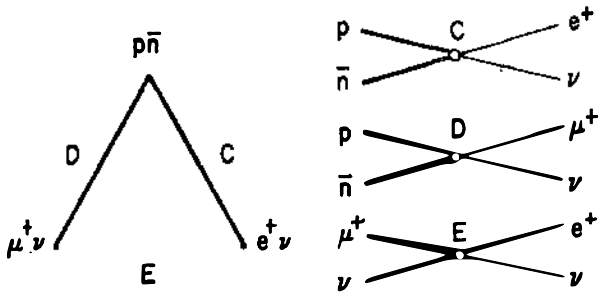

FIG. 2. Puppi triangle representing four-fermion interactions of positively charged pairs. Equivalent Feynman diagrams are drawn to the right. Leg C represents the β-decay interaction; leg D, muon capture, and leg E, muon decay.

reaction arrow and transposing a particle from one side of the arrow to the other while replacing it by its antiparticle. By convention, the antineutrino is the particle emitted in neutron decay along with the electron.

According to Fermi's theory of β-decay and its subsequent generalizations, we describe the coupling $e^+\nu$ and $p\bar{n}$ as a point interaction or δ function potential (with charge exchange). In the language of field theory, we write the interaction Lagrangian density in some such form as this (19):

$$C_V(\bar{p}\gamma_\alpha n)(\bar{e}\gamma_\alpha \nu) + C_V^*(\bar{n}\gamma_\alpha p)(\bar{\nu}\gamma_\alpha e).$$

Here ν is the field operator that destroys a neutrino or creates an antineutrino; e destroys an electron or creates a positron, etc. Correspondingly, the operator $\bar{\nu}$ creates a neutrino or destroys an antineutrino; mathematically, $\bar{\nu}=\nu\dagger\gamma_4$, where $\nu\dagger$ is the Hermitian conjugate of ν. There are similar relationships for the other fields. Thus the second term is the Hermitian conjugate of the first; the first term induces, for example, the β^--decay of the neutron and the second the β^+-decay of the proton.

Each operator is a spinor, of course, with four components, and each pair of spinors has a Dirac matrix sandwiched in between. In this case we have used the vector operator γ_α and constructed the vector interaction V. The nonconservation of parity in β-decay permits us to couple $\bar{p}\gamma_\alpha n$ also to the axial vector quantity $\bar{e}\gamma_5\gamma_\alpha\nu$ to form an additional "vector" interaction

$$C_V'(\bar{p}\gamma_\alpha n)(\bar{e}\gamma_\alpha\gamma_5\nu) + \text{Herm. conj.}$$

Eight more point interactions are possible: two forms each of the scalar interaction (S), the tensor (T), the axial vector (A) and the pseudoscalar (P). (The labels S, T, etc. describe the Dirac matrix for the nucleons.) The total β-decay coupling is then a linear combination of these ten forms:

$$\begin{aligned}\mathfrak{L} = {} & C_S(\bar{p}n)(\bar{e}\nu) + C_S'(\bar{p}n)(\bar{e}\gamma_5\nu) \\ & + C_V(\bar{p}\gamma_\alpha n)(\bar{e}\gamma_\alpha\nu) + C_V'(\bar{p}\gamma_\alpha n)(\bar{e}\gamma_\alpha\gamma_5\nu) \\ & + \tfrac{1}{2}C_T(\bar{p}\sigma_{\alpha\beta}n)(\bar{e}\sigma_{\alpha\beta}\nu) + \tfrac{1}{2}C_T'(\bar{p}\sigma_{\alpha\beta}n)(\bar{e}\sigma_{\alpha\beta}\gamma_5\nu) \\ & - C_A(\bar{p}\gamma_\alpha\gamma_5 n)(\bar{e}\gamma_\alpha\gamma_5\nu) - C_A'(\bar{p}\gamma_\alpha\gamma_5 n)(\bar{e}\gamma_\alpha\nu) \\ & + C_P(\bar{p}\gamma_5 n)(\bar{e}\gamma_5\nu) + C_P'(\bar{p}\gamma_5 n)(\bar{e}\nu) \\ & + \text{Herm. conj.}\end{aligned} \qquad 5.$$

If CP or T invariance holds, then all the coefficients C are real.

The line D connecting $p\bar{n}$ with $\mu^+\nu$ in Figure 2 represents a coupling analogous to β-decay but with the electron replaced by the muon. In this case, the only observed process is the capture of μ^- by a proton in a nucleus: $\mu^- + p \rightarrow n + \nu$. (We write ν rather than $\bar{\nu}$ for reasons discussed in 4.4. The other processes coupled by line D, $n \rightarrow p + \mu^- + \bar{\nu}$ and $p \rightarrow n + \mu^+ + \nu$, analogous to neutron and proton β-decay, are forbidden by conservation of energy and the reaction $\bar{\nu} + p \rightarrow n + \mu^+$ requires an intense source of high energy antineutrinos that is unavailable today.) Again there are ten possible forms of point interaction, such as $D_V(\bar{p}\gamma_\alpha n)(\bar{\mu}\gamma_\alpha\nu)$ + Herm. conj. The total "μ-capture coupling" is presumed to be some linear combination of these.

The third leg E of the Puppi triangle connects $\mu^+\nu$ and $e^+\nu$. Here the observed process is the decay of the muon $\mu^{\pm}\rightarrow e^{\pm}+\nu+\bar{\nu}$. The point interaction Lagrangian again may consist of ten different terms, such as $E_V(\bar{\nu}\gamma_\alpha\mu)(\bar{e}\gamma_\alpha\nu)+$Herm. conj., $E_V'(\bar{\nu}\gamma_\alpha\mu)(\bar{e}\gamma_\alpha\gamma_5\nu)+$Herm. conj., etc. The actual form of the coupling is taken up in 4.5.

The phenomenon of "universality of strength" can now be appreciated if we calculate the rates of three weak processes corresponding to the three legs of the triangle and compare these with experiments. The rate of decay of the free neutron is given, with neglect of recoil and of "Fierz terms," by the formula

$$\Gamma_n = \frac{0.47}{60\pi^3} c^4\hbar^{-7}\Delta^5(C_S^2 + C_S'^2 + C_V^2 + C_V'^2 + 3C_T^2 + 3C_T'^2 + 3C_A^2 + 3C_A'^2), \qquad 6.$$

where Δ is the difference in rest mass of the neutron and proton. The number 0.47 is computed from the ratio m_e/Δ and would be unity if the electron were massless; we may call it the "blocking factor" because the finite electron mass blocks up phase space. The rate of decay of the muon is given by the formula (20)

$$\Gamma_\mu = \frac{1}{1536\pi^3} c^4\hbar^{-7}m_\mu^5(E_S^2 + E_S'^2 + 4E_V^2 + 4E_V'^2 + 6E_T^2 + 6E_T'^2 + 4E_A^2 + 4E_A'^2 + E_P^2 + E_P'^2), \qquad 7.$$

where m_e/m_μ has been neglected. The rate of capture of μ^- from a 1s atomic orbit around a free proton is given, with neglect of recoil, by the formula

$$\Gamma = \frac{1}{2\pi^2}\left(\frac{1}{137}\right)^3 c^4\hbar^{-7}m_\mu^5[(D_S + D_V)^2 + (D_S' + D_V')^2 + 3(D_T + D_A)^2 + 3(D_T' + D_A')^2] \; 8.$$

Now in Table I we find Γ_n and Γ_μ in sec.$^{-1}$ and Δ and m_μ in Mev, which we may convert to grams. Then we obtain, substituting into Eqs. 5 and 6, the results

$$C_S^2 + \cdots \approx 10.4 \times (10^{-49}\text{ erg cm}^3)^2 = 2.62 \times 10^{-13}\hbar^2c^2\left(\frac{\hbar}{m_\pi c}\right)^4 \qquad 9.$$

and

$$E_S^2 + \cdots \approx 16.1 \times (10^{-49}\text{ erg cm}^3)^2 = 4.06 \times 10^{-13}\hbar^2c^2\left(\frac{\hbar}{m_\pi c}\right)^4 \qquad 9a.$$

The similarity in strength is striking. We have, for simplicity, used the Compton wave length of the charged pion to put the constants in dimensionless form. The third quantity Γ has not been measured but can be roughly estimated by extrapolating results on μ^- capture in nuclei. The result for $(D_S+D_V)^2+\cdots$, is then of the same order of magnitude as the quantities in Eqs. 9 and 9a.

4.2. *The two-component or longitudinal neutrino.*—The discovery of nonconservation of parity in weak processes has permitted an important advance in the theory of the neutrino. This particle seems to have the unique property of possessing no couplings but weak ones; it is possible, therefore, for the neutrino to be governed completely by laws that violate separate P and C invariance.

Such a possibility has been welcomed by theoretical physicists who have long felt that the idea of a massless neutrino is repugnant unless there is some principle guaranteeing the masslessness. In the case of the photon, the principle is gauge invariance. For the neutrino, there was such a principle available, but it violated conservation of parity! With the removal of this obstacle, however, it became possible to justify the masslessness of the neutrino. The theory that we shall set forth here has been proposed by Landau (6), Salam (21), and Lee & Yang (22).

Let us postulate that all physical laws are invariant under the replacement of the neutrino operator ν by $e^{i\alpha_\nu}\gamma_5\nu$, where $e^{i\alpha_\nu}$ is some fixed phase factor. Then the Dirac equation for a free neutrino[14]

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_\nu\right)\psi_\nu = 0, \tag{10}$$

where ψ_ν is the neutrino wave function, must be invariant under $\psi_\nu \to e^{i\alpha_\nu}\gamma_5\psi_\nu$. But γ_5 and γ_μ anticommute, so we have, multiplying Eq. 10 by $e^{i\alpha_\nu}\gamma_5$, the result

$$\left(\gamma_\mu \frac{\partial}{\partial x_\mu} - m_\nu\right) e^{i\alpha_\nu}\gamma_5\psi_\nu = 0 \tag{11}$$

The invariance is possible only if $m_\nu = 0$. We have guaranteed masslessness.

We must ask what happens to other fields under the transformation that affects the neutrino field as we have described. For the fermions, for example, it must not introduce multiplication by γ_5, for then these particles too would be massless. It must not carry one particle into another, for then the masses of these would be equal, and so forth. The only effect of the transformation on other fields than the neutrino can be multiplication of each one by some phase factor $e^{i\alpha}$ characteristic of the field.

Now let us apply the principle of invariance under this transformation to the Lagrangian (Eq. 5) of the β-decay interaction. The effect of the transformation is to interchange the roles of the primed and unprimed couplings and to multiply the whole interaction by $e^{i(\alpha_\nu+\alpha_n-\alpha_e-\alpha_p)}$. But according to the invariance principle the interaction must be unchanged by the transformation. There are then only two possibilities:

(*a*) $C_S = -C_S'$, $C_T = -C_T'$, etc. and $e^{i(\alpha_\nu+\alpha_n-\alpha_e-\alpha_p)} = -1$

(*b*) $C_S = +C_S'$, $C_T = +C_T'$, etc. and $e^{i(\alpha_\nu+\alpha_n-\alpha_e-\alpha_p)} = +1$.

Let us examine one of these alternatives, say (*a*), according to which the neutrino field in β-decay occurs only in the combination $(1-\gamma_5)\nu$ (and its Hermitian conjugate). Physically this means, as we shall see, that neutrinos can be created or destroyed in one spin state only—that in which the spin

[14] For the rest of this section we set $\hbar = c = 1$.

vector is aligned with the direction of motion (and antineutrinos, correspondingly, can be created or destroyed only in the case of spin aligned opposite to the direction of motion). It is physically clear why such a condition is coupled with rigorous masslessness of the neutrino—if there were any rest mass, the particle could be at rest, and the requirement that spin $\boldsymbol{\sigma}$ and momentum $\boldsymbol{p}$ be always aligned would have no meaning.

We must still show that coupling through $(1-\gamma_5)\nu$ corresponds to the interaction of right-handed neutrinos only. The wave function ψ_ν of the neutrino in β-decay occurs multiplied by $(1-\gamma_5)$. With $m_\nu=0$ the Dirac equation (Eq. 10) for ψ_ν can be written in the form

$$i\frac{\partial\psi_\nu}{\partial t}=\frac{1}{i}\,\boldsymbol{\alpha}\cdot\nabla\psi_\nu, \qquad 12.$$

where Dirac's $\boldsymbol{\alpha}$ is related to the spin $\boldsymbol{\sigma}$ by the relation $\boldsymbol{\alpha}=-\gamma_5\boldsymbol{\sigma}=-\boldsymbol{\sigma}\gamma_5$. For $(1-\gamma_5)\psi_\nu$, therefore, we have the equation

$$i\frac{\partial}{\partial t}(1-\gamma_5)\psi_\nu=\frac{1}{i}\,\boldsymbol{\sigma}\cdot\nabla(1-\gamma_5)\psi_\nu, \qquad 13.$$

or, in Fourier transform, frequency $=\boldsymbol{\sigma}\cdot\boldsymbol{p}$. For the positive frequency solution, which represents the neutrino, we have $+p=\boldsymbol{\sigma}\cdot\boldsymbol{p}$ or righthanded longitudinal polarization. The negative frequency solution has $-p=\boldsymbol{\sigma}\cdot\boldsymbol{p}$ or left-handed polarization; using the "hole theory" approach, we may say that the antineutrino is a hole in the sea of such "left-handed" negative frequency neutrinos. The hole has the opposite spin and momentum; the antineutrino is therefore left-handed.

In this theory, then, two states out of four have been eliminated from the interaction—the neutrino state with left polarization and the antineutrino state with right polarization. It is possible, therefore, to write the Dirac equation with a new wave function consisting of only two components instead of four. Hence the name "two-component theory." It is easier, however, to retain the four-component Dirac spinor together with the matrix $(1-\gamma_5)$ that strikes out any coupling of the two forbidden states.

We have looked so far at possibility (*a*) above, $(1-\gamma_5)\nu$, which couples right-handed ν and left-handed $\bar{\nu}$. Possibility (*b*), which is $(1+\gamma_5)\nu$, couples the other two states instead. We shall see below that (*b*) is excluded by experiment, while (*a*) is strongly supported. We may therefore discuss the longitudinal neutrino in terms of (*a*) only [see, however, (46)].

Now the question arises how this theory is to be applied to interactions other than β-decay. That depends, evidently, on the phase factors $e^{i\alpha_p}$, $e^{i\alpha_n}$, $e^{i\alpha_e}$, $e^{i\alpha_\mu}$, etc. One possible situation is that these factors are equal to unity for all fields except the neutrino. (According to (*a*) $e^{i\alpha_\nu}$ must then be -1.) In that case, it is always $(1-\gamma_5)\nu$ that occurs in the interaction, no matter what the process is (β-decay, μ-absorption, μ-decay, etc.). This is the simplest situation and seems, so far, to fit the facts. The other physically distinct possibility is that the product of phase factors is sometimes -1 and sometimes $+1$, depending on the particles involved. In that case, it is sometimes the right-handed ν and left-handed $\bar{\nu}$ that are coupled but other times the left-handed ν and right-handed $\bar{\nu}$. Fortunately, it has not been necessary to invoke this complicated alternative. We shall consider only the simple form of the longitudinal theory in which the same two neutrino states are involved in all processes. This is the true "two-component" theory.

The theory, then, can be summarized in this way: The neutrino field always occurs in the combination $(1-\gamma_5)\nu$ and its Hermitian conjugate, which couple right-handed ν and left-handed $\bar{\nu}$ only. We shall explore some experimental consequences in the next few paragraphs.

4.3. *The longitudinal neutrino in β-decay.* We have seen that the Fermi theory of β-decay may be described in terms of the ten parameters C_S, C_S', C_V, C_V', etc., and that CP invariance requires these to be real. In the longitudinal neutrino picture, there are only five independent constants since we have the relations $C_S' = -C_S$, $C_V' = -C_V$, etc.

The obsolete form of the theory, in which C and P are separately conserved, could be expressed by setting the coefficients C_S', C_V', etc. equal to zero; this would leave only "parity conserving" terms in the coupling.

Considerable support for the longitudinal neutrino theory was provided by the same experiment that first established the nonconservation of parity in β-decay: the work of Wu, *et al.* (5), who investigated the asymmetry in the angular distribution of electrons emitted by oriented Co^{60} nuclei. As we mentioned in 2.5, the angular distribution has the form $1-a(E)\cos\theta$ where E is the electron energy and θ is the angle between the electron momentum p and the spin direction $\langle J\rangle/J$ of the decaying nucleus. (For simplicity we consider the idealized case of fully oriented nuclei.)

Let us defer until later the comparison of the experimental result with the two-component theory. For the moment, we wish to emphasize that the novel feature of the experiment was the measurement of a pseudoscalar quantity. The probability of decay per unit solid angle and per unit electron energy may be written $I(E,\theta) = S(E)[1-a(E)\cos\theta] = S(E) - S(E)a(E)(\mathbf{p}/p)\cdot(\langle \mathbf{J}\rangle/J)$. The first term $S(E)$ is the electron spectrum (averaged over the direction of $\mathbf{p}$ or $\langle \mathbf{J}\rangle$) and is evidently a scalar. The second term, which contains the asymmetry, is a pseudoscalar, the dot product of a polar vector $\mathbf{p}$ and an axial vector $\langle \mathbf{J}\rangle$ times a constant. (With CP invariance, the constant in such a formula always has the opposite sign for the antinucleus and the symmetry of space between left and right is preserved.)

Now in any intensity formula in β-decay, we can divide the terms into scalar ones (such as those giving spectrum and reciprocal lifetime and $e-\nu$ angular correlation) and pseudoscalar ones (such as those giving electron asymmetry or longitudinal electron polarization). In the old theory with parity conserved, all the pseudoscalar quantities vanish. In a general theory with ten coupling constants, they arise from interference between "parity conserving" and "parity nonconserving" terms in the coupling; they depend on C_SC_S', C_VC_V', $C_SC_T'+C_TC_S'$, etc. In the longitudinal theory, of course, these become $-C_S^2$, $-C_V^2$, $-2C_SC_T$, etc.

The scalar quantities were the only ones measured until 1957. The formulas for scalars, unlike those for pseudoscalars, are essentially the same in the old theory and in the longitudinal theory. One need merely replace C_S^2 by $C_S^2+C_S'^2=2C_S^2$, C_SC_V by $C_SC_V+C_S'C_V'=2C_SC_V$, etc. (Even this doubling is, of course, a matter of definition of the constants C.)

The conclusions about the form of the β-decay coupling that are based on the measurements of these scalar quantities over the past several years should thus be essentially unchanged. It was concluded that S and T interactions are both present, with $|C_S|$ and $|C_T|$ of comparable magnitude, and $|C_V|$ and $|C_A|$ much smaller or zero (23). (The pseudoscalar interaction P is difficult to detect in β-decay since the operator γ_5 vanishes in the nonrelativistic limit and nucleons are not highly relativistic in the nucleus. In 4.6 we shall argue on other grounds that C_P is small or zero.)

We shall assume in what follows that the β-decay interaction does consist primarily of S and T. However, current experiments have cast some doubt on these time-honored assignments. In 4.6, we shall therefore discuss also the remote but attractive possibility that the interaction is instead a mixture of V and A.

Let us now return to the discussion of the Co^{60} decay. The initial and final nuclear spins are 5 and 4 respectively and the parities the same. This corresponds to an "allowed Gamow-Teller" transition in which only tensor and axial vector interactions are effective; since the latter is supposed to be absent (or nearly so) in the β-decay coupling, we are dealing with a transition induced purely by T. The longitudinal theory with $C_T=-C_T'$ then predicts a unique value for the asymmetry parameter a:

$$a = -v/c,$$

where v is the electron velocity. [This is in good agreement with the results of ref. (5).] If we had made the opposite choice of the neutrino spin direction ($C_T=+C_T'$) the sign of a would be changed. The measurement on Co^{60} thus confirmed the longitudinal theory and established the sign of the neutrino polarization.

Another important pseudoscalar quantity that has now been measured is the longitudinal polarization of the β-rays themselves. In a decay induced purely by the tensor interaction, the longitudinal theory predicts a fractional polarization $(-v/c)$ (the sign means the electrons are spinning to the left). It is clear that the polarization must vanish at zero electron velocity, since there is then no vector for the spin to point along. We can also see that the polarization must be -100 per cent for $v/c\approx 1$, by the following argument. In the longitudinal theory the tensor coupling has the form

$$[\bar{e}\sigma_{\alpha\beta}(1-\gamma_5)\nu][\bar{p}\sigma_{\alpha\beta}n] + \text{Herm. conj.}$$

We will recall that $\bar{e}=e^\dagger\gamma_4$ and that γ_5 commutes with $\sigma_{\alpha\beta}$ and anticommutes with γ_4. Thus we have, for the lepton factor in the coupling, the form $e^\dagger(1+\gamma_5)\gamma_4\sigma_{\alpha\beta}\nu$, or, in the Hermitian conjugate, the form $\nu^\dagger\sigma_{\alpha\beta}\gamma_4(1+\gamma_5)e$. Now when $v/c\approx 1$, the electron is effectively massless. Thus the expression $(1+\gamma_5)e$ is perfectly analogous to the expression $(1-\gamma_5)\nu$ that gives us a right-handed neutrino. We have a left-handed electron, i.e., polarization -100 per cent. The expression (v/c) for the polarization of the electrons in the decay of Co^{60} has been verified to an accuracy of around 20 per cent by Frauenfelder *et al.* (R2).

We have discussed the polarization in the case of a transition involving T only. The value at $v/c=1$ can be obtained for other interactions by the same argument we have used above for T. Since γ_5 commutes with 1 just as it does with $\sigma_{\alpha\beta}$, S and T behave alike. The operators γ_α and $\gamma_5\gamma_\alpha$ anticommute with γ_5, and thus the polarization of β^- in the case of V or A coupling is $+100$ per cent at $v/c, \approx 1$ whereas it is -100 per cent for S or T. If the true interaction really contains S and T only, then the polarization must be -100 per cent at $v/c\approx 1$ in *all* β^- transitions. The fact that the value -100 per cent is approximately verified for "Gamow-Teller" transitions like $Co^{60}\rightarrow Ni^{60}$ confirms, within the framework of the longitudinal theory with T dominant, that there is very little A compared to T. Current experiments on β-polarization in "Fermi" and "mixed" transitions, where S and V play a role, will reveal similarly whether or not there is a considerable admixture of V, assuming S dominant.

Note that for a given coupling the polarization of β^+ is opposite to that of β^-.

4.4. *Conservation of leptons.*—In our description of the weak couplings covered by the Puppi triangle (4.1), we made explicit assumptons about the roles of neutrino and antineutrino, which we promised to justify here. In the decay of the muon, we chose the scheme $\mu^-\rightarrow e^-+\nu+\bar{\nu}$ rather than $\mu^-\rightarrow e^-+\nu+\nu$ or $\mu^-\rightarrow e^-+\bar{\nu}+\bar{\nu}$ (with corresponding schemes for the positive muon). In the absorption of μ^- we assumed that a neutrino rather than an antineutrino is emitted. We must now discuss the motivation of these choices, which have been made possible by the success of the two-component theory, with its sharp physical distinction between ν and $\bar{\nu}$.

The electron spectrum in μ decay, according to a point interaction theory such as was discussed in 4.1, is characterized by a single parameter ρ, introduced by Michel (20). In general, ρ varies between 0 and 1 for the scheme $\mu^-\rightarrow e^-+\nu+\bar{\nu}$ employed in 4.1; if, instead, two neutrinos or two antineutrinos are emitted, ρ varies between 0 and $\frac{3}{4}$. The two-component theory removes all this freedom; it turns out that in this theory ρ must equal zero if the electron is accompanied by $\nu+\nu$ or $\bar{\nu}+\bar{\nu}$, while in the case of $\nu+\bar{\nu}$ the value of ρ is $\frac{3}{4}$ (21). Now the experimental value, corrected for inner bremsstrahlung effects, is 0.68 ± 0.02 (24), which selects the decay scheme $\mu^-\rightarrow e^-+\nu+\bar{\nu}$ and at the same time provides a check on the longitudinal theory of the neutrino, although there is still a discrepancy between .68 and .75 to be explained.

At this point an important theoretical principle should be introduced, the law of conservation of leptons. This law has not yet been fully established, but seems a very attractive hypothesis at the present time. It is exactly analogous to the law of conservation of baryons, discussed in 2.4; it states that the number of leptons minus the number of antileptons is a conserved quantity in all processes. The law is obviously incomplete, however, without a specification of which particles are leptons and which antileptons. To start with, we may certainly define the electron to be a lepton, as a matter of convention; the positron is then an antilepton. Next, we have defined the antineutrino to be the particle emitted along with the electron in neutron decay; we may say, then, that the antineutrino is an antilepton and the neutrino a lepton. (The neutron and proton are assumed to have the same lepton content, which we take to be zero.)

The nontrivial question is the assignment of the muon. Is the negative muon a lepton like e^- and ν or an antilepton like e^+ and $\bar{\nu}$? If leptons are really conserved, the answer is determined by the decay scheme for the muon that was established above: $\mu^- \to e^- + \bar{\nu} + \nu$. The negative muon must be a lepton like the negative electron. Our assignments are summarized in Table I, where ν, e^-, μ^- are labeled "leptons" and $\bar{\nu}$, e^+, μ^+, are labelled "antileptons." Finally, these assignments give for muon capture $\mu^- + p \to n + \nu$. The conservation of leptons requires that the particle on the right be ν rather than $\bar{\nu}$. This has not yet been tested experimentally.

All the assignments of ν and $\bar{\nu}$ in the reactions associated with the Puppi triangle have been justified. But it is clearly desirable to have more experimental tests of lepton conservation. One reaction that can be used for such a test is the decay of the charged pion. In 4.7 we shall speculate about the mechanism of this decay, but here we may simply refer to the experimental fact that it yields a muon and a neutrino or antineutrino. Conservation of leptons then requires that we have $\pi^- \to \mu^- + \bar{\nu}$ and $\pi^+ \to \mu^+ + \nu$, with consequences as described below. Another test of lepton conservation is the decay $K^\pm \to \mu^\pm \pm \nu$. Experiment (57) shows that the neutrino assignment is the same for K and π-decay (see 4.5).

4.5. *Polarization and decay of the muon.*—Consider the decay of a positive pion in its own rest frame and assume conservation of leptons. Since π^+ is spinless and the decay products μ^+ and ν travel in opposite directions, the neutrino's angular momentum of $+\frac{1}{2}$ about its direction of motion must be balanced by μ^+ also carrying an angular momentum of $+\frac{1}{2}$ about its direction of motion. In other words, μ^+ from the decay of π^+ is 100 per cent right polarized as a consequence of the longitudinality of the neutrino. Similarly μ^- from π^- decay is 100 per cent left polarized.

Eventually the sense of polarization of these muons will probably be measured "directly" (i.e., without recourse to the muon decay) but this has not yet been done. In order to understand the present evidence that the μ^+ is right polarized we shall discuss the decay $\mu^+ \to e^+ + \nu + \bar{\nu}$, using the two-component theory of the neutrino.

The Fermi type theory of μ-decay described in 4.1 contains ten parameters, but these are reduced to two in the two-component picture. Requiring that ν always occur in the combination $(1-\gamma_5)\nu$ and, correspondingly, $\bar{\nu}$ always occur in the combination $\bar{\nu}(1+\gamma_5)$, yields the conditions $E_V = -E_V' = -E_A = E_A'$, $E_S = -E_S' = -E_P = E_P'$, and $E_T = E_T' = 0$. Thus the complete interaction may be written in the form

$$E_V[\bar{\nu}(1+\gamma_5)\gamma_\alpha\mu][\bar{e}\gamma_\alpha(1-\gamma_5)\nu]+E_S[\bar{\nu}(1+\gamma_5)\mu][\bar{e}(1-\gamma_5)\nu]+\text{Herm. conj.} \quad 14.$$

The two coupling constants E_V and E_S are real in a theory invariant under CP.

If we now calculate the distribution of the angle θ between the spin direction $\hat{\sigma}_\mu$ of a stationary μ and the direction $\hat{p}_e$ of its decay electron, we find it to be given by the formula of Landau (6) and Lee & Yang (22),

$$I(\theta) \propto 1 \pm \lambda(\hat{\sigma}_{\mu}\pm\cdot\hat{p}_e\pm)(2\epsilon-1)(3-2\epsilon)^{-1}, \quad 15.$$

where ϵ is the electron energy in units of the maximum electron energy and λ is $(E_S^2-E_V^2)/(E_S^2+E_V^2)$. Note that for a μ^- with the same spin direction, the coefficient of $\hat{\sigma}\cdot\hat{p}$ changes sign.

Now the quantity $\hat{\sigma}\cdot\hat{p}$ of Eq. 15 is not directly observable, since the muon spin direction has not yet been measured. Instead, let us consider the $\pi\rightarrow\mu\rightarrow e$ chain, with conservation of leptons. We want to rewrite 15 in terms of the observable angle $\varphi=\cos^{-1}\hat{p}_\mu\cdot\hat{p}_e$. We remember that when a π^+ decays at rest, the μ^+ emitted is fully right polarized; if the μ^+ is now brought to rest, it is still polarized in the same direction. There may, however, be some depolarization, in an amount depending on the medium. Say the muon retains a fraction $|P|$ of its polarization along its former direction of motion. Then the angular distribution of the decay positrons relative to this direction is

$$I(\varphi) \propto 1+|P|\lambda(\hat{p}_\mu\cdot\hat{p}_e)(2\epsilon-1)(3-2\epsilon)^{-1}. \quad 16.$$

For the negative $\pi\rightarrow\mu\rightarrow e$ chain, there are two changes of sign in this formula. In the first place, μ^- is accompanied by $\bar{\nu}$ instead of ν and μ^- is therefore fully left polarized. In the second place, the $\hat{\sigma}\cdot\hat{p}$ term in Eq. 15 (for the electron distribution relative to the spin direction) changes sign as we go from μ^+ to μ^-. The two sign changes cancel each other and we are left with the same formula (Eq. 16) for $\pi\rightarrow\mu\rightarrow e$ whether the charge is positive or negative. Of course the factor $|P|$ depends on the sign of charge; experimentally μ^- is depolarized much more than μ^+.

The historic experiments confirming the asymmetry in the $\pi\rightarrow\mu\rightarrow e$ chain were carried out by two groups: Friedman & Telegdi (25) looked at $\pi^+\rightarrow\mu^+\rightarrow e^+$ decays in emulsion; Garwin, Lederman & Weinrich (26) stopped $\mu^\pm$ in many materials and counted the decay electrons with scintillation counters, verifying approximately the energy dependence of Eq. 16. For both μ^+ and μ^- the highest energy decay electrons ($\epsilon=1$) come off preferentially backwards (i.e., the distribution for $\epsilon=1$ is of the form $1+|P|\lambda\hat{p}_\mu\cdot\hat{p}_e$ with $|P|\lambda<0$). Because μ^- are depolarized while making atomic transitions before they decay, their asymmetry coefficient $|P\lambda|$ is never >10 per cent, but for μ^+ stopping in many materials it is nearly unity ($|P|\lambda\approx-1$). We can then say that for these materials there is little depolarization ($|P|\approx 1$) and that the muons from π decay are fully polarized. Moreover, λ is then roughly -1; since we know $\lambda=(E_S^2-E_V^2)/(E_S^2+E_V^2)$, we have established that μ decay proceeds predominantly through a vector-axial vector interaction.

Now that we know the form of the interaction ($E_V{}^2 \gg E_S{}^2$) we can finally calculate the direction of polarization of the electron in μ decay and compare it with experiment. We proceed just as in 4.3 where we calculated the polarization of nuclear β-rays. In Eq. 1, the E_V term contains e in the combination $\bar{e}\gamma_\alpha(1-\gamma_5)=e^+\gamma_4\gamma_\alpha(1-\gamma_5)=e^+(1-\gamma_5)\gamma_4\gamma_\alpha$ since γ_5 anticommutes with both γ_4 and γ_α. Thus this vector-axial vector interaction involves e coupled only through $(1-\gamma_5)e$, which corresponds to electrons spinning to the right and positrons spinning to the left.

At the time of writing, the positron polarization is in the process of being measured (27); and it seems to be coming out lefthanded, consistent with conservation of leptons in $\pi\rightarrow\mu$ decay. Let us emphasize that if the decay scheme of π^+ were $\pi^+\rightarrow\mu^++\bar{\nu}$, in violation of conservation of leptons, the result would be the opposite: the positrons from μ^+ decay would be spinning to the right. The point is that Eq. 15, giving the angular distribution of positrons from muons of known spin direction, is independent of the pion decay and would still hold; however, in Eq. 16, where we have expressed the angle in terms of the initial muon momentum, we have made use of the decay scheme $\pi^+\rightarrow\mu^++\nu$. If it were $\bar{\nu}$ instead, μ^+ would be left polarized instead of right and the coefficient of $\hat{p}_e\cdot\hat{p}_\mu$ in Eq. 16 would change sign. The experimental asymmetry of $\pi\rightarrow\mu\rightarrow e$ decay would then indicate $\lambda=+1$ or scalar-pseudoscalar interaction. Now 1 and λ_5 both commute with γ_5, as opposed to γ_α and $\gamma_5\gamma_\alpha$, which anticommute with it. Thus the positron polarization would change sign.

We conclude our discussion of the π-μ-e chain by repeating that if leptons are conserved then the K-μ-e chain must give the same μ polarization; consequently the $\widehat{p_\mu/p_e}$ asymmetry must again be given by Eq. 16. This asymmetry has recently been experimentally verified at Berkeley (57). It was mentioned at the end of 4.4 that conservation of leptons must eventually pass three experimental tests. This experiment is the only unambiguous positive result so far reported.

4.6. *The "universal Fermi interaction."*—Since the "universality of strength" of the three sides of the Puppi triangle was remarked (18), there has been speculation that the form of the interaction might also be "universal." Such a situation seems to be ruled out if the β-decay coupling is primarily S and T and the μ decay coupling V and A, as we have stated in 4.3 and 4.5 respectively. Since the β-decay picture is somewhat confused at the moment, let us discuss briefly the possibility that we may have V and A there too, instead of S and T with a possible admixture of V. We may call this V, A hypothesis the "last stand" of the UFI (universal Fermi interaction) (28).

We must first of all disregard much of the evidence on e-ν angular correlation in β-decay, especially the result of Rustad & Ruby (29) on He^6, which clearly indicates T rather than A. This is already a very serious objection to the UFI.

The evidence from β-decay spectra and rates is perfectly consistent with

V and A, as well as with S and T. That exhausts the information available from scalar quantities. We must go on, therefore, to discuss the recent measurements of pseudoscalars. The results of Wu *et al.* (5) and Fraunfelder *et al.* (R2) on the electron asymmetry and longitudinal polarization respectively in the decay of Co^{60} are also perfectly consistent with A instead of T in Gamow-Teller transitions provided the sign of neutrino polarization is changed in the longitudinal theory. We recall from 4.3 that the electron polarization changes sign if we replace T by A; if we also replace $(1-\gamma_5)\nu$ by $(1+\gamma_5)\nu$, i.e., we have left-handed ν instead of right-handed ν, then the sign of the electron polarization changes back to the observed one.

With a longitudinal but left-handed neutrino and a V, A coupling, all β^- polarizations are -100 per cent v/c, just as they are for S, T coupling with a right-handed neutrino. As we mentioned in 4.3, current experiments are testing this in "Fermi transitions" where S and V are important. If -100 per cent v/c polarization is confirmed, it will still not distinguish between pure S, T with right-handed ν and pure V, A with left-handed ν. If it is not confirmed, then there must be an admixture of V in the former case or of S in the latter case. For the moment, let us suppose that the β^- polarization is always -100 per cent v/c and continue to explore the consequences of a universal V, A coupling with left-handed ν and right-handed $\bar{\nu}$.

The value of the spectrum parameter ρ in muon decay still requires the scheme $\mu^\pm \rightarrow e^\pm + \nu + \bar{\nu}$, as in 4.4, and the assignments of lepton and antilepton are unchanged. In pion decay, the conservation of leptons still gives $\pi^+ \rightarrow \mu^+ + \nu$ and $\pi^- \rightarrow \mu^- + \bar{\nu}$. However, μ^+ must now be left polarized and μ^- right polarized instead of the other way around. In Eq. 15 for the electron angular distribution from muons of given spin direction, the sign of the asymmetry term must be changed; this is because in Eq. 14 for the coupling, the parity nonconserving term changes sign when we replace $(1-\gamma_5)\nu$ by $(1+\gamma_5)\nu$ and $\bar{\nu}(1+\gamma_5)$ by $\bar{\nu}(1-\gamma_5)$, while the parity conserving term is unchanged. Thus in Eq. 16 for the asymmetry in the $\pi \rightarrow \mu \rightarrow e$ chain, we have two changes in sign which cancel each other; one from the muon spin direction and one from Eq. 15. Thus the experiments on the $\pi \rightarrow \mu \rightarrow e$ chain still show that $\lambda \equiv (E_S{}^2 - E_V{}^2)/(E_S{}^2 + E_V{}^2)$ is about -1 and that the muon coupling has the form V, A. However, the positron polarization in $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ is now predicted to be positive (right-handed) rather then negative as in 4.5. The positive polarization is in disagreement with preliminary experimental results (27).

In 4.7 we shall see that there is one more piece of evidence against the UFI: the very small or vanishing branching ratio of $(\pi^+ \rightarrow e^+ + \nu)/(\pi^+ \rightarrow \mu^+ + \nu)$.

Despite all of the objections to it at the present time, the UFI hypothesis with V, A coupling seems so attractive that it should perhaps be borne in mind until definitely disproved by experiment. It would correspond to a coupling

$$E_V[\bar{\nu}\gamma_\alpha(1+\gamma_5)\mu][\bar{e}\gamma_\alpha(1+\gamma_5)\nu] + \text{Herm. conj. in muon decay,}$$
$$C_V[\bar{p}\gamma_\alpha(1+\gamma_5)n][\bar{e}\gamma_\alpha(1+\gamma_5)\nu] + \text{Herm. conj. in } \beta\text{-decay, and}$$
$$D_V[\bar{p}\gamma_\alpha(1+\gamma_5)n][\bar{\mu}\gamma_\alpha(1+\gamma_5)\nu] + \text{Herm. conj. in muon capture.}$$

The coefficients C_V and D_V would have to be equal, but they could differ somewhat from E_V since the C and D interactions may be subject to corrections from pion effects in the nucleon. Using Eqs. 9 and 9a, we find, with the couplings listed above,

$$|E_V|\, m_\pi{}^2 \approx 1.6 \times 10^{-7} \quad \text{and} \quad |C_V|\, m_\pi{}^2 = |C_A|\, m_\pi{}^2 \approx 1.6 \times 10^{-7}.$$

The scheme that we have used previously for β-decay, with roughly equal amounts of S and T and a right-handed neutrino, yields in the same way $|C_S|\, m_\pi{}^2 \approx |C_T|\, m_\pi{}^2 \approx 1.6 \times 10^{-7}$. The result for the muon decay is of course the same as above, $|E_V|\, m_\pi{}^2 \approx 1.6 \times 10^{-7}$.

4.7. *Mechanism of pion decay.*—We have sketched so far the present theory of β-decay, μ-decay, and μ-absorption, based on four-fermion contact interactions as indicated schematically by the Puppi triangle. Experiments to date are consistent with CP invariance, the longitudinality of the neutrino and conservation of leptons, although these principles must be subjected to further experimental tests. More experimental work is needed also to determine the values of the five independent real coupling constants in μ-absorption, the five in β-decay (of which two or three may well be zero), and the two in Eq. 14 for μ-decay (of which one, E_S, may be zero). It is known, as mentioned in 4.1, that all three legs of the triangle have about the same strength, but the present experimental situation, indicating S and T (and perhaps V) for β-decay and V and A for μ decay, is hardly suggestive of any universality of form for the four-fermion couplings, unless the β-decay evidence should change, as discussed in 4.6, so as to permit a universal coupling of the V, A type.

We must now take up the question of other weak processes and how they fit into the scheme we have outlined. Let us discuss first the decay of the charged pion, which we have referred to extensively as a source of polarized muons.

An obvious explanation of the decay $\pi^+ \rightarrow \mu^+ + \nu$ is available within the framework of the Puppi triangle: we may suppose that the virtual Yukawa process $\pi^+ \Rightarrow p + \bar{n}$ is followed by the virtual weak process $p + \bar{n} \rightarrow \mu^+ + \nu$ induced by the coupling responsible for μ-absorption. (See the top Feynman Diagram of Fig. 3, sect. 4.9.) Let us use this possible explanation as the starting point of our discussion.

It is important to notice that only two of the five possible μ absorption couplings can induce the decay of the pseudoscalar pion: these are the pseudoscalar coupling

$$D_P[\bar{p}\gamma_5 n][\bar{\mu}\gamma_5(1-\gamma_5)\nu] + \text{Herm. conj.}$$

and the axial vector coupling

$$-D_A[\bar{p}\gamma_\alpha\gamma_5 n][\bar{\mu}\gamma_\alpha\gamma_5(1-\gamma_5)\nu] + \text{Herm. conj.}$$

For the other three couplings (scalar, vector, and tensor) the decay of the charged pion is forbidden (apart from electromagnetic corrections). We can derive this result by means of field theory. Imagine the most general Feynman diagram for the process $\pi^+ \Rightarrow p + \bar{n} \Rightarrow ? \Rightarrow p + \bar{n} \rightarrow \mu^+ + \nu$, where the question mark stands for all the possible strong processes that may intervene between the creation of the virtual $(p\bar{n})$ pair by the pion at some space-time point x and its annihilation into leptons at another space-time point y. No matter what these intervening processes are, they give an effective coupling between the pion destroyed at x and the nucleon-antinucleon operator at y that is involved in the interaction with leptons. This effective coupling is nonlocal, but can depend on only one vector in space-time, namely $(x-y)$. From the pseudoscalar pion field and one four-vector, the only couplings we can form are the pseudoscalar P and the axial vector A, as was stated above.

In the following discussion, we shall ignore the UFI hypothesis of 4.6 and adopt the point of view of 4.1–4.5 that there is no universality of form.

It is not yet known experimentally whether or not the μ capture interaction contains appreciable amounts of P or A. Suppose, for the moment, that a large fraction of the μ capture interaction is A. Then $|D_A| \sim (10^{-7}/m_\pi^2)$. We may then try to estimate (very roughly) the rate of $\pi \to \mu\nu$ decay through the axial vector coupling by treating the Yukawa process in the lowest order of perturbation theory. The result is logarithmically divergent and must be cut off at some virtual mass λ:

$$\Gamma_\pi \approx \frac{1}{2\pi^4} m_\pi [D_A m_\pi^2]^2 \frac{g^2}{4\pi} \left(\ln \frac{\lambda}{m_N}\right)^2 \left(\frac{m_N}{m_\pi}\right)^2 \left(\frac{m}{m_\pi}\right)^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 \approx 7 \times 10^8/\text{sec.} \quad 17.$$

if we estimate D_A as above and $\ln \lambda/m_N$ as unity. (We recall that $g^2/4\pi \approx 15$.) This result is about 15 times greater than the experimental rate in Table I. Such a discrepancy is not surprising in view of the extreme crudity of the calculation, and we have no particular reason, therefore, for discarding the simple explanation of $\pi \to \mu\nu$ decay. If the μ capture interaction contains P, the formula corresponding to Eq. 17 contains a quadratic divergence, and the argument is even less reliable.

We must, of course, understand not only the occurrence of the decay $\pi^\pm \to \mu^\pm \pm \nu$ but also the absence or extreme rarity of the analogous process $\pi^\pm \to e^\pm \pm \nu$, for which the branching ratio is quoted experimentally as $<10^{-5}$ (30). We have mentioned in 4.3 that there seems to be little or no A in the β-decay coupling but that essentially no information is available about P. If P and A are both lacking in β-decay, then $\pi^\pm \to e^\pm \pm \nu$ is forbidden. At the moment, this seems the likeliest explanation of the situation. The forbidden process $\pi^\pm \to e^\pm \pm \nu$ can still occur through electromagnetic effects: the virtual proton or antiproton emits a virtual photon that is absorbed by the final electron. The rate of the decay is then expected to be so small that it is inaccessible to present experimental techniques.

There is, however, another electromagnetic process that should occur more rapidly, in which the virtual proton or antiproton emits a real photon. The decay scheme is $\pi^\pm \to e^\pm \pm \nu + \gamma$. This is allowed, for example, for the tensor interaction in β-decay. Again we may estimate the rate using first order perturbation theory for the Yukawa process, and again the result is logarithmically divergent:

$$\Gamma_{\pi\to e\nu\gamma} \approx \frac{C_T^2 m_\pi^5}{384\pi^5}\frac{e^2}{4\pi}\frac{g^2}{4\pi}\left(\ln\frac{\lambda}{m_N}\right)^2 \approx 5\times 10^3/\text{sec.} \qquad 18.$$

using the value $|C_T|m_\pi^2 \approx 1.6\times 10^{-7}$ given by β-decay phenomenology and estimating $\ln\lambda/m_N$ as unity as before. Experimental upper limits for the rate of the process are given by Cassels (31) and by Lokanathan (32) as 4×10^2/sec. and 10^3/sec., respectively. Just as for $\pi^\pm\to\mu^\pm\pm\nu$, the crude calculation has overestimated the rate by at least a factor of ten, but the general theoretical picture does not seem to be in serious trouble. Of course, if further measurements show that the rate of the radiative decay is even lower, we may have to revise the theory.

Besides the obvious objections to a calculation using a cutoff and perturbation theory in a strong interaction, there is an additional point to be made about the theory. The pion (say π^+) may dissociate virtually not only into $p+\bar{n}$ but also into the following baryon-antibaryon pairs: $\Sigma^+ + \overline{\Lambda}$, $\Sigma^+ + \overline{\Sigma^0}$, $\Lambda + \overline{\Sigma^-}$, $\Sigma^0 + \overline{\Sigma^-}$, and $\Xi^0 + \overline{\Xi^-}$. All of these processes are allowed by charge independence and can presumably be induced by the strong interactions. It may easily turn out that these baryons are particles of spin $\frac{1}{2}$ and possess weak couplings to the pairs $\mu\nu$ and $e\nu$ analogous to the μ-absorption and β-decay couplings of the nucleons. In the Puppi triangle, we would have to replace the label "$(p\bar{n})$" at one vertex by the label "$(p\bar{n})$, $(\Sigma^+\overline{\Lambda})$, etc." In the decay of the pion, then, all six of these baryon-antibaryon pairs may appear as intermediate states and interfere with one another. If the interference should turn out to be destructive, it might help to explain the overestimates obtained in Eqs. 17 and 18. However, the calculations are so crude that at present this is pure speculation.

We have ignored completely the possibility of a direct coupling of π to μ and ν, relying entirely on four-fermion interactions to explain the decay of the charged pion. Such a point of view may, of course, be wrong. However, there is not much practical difference between the two theories. We have seen that a pseudoscalar or axial vector coupling of $p\bar{n}$ to $\mu^+\nu$ implies an effective coupling of π to $\mu\nu$. Conversely a direct coupling of π to $\mu\nu$ implies an effective pseudoscalar or axial vector interaction in μ capture. Experiment cannot distinguish one theory from the other without reliable calculations involving the strong interactions in virtual baryon-antibaryon pair states, and such calculations are far from possible at the present time.

Practically the same situation applies to the weak interactions of the strange particles. We shall therefore discuss all weak interactions in terms of four-fermion couplings, leaving in abeyance the question of whether direct weak couplings of mesons exist as well.

Let us now return briefly to the hypothesis of the UFI with V, A coupling, as treated in 4.6. We promised to show that the experimental ratio $(\pi^+\to e^+ + \nu)/(\pi^+\to\mu^+ + \nu) < 10^{-5}$ is inconsistent with the hypothesis. With V, A as the coupling, only A is effective in inducing the decays. We may refer to Eq. 17 for the $\pi\to\mu\nu$ rate using A. The $\pi\to e\nu$ rate is given by the same formula with m_μ replaced by m_e, so that the ratio is

$$\frac{\Gamma(\pi^+ \to e^+ + \nu)}{\Gamma(\pi \to \mu^+ + \nu)} = \frac{m_e^2}{m_\mu^2} \cdot \frac{\left(1 - \frac{m_e^2}{m_\pi^2}\right)^2}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} \approx 1.3 \times 10^{-4}.$$

This result was remarked by Ruderman & Finkelstein (33), who showed that it is exact to all orders in the strong couplings.

4.8. *Classification of weak interactions; the tetrahedron.*—What changes must we make in the Puppi triangle in order to accommodate all known weak processes?

In 4.6 we have already suggested that it may be necessary to modify the vertex $p\bar{n}$ by adding some or all of the five additional baryon pairs $\Sigma^+\bar{\Lambda}$, $\Sigma^+\overline{\Sigma^0}$, etc. It is difficult to test whether or not this replacement is necessary, since interconversion among all these baryon-antibaryon pairs is possible by means of the strong couplings. For example, if someone should observe an event $\Sigma^+ \to \Lambda + e^+ + \nu$, this might be interpreted either as caused by a direct coupling of these four particles or else as proceeding indirectly through the usual β-decay coupling by means of the virtual reactions $\Sigma^+ \Leftrightarrow \Lambda + p + \bar{n} \to \Lambda + e^+ + \nu$. In the absence of quantitative arguments, we must regard the above-mentioned six baryon-antibaryon pairs as a class and not attempt at the moment to resolve the class into its individual members. Let us call it Class I of baryon-antibaryon pairs.

The observed weak processes not accounted for by the Puppi triangle all involve strange particles. We may start with the decay $K^\pm \to \mu^\pm \pm \nu$. (We assume conservation of leptons.) The selection rules obeyed by the strong interactions allow K^+ to dissociate into any of the following six charged pairs of baryons and antibaryons: $p + \bar{\Lambda}$, $p + \overline{\Sigma^0}$, $n + \overline{\Sigma^-}$, $\Lambda + \overline{\Xi^-}$, $\Sigma^+ + \overline{\Xi^0}$, $\Sigma^0 + \overline{\Xi^-}$. Let us refer to these as Class II and again not attempt to distinguish carefully among them. If some or all of the pairs of Class II are coupled by appropriate four-fermion couplings to μ and ν, then the decay of the charged K into muon and neutrino can be understood. By "appropriate," we mean, of course, "consistent with selection rules"; for example, if K is pseudoscalar relative to Λ and N, then we refer to a pseudoscalar or axial vector coupling of $p\bar{\Lambda}$ to $\mu^+\nu$ as "appropriate." (The situation is then exactly analogous to that in $\pi^\pm \to \mu^\pm \pm \nu$ decay.)

The decay $K^\pm \to \pi^0 + \mu^\pm \pm \nu$ also requires couplings of Class II to μ and ν. although here the selection rules on the form of the coupling are different. (See 4.9.) The point is that any of the virtual baryons or antibaryons can easily emit a π^0.

Similarly the decay $K^\pm \to \pi^0 + e^\pm \pm \nu$ corresponds to four-fermion couplings between Class II and the electron-neutrino pair. So does the unobserved decay $K^\pm \to e^\pm \pm \nu$, but this can be forbidden, like the analogous decay of the pion, by omitting some of the possible forms of the coupling.

The purely pionic decays of K particles can be understood in terms of four-fermion couplings between Class I and Class II. For example, the process $K^+ \to \pi^+ + \pi^0$ can be thought of in terms of the sequence of virtual steps $K^+ \Rightarrow p + \bar{\Lambda} \to p + \bar{n} \Rightarrow p + \bar{n} + \pi^0 \Rightarrow \pi^+ + \pi^0$ or else, say, $K^+ \Rightarrow \Lambda^0 + \bar{\Xi}^- \to \Sigma^+ + \bar{\Sigma}^0 \Rightarrow \Sigma^+ + \bar{\Sigma}^0 + \pi^0 \Rightarrow \pi^+ + \pi^0$, etc.

The reader may now satisfy himself that all of the weak decays of strange particles listed in Tables II and VI are accounted for by the interactions we have just listed; namely, those of Class II with $e\nu$, $\mu\nu$, and Class I. Let us give some further examples. For the decay $\Sigma^+ \to p + \pi^0$ we may have a chain like $\Sigma^+ \to \Sigma^0 + p + \bar{\Lambda} \Rightarrow p + \pi^0$. For $\Xi^- \to \Lambda + \pi^-$ one possibility is $\Xi^- \to \Lambda + n + \bar{p} \Rightarrow \Lambda + \pi^-$, and so forth.

A number of so far unobserved processes are predicted also. For example, the coupling of Class II to $e\nu$ leads to at least one of the following leptonic decays of hyperons: $\Lambda^0 \to p + e^- + \bar{\nu}$, $\Sigma^0 \to p + e^- + \bar{\nu}$ (undetectable because of competition from $\Sigma^0 \Rightarrow \Lambda^0 + \gamma$), $\Sigma^- \to n + e^- + \bar{\nu}$, $\Xi^- \to \Lambda^0 + e^- + \bar{\nu}$, $\Xi^- \to \Sigma^0 + e^- + \bar{\nu}$, and $\Xi^0 \to \Sigma^+ + e^- + \bar{\nu}$. We discuss these hypothetical processes further in 4.9.

The picture we have outlined is summarized in Figure 2a. The Puppi triangle is replaced by a tetrahedron, with vertices occupied by $e^+\nu$, $\mu^+\nu$, Class I and Class II, respectively. We may think of the K particles as attached by means of strong interactions to the Class II vertex (for example through $K^+ \Leftrightarrow p + \Lambda$), while one or more pions are similarly attached to the Class I vertex (for example through $\pi^+ \Leftrightarrow p + \bar{n}$).

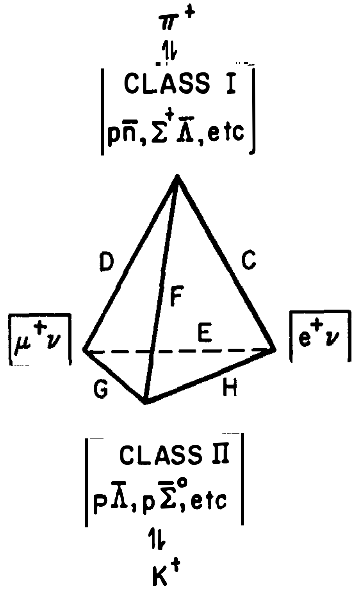

FIG. 2a. Tetrahedron representation of four-fermion interaction of positively charged pairs. Legs C, D, and E form the Puppi triangle of Fig. 2. Strange particles are introduced at the Class II vertex; their decays are represented by legs F, G, and H.

The physical ideas behind the tetrahedron scheme have been put forward by Dallaporta and collaborators (34) and by Gell-Mann (R1).

In our subsequent discussion of the weak decays of strange particles, we shall refer to Figure 2a and to the rough theory that it represents. Let us therefore make clear what is involved in such a description of the weak couplings.

We have already emphasized that it is not worth while at the present moment to quibble about whether some of the four-fermion interactions should be replaced by, say, boson-fermion interactions, nor about which members of Classes I and II actually possess direct weak couplings. We have seen, moreover, that the tetrahedron expresses the general features of the experimental situation. In 4.9 we shall discuss the "universality of strength" of the six edges of the tetrahedron. The principal point to be clarified, therefore, is the nature of the conceivable weak processes not included in Figure 2a.

In effect, by restricting the weak interactions to the *minimal set* (those represented in Fig. 2a) that can explain known processes, we are constructing an elaborate set of selection rules forbidding other weak couplings. Some of these additional couplings are known to be absent or nearly so, others can be looked for in the near future, while still others are inaccessible to investigation with present techniques. Let us list some of the more interesting hypothetical couplings omitted from the tetrahedron: (*a*) The processes $\mu^\pm \to e^\pm + e^\pm + e^\mp$, $\mu^\pm \to e^\pm + \gamma$, and $\mu^- + p \to e^- + p$ have been searched for and found to be very rare or absent (35). Nothing in the tetrahedron is known to induce these. In fact the only interaction in our scheme that couples μ and e together is the muon decay interaction. (In Table VI an experimental upper limit is given for the rate of the hypothetical decay $K^+ \to \mu^\pm + e^\mp + \pi^+$, a typical reaction in which μ and e might occur together.) (*b*) The muon decay interaction is also the only one listed that couples a neutrino-antineutrino pair. Now it is conceivable that other systems also can decay by $\nu - \bar{\nu}$ pair emission. For example, weak virtual processes like $n \to n + \nu + \bar{\nu}$ or $p \to p + \nu + \bar{\nu}$ might exist. They are completely undetectable at present, however, because in the decay of nuclear excited states or the decay of π^0, where such couplings might show up, they are overwhelmingly dominated by electromagnetic decays. Other decays involving $\nu - \bar{\nu}$ pairs could be detected if they exist: For example, $K^\pm \to \pi^\pm + \nu + \bar{\nu}$ or $\Sigma^+ \to p + \nu + \bar{\nu}$ could be distinguished from other decay schemes, and all competing processes, moreover, are weak. An upper limit for the rate of the former process is given in Table VI. The decays $K_1^0 \to \nu + \bar{\nu}$ and $K_2^0 \to \nu + \bar{\nu}$, it is interesting to note, are forbidden by conservation of angular momentum if K is spinless and the neutrino longitudinal. (*c*) It is convenient to introduce a Class III of baryon-antibaryon pairs that might form an additional fifth vertex of our figure that was first a triangle and then a tetrahedron. This class consists of the two pairs $\Sigma^+\bar{n}$ and $\Xi^0\overline{\Sigma^-}$; these are the pairs into which a hypothetical meson of positive charge but negative strangeness might dissociate. (We recall that Classes I and II are the possible dissociation products of π^+ and K^+, respectively.) We may now generate a group of weak couplings excluded from the tetrahedron: we couple the pairs of Class III to $e\nu$ or to $\mu\nu$. This will lead to such decays as $\Sigma^+ \to n + \mu^+ + \nu$ and $\Xi^0 \to \Sigma^- + e^+ + \nu$, which are forbidden in the tetrahedron scheme.

A subtle point now arises in connection with the leptonic decay of K^0 and $\overline{K}^0$. We note that in the tetrahedron picture we can have only $\overline{K}^0 \to e^- + \bar{\nu} + \pi^+$, or $\mu^- + \bar{\nu} + \pi^+$, $K^0 \to e^+ + \nu + \pi^-$ or $\mu^+ + \nu + \pi^-$. These processes are essentially implied by the corresponding decays of K^- and K^+.

The hypothetical couplings of Class III to the $e\nu$ and $\mu\nu$ vertices would lead to the additional processes $\overline{K}^0 \to e^+ + \nu + \pi^-$ or $\mu^+ + \nu + \pi^-$, and $K^0 \to e^- + \bar{\nu} + \pi^+$ or $\mu^- + \bar{\nu} + \pi^+$.

The subtlety enters when we remember that the decay of neutral K mesons must be discussed as in 3.9, in terms of $K_1{}^0$ and $K_2{}^0$, which are linear combinations of K^0 and $\overline{K}^0$. We shall return to this very interesting question in 6.9. (*d*) If we introduce Class III as above and couple it to Class II, we obtain interactions involving *SIP*s only, but with ΔS not restricted to ± 1. Such processes as $\Xi^- \to n + \pi^-$, involving $|\Delta S| = 2$, become possible. In 3.8, we have mentioned that there is some evidence against such decays. (*e*) The tetrahedron excludes also decays of Ξ into N with lepton pair emission, such as $\Xi^- \to n + e^- + \bar{\nu}$.

In concluding this survey, let us call attention to one further set of possible weak interactions of the four-fermion type, a set which is suggested by the tetrahedron scheme although not included in it as we have described it so far. These are couplings of the four vertices to themselves. For example, we may have such interactions as $[\bar{e}(1-\gamma_5)\nu][\bar{\nu}(1+\gamma_5)e]$ or $(\bar{p}n)(\bar{n}p)$ or $(\bar{p}\Lambda)(\overline{\Sigma^0}p)$.

The first one represents electron-neutrino scattering and may be experimentally detectable by means of the recoils of atomic electrons struck by the antineutrinos from a large nuclear reactor. The cross section should be slightly smaller than the cross section for the reaction $\bar{\nu} + p \to e^+ + n$, which has been observed (12). It should be noted that in the longitudinal theory the neutrino possesses no magnetic or electric moments and any neutrino-electron scattering that may be observed is presumably to be ascribed to the interaction we are discussing.

A weak coupling of the pair $\bar{p}n$ to itself might some day be detectable to the extent that it violates conservation of parity. If it is of the same strength as other weak couplings, it could give a violation of parity conservation by one part in 10^{13} or 10^{14} in intensity in a nuclear experiment such as Tanner's (17) (mentioned in 3.10). Present measurements exclude any violation by more than one part in 10^7. If no coupling of Class I (or Class II) to itself exists, then parity may be conserved in nuclear processes to one part in 10^{26} or 10^{28}. At this level, second order weak processes come into play such as virtual β-decay followed by virtual inverse β-decay.

4.9. *Decay rates of strange particles.*—We have sketched in 4.8 the tetrahedron scheme that provides a minimal set of four-fermion couplings that can account for known weak processes. Three of the six edges of the tetrahedron are simply the three sides of the Puppi triangle. The remaining three edges correspond to the coupling of Class II ($p\bar{\Lambda}$, etc.) to $e^+\nu$, $\mu^+\nu$, and Class I ($p\bar{n}$, etc.); these three are the ones responsible for strange particle decays. We shall now present some crude estimates of the rates of decay of strange particles on the assumption that these edges have about the same strength as those in the Puppi triangle. The estimates are in reasonable accord with experiment.

We have supposed that the baryons have spin $\frac{1}{2}$. Let us further suppose that the spin of K is zero. (Experimental evidence for this is adduced in 6.1.) The relative parity of Λ and N is not a determinate quantity, since any process such as $\Lambda \to \pi + N$, by means of which it might be measured, is weak and need not conserve parity. However, as pointed out in 3.10, the parity of K relative to Λ and N is unique if parity is conserved by the strong interactions. Let us therefore define the parity of Λ to be the same as that of N and call K scalar or pseudoscalar according to its coupling to Λ and N. (It is in the same spirit that we conventionally define the parities of p and n to be the same and call $\pi^\pm$ pseudoscalar because of its pseudoscalar coupling to p and n.)

For simplicity let us consider in Class I only the pair $p\bar{n}$ and in Class II only the pair $p\bar{\Lambda}$. In this way we discard many interesting effects, especially possible cancellations among the various members of a class, but we are concerned here only with very rough estimates.

Let us look first at the decays of Λ. The coupling of $p\bar{\Lambda}$ and $p\bar{n}$ can lead to the decays $\Lambda \to p + \bar{p} + n \Rightarrow p + \pi^-$ and $\Lambda \to p + \bar{p} + n \Rightarrow n + \pi^0$, which have been observed. The couplings of $p\bar{\Lambda}$ to $e^+\nu$ and to $\mu^+\nu$ give the processes $\Lambda \to p + e^- + \bar{\nu}$ and $\Lambda \to p + \mu^- + \bar{\nu}$ respectively; these have never been detected with certainty and seem to form $\lesssim 2$ per cent of all decays of Λ (Table II). We must, of course, try to account for this situation.

The absolute rates of the leptonic decays are easily estimated. There is a direct analogy between these processes and the decay of the neutron, $n \to p + e + \bar{\nu}$. We may refer to Eq. 6 for the rate of neutron decay. Assuming that $C_s^2 + C_s'^2 + \cdots$, is about the same for Λ couplings as for the neutron, we see that the only differences are these: Δ, which is now the mass difference of Λ and p, is 177 Mev instead of 1.293 Mev; and the blocking factor is 1 for $\Lambda \to p + e^- + \bar{\nu}$ and 0.16 for $\Lambda \to p + u^- + \bar{\nu}$. With the couplings roughly the same, therefore, we have for the rates

$$\Gamma(\Lambda \to p + e^- + \bar{\nu}) = \left(\frac{177}{1.293}\right)^5 \frac{1}{.47} \Gamma_n \sim 10^8/\text{sec.}$$

$$\Gamma(\Lambda \to p + \mu^- + \bar{\nu}) \sim .16\Gamma(\Lambda \to p + e^- + \bar{\nu}) \sim .16 \times 10^8/\text{sec.}$$

Here Γ_n is, of course, the rate of neutron decay. We have neglected recoil effects, which may reduce these estimates by as much as a factor of two. The experimental upper limit on the rate of leptonic decays (Table II) is in the vicinity of 10^8 /sec. If these rates are really not much lower, the theory is not in trouble.

We must somehow understand, however, why the pionic decays go much faster. Let us try to estimate the rate of $\Lambda \rightarrow p+\pi^-$. We might simply use the lowest order of perturbation theory for the strong couplings in the process $\Lambda \rightarrow p+n+\bar{p} \Rightarrow p+\pi^-$. It is helpful to note, however, the similarity between the decays $\Lambda \rightarrow p+\pi^-$ and $\pi^\pm \rightarrow \mu^\pm \pm \nu$, exhibited in the lowest order Feynman diagrams for these two processes (Fig. 3). We must recall that in 4.7, when we treated the pion decay using an axial vector coupling of typical strength, we found that the nucleon-antinucleon loop in the lowest order Feynman diagram gave rise to a logarithmic divergence. When we estimated

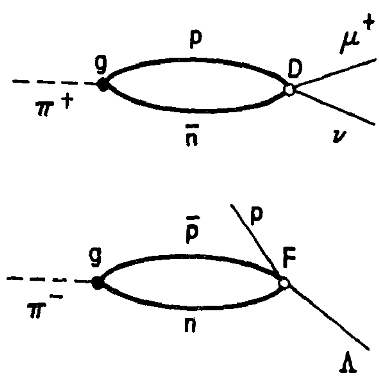

FIG. 3. Feynman diagrams for $\pi^+ \rightarrow \mu^+ + \nu$ and $\Lambda \rightarrow p+\pi^-$. The solid circles labeled g represent the strong πN interaction. The open circles labeled D and F represent the weak four-fermion interactions (legs D and F of the tetrahedron).

the logarithm by means of a cutoff in the vicinity of the nucleon mass, we found that the rate of pion decay came out too high by a factor of about 15.

In view of the many uncertainties in the pion decay calculation (perturbation theory, cutoff, possible contributions of other pairs in Class I, etc.) it is probably wisest to let the rate of $\pi^+ \rightarrow \mu^+ + \nu$ serve as an experimental calibration of the effect of the nucleon-antinucleon loop. We mentioned in 4.7 that the loop gives rise to an effective nonlocal interaction of π with the pair $\mu\nu$. Actually, since the mass of the nucleon pair in the loop is much greater than the mass of the pion, we may approximate this by a local coupling. Thus the axial vector interaction

$$\mathfrak{L}_A = - D_A[\bar{p}\gamma_\alpha\gamma_5 n][\bar{\mu}\gamma_\alpha\gamma_5(1-\gamma_5)\nu] + \text{Herm. conj.} \qquad 19.$$

of 4.7 gives rise effectively to a direct interaction of the form

$$\mathfrak{L}_{\text{eff.}} = \frac{id}{m_\pi}\frac{\partial\pi}{\partial x_\alpha}[\bar{\mu}\gamma_\alpha\gamma_5(1-\gamma_5)\nu] + \text{Herm. conj.} \qquad 20.$$

where we have formed an axial vector by taking the gradient of the pseudoscalar pion field operator π that destroys π^- or creates π^+. The Hermitian conjugate operator $\pi^\dagger$ destroys π^+ or creates π^-. In 4.7, we tried to calculate d from the loop. Now let us take it from experiment. The rate of charged pion decay, with the coupling of Eq. 20, is

$$\Gamma_\pi = \frac{d^2}{4\pi} m_\pi \frac{m_\mu^2}{m_\pi^2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2. \qquad 21.$$

Using the known rate Γ_π, we find $(d^2/4\pi) \approx 1.8 \times 10^{-15}$.

Suppose we try a coupling of $p\bar{n}$ to $p\bar{\Lambda}$ analogous to Eq. 19 for $p\bar{n}$ to $\mu^+\nu$. In the case of $p\bar{\Lambda}$, we have no longitudinal theory that forces us to the expression $(1-\gamma_5)$; the "parity conserving" and "parity nonconserving" terms need not have equal strength. Let us therefore imagine an interaction like

$$-F_A[\bar{p}\gamma_\alpha\gamma_5 n][\bar{\Lambda}\gamma_\alpha\gamma_5 p] - F_A'[\bar{p}\gamma_\alpha\gamma_5 n][\bar{\Lambda}\gamma_\alpha p] + \text{Herm. conj.}$$

Then the effective coupling analogous to Eq. 20 will be

$$\frac{if}{m_\pi}\frac{\partial\pi}{\partial x_\alpha}[\bar{\Lambda}\gamma_\alpha\gamma_5 p] + \frac{if'}{m_\pi}\frac{\partial\pi}{\partial x_\alpha}[\bar{\Lambda}\gamma_\alpha p] + \text{Herm. conj.} \qquad 22.$$

The first (parity conserving) term gives rise to $\Lambda \to p + \pi^-$ with the pion in a p state and the second (parity nonconserving) term gives $\Lambda \to p + \pi^-$ with the pion in an s state. (Remember we have defined the parities of Λ and p to be the same.) Using Eq. 22, we may calculate the rates of decay of Λ into $p + \pi^-$ in an s or a p state respectively, neglecting recoil for simplicity:

$$\Gamma_{\Lambda^s} \to p + \pi^- = \frac{f'^2}{4\pi}\cdot 2p_\Lambda\left(\frac{m_\Lambda - m_p}{m_\pi}\right)^2, \qquad 23.$$

$$\Gamma_{\Lambda^p} \to p + \pi^- = \frac{f^2}{4\pi}\cdot 2p_\Lambda\left(\frac{p_\Lambda}{m_\pi}\right)^2, \qquad 24.$$

where p_Λ is the momentum of the emitted pion (see Table III). If the order of magnitude of $F_A'^2$ and D_A^2 is the same, then we should expect $f'^2/4\pi \approx (d^2/4\pi)$; similarly, if F_A^2 and D_A^2 are comparable, so are $f^2/4\pi$ and $d^2/4\pi$. Under the former assumption, using our experimental value of $d^2/4\pi$, we find $\Gamma_{\Lambda \to p + \pi^-} \sim 10^9$/sec. If $(f^2/4\pi) \sim (d^2/4\pi)$, we have $\Gamma_{\Lambda \to p + \pi^-} \sim 3 \times 10^8$/sec. The experimental value of the total rate of $\Lambda \to p + \pi^-$ is $\approx 2 \times 10^9$/sec. (see Table II), in excellent and probably fortuitous agreement with the calculated values.

If we had used pseudoscalar rather than axial vector coupling, the agreement would have been somewhat worse and the ratio of s to p wave emission would have been much larger for equivalent coupling constants.

Let us regard our discussion of Λ as typical of the situation for hyperons, and turn to the decays of K particles. The process $K^+ \to \mu^+ + \nu$ is also analogous to $\pi^+ \to \mu^+ + \nu$, although here the analogy is of a different nature. The Feynman diagrams to lowest order in the strong couplings are shown in Figure 4. In the pion decay, the loop is composed of, say, nucleon and antinucleon, while in K^+ decay it is, say, a $p\bar{\Lambda}$ loop. The coupling of K to $\bar{\Lambda}$ and N need not have precisely the same strength as that of π to N and $\bar{N}$; in

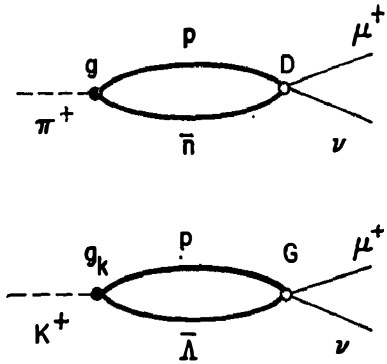

FIG. 4. Feynman diagrams for $\pi^+ \to \mu^+ + \nu$ and $K^+ \to \mu^+ + \nu$. The solid circles labeled g represent the strong meson-nucleon interactions. The open circles represent weak four-fermion interactions (legs D and G of the tetrahedron).

fact, K might be scalar rather than pseudoscalar so that the coupling would even have a different form. Also the K mass is greater than that of π and not nearly so negligible compared to baryon masses.

These differences are in addition to possible differences between the weak coupling of $p\bar{\Lambda}$ to $\mu^+\nu$ and that of $p\bar{n}$ to $\mu^+\nu$. Say we write the former as

$$G_S[\bar{p}\Lambda][\bar{\mu}(1 - \gamma_5)\nu] + G_V[\bar{p}\gamma_\alpha\Lambda][\bar{\mu}\gamma_\alpha(1 - \gamma_5)\nu] + \cdots + \cdots + \text{Herm. conj.} \quad 25.$$

where the labels S, V, etc., refer to the Dirac matrices for the baryons. We have assumed the two component theory of the neutrino and conservation of leptons. Supposing that the coefficients G are of the same order of magnitude as the constants D in the muon capture interaction, let us pursue our rough analogy between $K^\pm \to \mu^\pm \pm \nu$ and $\pi^\pm \to \mu^\pm \pm \nu$.

If K, like π, is pseudoscalar, then only the P and A interactions can induce the decay $K^\pm \to \mu^\pm \pm \nu$. (The reason is the same as that given in 4.7 for the pion case.) By the same argument, if K is scalar, only the S and V interactions can induce it. Suppose that the interaction responsible for the decay is A (for a pseudoscalar K) or V (for a scalar K). This corresponds to our assumption of an A interaction in treating the pion decay. The $p\bar{\Lambda}$ loop will produce an effective coupling of K to $\mu\nu$ analogous to the effective coupling between π and $\mu\nu$ displayed in Eq. 20. The coefficient of the effective interaction is called d/m_π in Eq. 20; for the K particle let us call the corresponding coefficient γ/m_K. Then in place of Eq. 21 for the pion decay rate we have for the rate of $K^\pm \to \mu^\pm \pm \nu$ the expression

$$\Gamma_{K\to\mu\nu} = \frac{\gamma^2}{4\pi} m_K \frac{m_\mu^2}{m_K^2}\left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 \quad 26.$$

Since experimentally $\Gamma_{K\to\mu\nu} \approx \Gamma_{\pi\to\mu\nu}$, we see that $\gamma^2/m_K^2 \approx 1/17\ d^2/m_\pi^2$. Why is the effective coupling in K decay apparently much smaller than that in π decay if the weak four-fermion couplings have about the same strength? The explanation may be that the strong coupling of K to Λ, N is somewhat weaker than the Yukawa coupling of π to the nucleon. Experiments on the photo-K effect ($\gamma+p\Rightarrow\Lambda+K^+$) should soon decide whether this is the case.

As in pion decay, we must understand why $\mu^\pm \pm\nu$ is preferred over $e^\pm\pm\nu$. The experimental upper limit on the ratio $(K^\pm\rightarrow e^\pm\pm\nu)/(K^\pm\rightarrow\mu^\pm\pm\nu)$ is only 0.02 (Table VI) as compared with 10^{-5} for the pion, but the problem still exists. We mentioned in discussing the UFI (4.6) that with identical couplings of Class I to $\mu^+\nu$ and $e^+\nu$ the lowest ratio $(\pi^\pm\rightarrow e^\pm\pm\nu)/(\pi^\pm\rightarrow\mu^\pm\pm\nu)$ is attained for the A interaction, namely

$$\frac{\Gamma(\pi\rightarrow e+\nu)}{\Gamma(\pi\rightarrow\mu+\nu)} = \frac{m_e^2}{m_\mu^2}\,\frac{\left(1-\dfrac{m_e^2}{m_\pi^2}\right)^2}{\left(1-\dfrac{m_\mu^2}{m_\pi^2}\right)^2} \approx 1.3\times10^{-4}.$$

Since according to Anderson and Lattes (30) the actual ratio is less than 10^{-5}, we must apparently give up the idea of identical couplings.

This result makes speculation about identical couplings of Class II to $e^+\nu$ and $\mu^+\nu$ seem rather unprofitable. For purposes of orientation, however, we may remark that for equal A couplings the ratio $(K^\pm\rightarrow e^\pm\pm\nu)/(K^\pm\rightarrow\mu^\pm\pm\nu)$ would be $\approx 2.5\times10^{-5}$ while for equal P couplings it would be ≈ 1. (We are taking K to be pseudoscalar.) Probably the best we can say at the moment is the following: The coupling of Class II to $\mu^+\nu$ contains P and/or A while the coupling to $e^+\nu$ probably lacks P but may or may not contain A. If K is scalar, we must replace P by S and A by V in this statement.

The decays $K\rightarrow\pi+e^\pm\pm\nu$ and $K\rightarrow\pi+\mu^\pm\pm\nu$ occur with about equal frequency. This is in no sense a paradox, despite the smallness of the ratio $(K^\pm\rightarrow e^\pm\pm\nu)/(K^\pm\rightarrow\mu^\pm\pm\nu)$, since it turns out that the leptonic decay with single pion emission occurs through just those interactions that cannot induce plain leptonic decay. For a pseudoscalar K, the processes $K\rightarrow\pi+$leptons are induced by S, T, and V; for a scalar K, by P, T, and A.

The rate of $K^\pm\rightarrow\pi^0+\mu^\pm\pm\nu$ is about 15 times smaller than that of $K^\pm\rightarrow\mu^\pm\pm\nu$. Simple considerations of available phase space may explain this qualitatively. The uncertainties in quantitative arguments seem to be so great as to make a detailed calculation not worthwhile.

The same applies to the rates of $K\rightarrow3\pi$ and $K\rightarrow2\pi$, which occur through the coupling of Class I with Class II in the tetrahedron scheme. We shall see in 6.5 that for a spinless K the 3π are in a 0^- state of angular momentum and parity, while the 2π are in a 0^+ state. For a pseudoscalar K, then, the former decay is induced by the "parity conserving" terms in the coupling and the latter by the "parity nonconserving" terms. For a scalar K, the reverse is true.

5. The Weak Decays of Hyperons

5.1. *General features*—In Table II we show what is known about hyperon decays. We have already referred to the principal features of the situation. Let us review them here.

TABLE II

BRANCHING RATIOS AND DECAY RATES OF Λ AND $\Sigma^{\pm}$

Mode	Branching ratio (%)	Rate* (number per sec)	Momentum p† (Mev/c)	$h\Gamma/2pc$‡
$\Lambda \to p + \pi^+$	65 ± 5 (a)	0.234×10^{10}	99.9	0.78×10^{-14}
$\to n + \pi^0$	35 ± 5 (a)	0.126×10^{10}	103.3	0.41×10^{-14}
$\to p + \begin{Bmatrix} e \\ \mu \end{Bmatrix}^- + \bar{\nu}$	<2 (a)	$(<0.007 \times 10^{10})$		
	100	0.36×10^{10}		1.19×10^{-14}
$\Sigma^+ \to p + \pi^0$	46 ± 6 (b)	0.59×10^{10}	189.0	1.04×10^{-14}
$\to n + \pi^+$	54 ± 6 (b)	0.69×10^{10}	185.0	1.24×10^{-14}
$\to n + \begin{Bmatrix} e \\ \mu \end{Bmatrix}^+ + \nu$	<3 (b)	$(<0.04 \times 10^{10})$		
	100	1.28×10^{10}		2.28×10^{-14}
$\Sigma^- \to n + \pi^-$	100	0.64×10^{10}	192.1	1.06×10^{-14}
$\to n + \begin{Bmatrix} e \\ \mu \end{Bmatrix}^- +$	<5 (b)	$(<0.04 \times 10^{10})$		
	100	0.64×10^{10}		1.06×10^{-14}

* The decay rates are based on the mean lives given in Table I.

† From Table III.

‡ $h\Gamma/2pc = |S|^2 + |P|^2$, the sum of the squares of the amplitudes, as defined in 5.3 and 5.6.

(a) Plano, Samios, Schwartz, and Steinberger, *Nuovo cimento* **5**, 1700 (1957).

(b) Alvarez, Bradner, Falk-Vairant, Gow, Rosenfeld, Solmitz, and Tripp, Interactions of K^- Mesons in Hydrogen, UCRL-3775, May 1957; see also *Nuovo cimento*, **5**, 1026 (1957).

This branching ratio $f_0{}^+ = 46 \pm 6\%$ comes from 58 Σ^+ decays in a hydrogen bubble chamber ($27 \to \pi^0$, $31 \to \pi^+$). Six times as many Σ^+ decays have been seen in emulsion, namely 291 Σ^+ decays from rest ($137 \to \pi^0$, $154 \to \pi^+$), giving $f_0{}^+ = (47 \pm 3)\%$, where the 3% is based on statistical errors only. However, the true uncertainty in $f_0{}^+$ is no smaller than that for the bubble chamber events, because in emulsion the detection efficiency for $\Sigma^+ \to n + \pi^+$ is probably 10% less than for $\Sigma^+ \to p + \pi^0$. A correction for this bias of $(10 \pm 10)\%$ against $\Sigma^+ \to p + \pi^0$ reduces $f_0{}^+$ from emulsion to $(45 \pm 5)\%$. The emulsion events were gathered from all laboratories by G. Snow (private communication) for presentation to the Seventh Rochester Conference.

(*a*) Leptonic Decays. So few leptonic decays have been reported (36) that it is hard to say that any has ever been identified with certainty. We have seen in 4.9 that an estimate of the rate of $\Lambda \to p + e^- + \bar{\nu}$ on the basis of the tetrahedron is roughly equal to the experimental upper limit (Table II) for this rate. A similar estimate for $\Sigma^- \to n + e^- + \bar{\nu}$ yields the same conclusion. The decay $\Sigma^+ \to n + e^+ + \nu$ has a comparable upper limit experimentally; of course it may be absent as in the tetrahedron scheme. However, in the case of Λ and Σ^-, it is difficult to conceive that the leptonic modes could be absent, since even without the tetrahedron they could proceed by the virtual emission of K^-, which is known to undergo leptonic decay. Of course such an indirect process might give lower rates.

We have mentioned in 4.8 that the decays of strange particles present an opportunity to test the forbiddenness of ν-$\bar{\nu}$ and $\mu^{\pm}$-$e^{\mp}$ lepton pair emission.

However, the K particle decays, in which at least *charged* lepton pairs occur frequently, are probably a better place to look for such an effect.

(*b*) Decays into *SIP*s. All energetically possible pionic decays of hyperons have been observed, except three: the decay of Ξ into π and N, which involves a change in strangeness of two units; the weak decay of Σ^0 that is unobservable because it cannot compete with the electromagnetic process $\Sigma^0 \rightarrow \Lambda + \gamma$; and the decay of the so-far unseen Ξ^0, presumably into π^0 and Λ.

The rates of the known pionic decays of Σ and Λ are given in Table II and are all of the same order of magnitude; if we try to make a small correction for the available phase space by dividing out the pion momentum as in the fourth column, the effective strengths come out even closer. It seems that the intrinsic strengths of the weak couplings involved are very similar. We do not know, however, exactly what the law of variation of the strength is, neither between Σ and Λ nor even with the charge multiplet Σ. The law of variation within a multiplet is not charge independence, of course, since it is precisely by violating charge independence that the weak couplings induce these decays. It has been suggested, as we mentioned in 3.8, that the violation of isotopic spin conservation in weak decays of *SIP*s into *SIP*s may be subject to limitations, for example $|\Delta I| = \frac{1}{2}$.

No suggestion has yet been made of a possible law of variation of the decay rate from multiplet to multiplet.

5.2. *Spins of* Λ *and* Σ.—We shall assume that both Λ and Σ have spin $\frac{1}{2}$. The experimental evidence to date by no means either confirms or contradicts this assumption.

An argument in favor of spin $\frac{1}{2}$ for the Λ, based on the decay of hyperfragments, has been given by Ruderman & Karplus (37); it is taken up in Appendix D.

For both Λ and Σ the remaining evidence concerns the absence of anisotropy in the decay of these particles. Appendix B contains a description of the anisotropies and asymmetries to be expected under various assumptions. It also summarizes the present experimental evidence.

5.3. *Decay of* Λ.—The pionic decay of Λ can take place through four channels. There are two charge states, which we may list as $\pi^- + p$ and $\pi^0 + n$ or as eigenstates of isotopic spin with $I = \frac{1}{2}$ and $I = \frac{3}{2}$. There are also two states of pion orbital angular momentum: the final pion-nucleon system with angular momentum $J = \frac{1}{2}$ can be in either a $^2S_{1/2}$ or a $^2P_{1/2}$ state. These have opposite parity, but presumably parity need not be conserved in the decay.

Let us define amplitudes for the decays into the four channels. For the $^2S_{1/2}$ state with $I = \frac{3}{2}$, call the amplitude S_3; for the $^2P_{1/2}$ state with $I = \frac{1}{2}$, call it P_1, etc. To make the amplitudes dimensionless, let us normalize them so that the decay rate into a given channel, say $^2S_{1/2}$ with $I = \frac{3}{2}$, is given by

$$\Gamma(^2S_{1/2}, I = 3/2) = \frac{2p_\Lambda c}{\hbar} \, | S_3 |^2. \qquad 27.$$

TABLE III

KINEMATICS OF HYPERON DECAYS

Decay mode	Momentum, p (Mev/c)	Total energy, $w=T+mc^2$ (Mev)	Available kinetic energy, Q (Mev)
p (mass $=938.21\pm0.01$ Mev) n (mass $=939.51\pm0.01$ Mev)			
Λ (mass $=1115.2\pm.14$ Mev)			
$\Lambda\rightarrow p+\pi^-$	99.9	$w_p=943.5$ $w_\pi=171.7$	37.2
$\Lambda\rightarrow n+\pi^0$	103.3	$w_n=945.2$ $w_\pi=170.0$	40.5
Σ^0 (mass $=1188.8_{-3}^{+2}$ Mev)			
$\Sigma^0\rightarrow\Lambda+\gamma$	71.2	$w_\Lambda=1117.3$	73.5
Σ^+ (mass $=1189.3\pm.25$ Mev)			
$\Sigma^+\rightarrow p+\pi^0$	189.0	$w_p=957.1$ $w_\pi=232.2$	116.1
$\Sigma^+\rightarrow n+\pi^+$	185.0	$w_n=957.5$ $w_\pi=231.8$	110.2
Σ^- (mass $=1196.4\pm.5$ Mev)			
$\Sigma^-\rightarrow n+\pi^-$	192.2	$w_n=958.9$ $w_\pi=237.5$	117.3
Ξ^- (mass $=1321\pm3.5$ Mev)			
$\Xi^-\rightarrow\Lambda+\pi^-$	139.4	$w_\Lambda=1123.6$ $w_\pi=197.3$	66.4

Here p_Λ is the momentum of the pion emitted in Λ decay, about 100 Mev/c. (See Table III.) (Comparing Eq. 27 with Eqs. 23 and 24, we see that the quantities $|S|^2$ and $|P|^2$ are related to the "effective coupling constants" $f'^2/4\pi$ and $f^2/4\pi$ introduced in 4.9.) Evidently the total rate of pionic decay of Λ is

$$\Gamma(\Lambda\rightarrow N+\pi)=\frac{2p_\Lambda c}{\hbar}\left\{|S_1|^2+|S_3|^2+|P_1|^2+|P_3|^2\right\} \qquad 28.$$

and, neglecting possible rare additional modes of decay, we may equate this to the reciprocal lifetime of Λ (Table II) and obtain

$$|S_1|^2+|S_3|^2+|P_1|^2+|P_3|^2\approx 1.2\times10^{-14}. \qquad 29.$$

We may expand the states π^-+p and π^0+n as linear combinations of the isotopic spin eigenstates. The coefficients are given in Table IV which displays a real, unitary matrix of Clebsch-Gordan coefficients. Say the

TABLE IV

CLEBSCH-GORDAN COEFFICIENTS*

z Component of isotopic spin		Mode	$I=\frac{1}{2}$ (S_1 or P_1)	$I=\frac{3}{2}$ (S_3 or P_3)
$-\frac{1}{2}$	Λ	π^-+p (S_- or P_-)	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
		π^0+n (S_0 or p_0)	$-\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$
$+\frac{1}{2}$	Σ^+	π^0+p (S_0^+ or P_0^+)	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$
		π^++n (S_+^+ or P_+^+)	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
$-\frac{3}{2}$	Σ^-	π^-+n (S_-^- or P_-^-)	0	1

* Taken from E. U. Condon and G. H. Shortley, "Theory of Atomic Spectra" (Cambridge University Press, Cambridge, England, 1953), Table 2^3, p. 76.

s-wave amplitudes for final states π^-+p and π^0+n are S_- and S_0 respectively; for the p-waves we have P_- and P_0. Then we have the formulae

$$S_- = \sqrt{\tfrac{2}{3}}S_1 + \sqrt{\tfrac{1}{3}}S_3, \qquad P_- = \sqrt{\tfrac{2}{3}}P_1 + \sqrt{\tfrac{1}{3}}P_3, \text{ etc.}$$

The unitarity of the Clebsch-Gordan matrices means that

$$|S_1|^2 + |S_3|^2 = |S_-|^2 + |S_0|^2 = \Gamma_S \frac{\hbar}{2p_\Lambda c}$$

and the same for the P's.

The fraction of all pionic decays leading to π^-+p is then

$$f_- = \frac{|S_-|^2 + |P_-|^2}{\Gamma_\Lambda \dfrac{\hbar}{2p_\Lambda c}} = \frac{|\sqrt{\frac{2}{3}}S_1 + \sqrt{\frac{1}{3}}S_3|^2 + |\sqrt{\frac{2}{3}}P_1 + \sqrt{\frac{1}{3}}P_3|^2}{|S_1|^2 + |S_3|^2 + |P_1|^2 + |P_3|^2} \qquad 30.$$

Experiments on associated production of the $\Lambda - K^0$ pairs show that the fraction of all Λ's decaying by this mode is 0.65 ± 0.05 (Plano 57); if the residue is virtually all π^0+n, then we may say that $f_- \sim \frac{2}{3}$.

5.4. *Up-down asymmetry in the decay of* Λ.—The decay rates of Eqs. 27 or 28 involve the S and P amplitudes incoherently. An experiment that involves them coherently is one that explicitly tests the nonconservation of parity. Suppose that we have produced Λ particles in a reaction that leaves them polarized in the x direction. For example, in a two-body collision the

x-axis would be normal to the plane of production. In the center of mass system of the collision, let the Λ come off at an angle θ to the z-axis, which is along the direction of the incident beam that produces the reaction. Say that at this angle the amount of polarization in the x-direction is $p(\theta) \neq 0$. Then nonconservation of parity in the decay of Λ permits the existence of an "up-down" asymmetry of the decay pions. Let us write the intensity $W_-(\theta, \xi)$ of negative decay pions (coming from Λ produced at angle θ) as a function of $\xi = \cos\psi$, where ψ is the angle (in the rest system of Λ) between the momentum of the decay pion and the x-axis. We have

$$W_-(\theta, \xi)d\Omega d\xi = f_- I(\theta) d\Omega \tfrac{1}{2}[1 + \alpha_- p(\theta)\xi]d\xi, \tag{31}$$

where $I(\theta)\ d\Omega$ is the intensity of Λ production in the element of solid angle $d\Omega$ and α_- is the parameter of up-down asymmetry, which can range from -1 to 1. For neutral decay pions we replace f_- by $1-f_-$ and α_- by the corresponding parameter α_0.

If we regard the production reaction as a polarizer of Λ's then α measures the efficiency of the decay asymmetry as an analyzer. It is easy to express each α in terms of the S and P amplitudes (38):

$$\alpha_- = \frac{2\ Re\ S_-^* P_-}{|S_-|^2 + |P_-|^2}; \qquad \alpha_0 = \frac{2\ Re\ S_0^* P_0}{|S_0|^2 + |P_0|^2} \tag{32}$$

So far the S and P amplitudes are complex numbers of unknown phase. If CP or T invariance holds, then the phases are determined (see 2.6) in terms of pion-nucleon phase shifts. We can take

$$S_1 = |S_1| e^{i\delta_1}, \quad S_3 = \pm |S_3| e^{i\delta_3}, \quad P_1 = \pm |P_1| e^{i\delta_{11}}, \quad P_3 = \pm |P_3| e^{i\delta_{31}}, \tag{33}$$

where the δ's are the π-N phase shifts at center-of-mass momentum p_Λ in the states $(S_{1/2}, I=\frac{1}{2})$, $(S_{1/2}, I=\frac{3}{2})$, $(P_{1/2}, I=\frac{1}{2})$, and $(P_{1/2}, I=\frac{3}{2})$ respectively. The $\pm$ signs in Eq. 33 are all independent. Experimental values of the phase shifts are given in Table V. Since all these phases are quite small, it is a fair approximation to take all of the S and P amplitudes real, assuming T invariance.

An up-down asymmetry has indeed been observed by the Berkeley hydrogen bubble chamber group (58). They have produced Λ by bombarding protons with pions according to the reactions $\pi^- + p \Rightarrow \Lambda + K^\circ$. Preliminary results indicate a large positive value of α_-. We can see from Eq. 32 that the search for up-down asymmetry represents a measurement of the extent to which one parity state (s- or p-wave) predominates over the other. The large asymmetry thus shows that neither s- or p-wave emission is dominant. However in Appendix D on hyperfragments we mention some evidence that $|P_-/S_-|^2 \lesssim 1/2$.

5.5. *Possible rule* $|\Delta I| = \frac{1}{2}$ *in* Λ *decay.*—We have mentioned in 3.8 that a selection rule on the change if isotopic spin has been suggested for the decay of *SIP*s into *SIP*s, namely $|\Delta I| = \frac{1}{2}$ (apart from electromagnetic corrections).

TABLE V

$\pi-N$ PHASE SHIFTS AT THE MOMENTA OF Λ AND Σ DECAY

Momentum and type of decay	Phase shift			
	δ_1	δ_3	δ_{11}	δ_{31}
99.5 Mev/c (Λ)	$+8°$	$-4°$	≈ 0	≈ 0
189.0 Mev/c (Σ)	$+8°$	$\underline{-14°}$	$-3°$	$\underline{-3°}$

The subscripts used for the phase shifts are explained below, Eq. 33.

All the phase shifts except the underscored ones are taken from H. L. Anderson, Sixth Rochester Conference, 1956. The underscored numbers come from the more recent compilation of Anderson and Davidon, *Nuovo cimento*, **5**, 1238 (1957).

For decay of Λ, which has $I=0$, the consequences of this rule are extremely simple. It requires that the decay leave pion and nucleon in the $I=\frac{1}{2}$ state, so that both amplitudes P_3 and S_3 must vanish; according to Table IV we then have $S_-=-\sqrt{2}S_0$ and $P_-=-\sqrt{2}P_0$. Eq. 30 then tells us that the fraction f_- of charged decays must be $\frac{2}{3}$, which is in good agreement with experiment (see Table II).

The rule has the further consequence (as we see from Eq. 32) that the asymmetry parameters α_- and α_0 are equal. Of course α_0 is intrinsically difficult to deal with because only neutral particles are involved, and there is no experimental information on this point at present. Nevertheless, it is perhaps interesting to remark that the ratio α_-/α_0 can be determined without a knowledge of the Λ polarization $p(\theta)$.

5.6. *The decay of* $\Sigma^{\pm}$.—Just as we described the pionic decay of Λ in terms of four parameters, we can use six to describe the decay of Σ^+ and Σ^-.

In Σ^+ decay there are again four channels corresponding to s- and p-waves and to $I=\frac{1}{2}$ and $I=\frac{3}{2}$. We may call the corresponding amplitudes S_1^+, S_3^+, P_1^+, and P_3^+ in an obvious notation. If we use the alternative description in terms of the states π^0+p and π^++n (see Table IV) we must use (say in the s-state) the amplitudes S_0^+ and S_+^+ respectively, given by the relations

$$S_0^+ = \sqrt{\tfrac{1}{3}}S_1^+ + \sqrt{\tfrac{2}{3}}S_3^+, \qquad S_+^+ = -\sqrt{\tfrac{2}{3}}S_1^+ + \sqrt{\tfrac{1}{3}}S_3^+. \qquad 34.$$

In Σ^- decay the only possible final state is π^-+n with $I=\frac{3}{2}$. Thus we have only two amplitudes $S_3^-=S_-^-$ and $P_3^-=P_-^-$.

The normalization of the S and P amplitudes will be as in Eq. 27 for the Λ case, except that here in place of p_Λ we use $p_\Sigma \approx 200$ Mev/c, the momentum of a pion in Σ decay. The quantities $|S_0^+|^2+|P_0^+|^2=\hbar\Gamma_0^+/2p_\Sigma c$, $|S_+^+|^2+|P_+^+|^2=\hbar\Gamma_+^+/2p_\Sigma c$, and $|S_-^-|^2+|P_-^-|^2=\hbar\Gamma_-^-/2p_\Sigma c$ are displayed in Table II; they are all approximately equal.

For each of the three decay modes we can define an asymmetry parameter α as we did in Eq. 32 for the Λ. Let us call these $\alpha_-{}^-$, $\alpha_0{}^+$, $\alpha_+{}^+$, where as usual the upper index is the charge of the Σ and the lower one is the charge of the emitted pion. In each case we have as before

$$\alpha = \frac{2\,Re\,(S^*P)}{|S|^2+|P|^2}, \tag{35}$$

where S and P carry the same indices as α.

Associated production experiments with bubble chambers have shown an up-down asymmetry for Λ, (as mentioned above) but not for Σ. However most of the experiments have used negative pion beams, and we must remember that π^-+p can yield Σ^-+K^+, but not Λ^++K^-. We conclude that the asymmetry is small for $\Sigma^-\to\pi^-+n$, and we must look elsewhere for information on the two decay modes of Σ^+. Now in nuclear emulsion many K^- have been brought to rest and captured from atomic orbits according to the reaction $K^-+p\Rightarrow\Sigma^\pm+\pi^\mp$. The Σ and π tracks are in general not collinear for at least two reasons: first, the proton p is in motion in the nucleus; second, the Σ may be scattered before it leaves the nucleus. Thus one can define a "production plane" and its normal, $\hat{n}=\hat{p}_\Sigma\times\hat{p}_\pi$. Now consider the decay $\Sigma\to N+\pi'$, and classify it "up" if $\hat{p}_{\pi'}\cdot\hat{n}>0$. The present data are: $\Sigma^+\to p+\pi^0$, 48 up and 69 down; $\Sigma^\pm\to n+\pi^\pm$, 67 up and 88 down. In the first process there seems to be an effect.[15]

If T invariance holds, the quantities $S_1{}^+$, $P_1{}^+$, etc. have determined phases as before (compare Eq. 33). The relevant phase-shifts at the energy of Σ decay are given in Table V along with the corresponding ones for the Λ decay energy. Even at the higher energy the phase shifts are small enough so that in a rough approximation all S and P amplitudes are real.

5.7. *The $|\Delta I|=\frac{1}{2}$ rule in $\Sigma^\pm$ decay.*—We shall now examine the restrictions imposed on the experimental quantities by the proposed selection rule $|\Delta I|=\frac{1}{2}$. We employ the "spurion" notation of Wentzel. (See 3.8.)

In $\Sigma\to N+\pi$, the value of I_z decreases by $\frac{1}{2}$; we can say that a spurion with $I=\frac{1}{2}$, $I_z=-\frac{1}{2}$ is absorbed by Σ^+ or Σ^-, which has $I=1$ and $I_z=+1$ or -1 respectively, to give $\pi+N$, which can have $I=\frac{3}{2}$ or $I=\frac{1}{2}$.

We then have the following Clebsch-Gordan coefficients:

	$I=1/2$ (S_1 and P_1)	$I=3/2$ (S_3 and P_3)
Σ^+ + Spurion	$-\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
Σ^- + Spurion	0	1

In our treatment of hyperon decays so far we have defined, for each isotopic spin state, a pair of amplitudes S and P corresponding to the two

[15] This possible source of polarized Σ's was pointed out to us by R. Gatto and R. D. Tripp and by M. Ceccarelli in private communications. The data have been supplied by many emulsion groups.

values of orbital angular momentum. Here we shall find it more convenient to define for the $I=\frac{1}{2}$ state, say, a vector N_1 with components S_1 and P_1 along orthogonal "s" and "p" axes (see Fig. 5). In this notation the rule $|\Delta I|=\frac{1}{2}$ gives us the restriction

$$N_3^- = \sqrt{3}N_3^+, \qquad 35a.$$

as we can see from the Clebsch-Gordan coefficients above. There is no restriction on N_1^+. We define analogous vectors N_0, N_+, and N_-, where as usual the subscript refers to the charge of the emitted pion and where we have dropped the redundant superscript giving the charge of Σ. Using Table IV we can then write:

$$N_0 = \sqrt{\tfrac{1}{3}}\,N_1^+ + \sqrt{\tfrac{2}{3}}\,N_3^-; \qquad |N_0|^2 = \frac{\hbar}{2p_\Sigma c}\Gamma_0$$

$$N_+ = -\sqrt{\tfrac{2}{3}}\,N_1^+ + \sqrt{\tfrac{1}{3}}\,N_3^+ \qquad |N_+|^2 = \frac{\hbar}{2p_\Sigma c}\Gamma_+$$

$$N_- = \quad 0 \quad + \quad \sqrt{3}\,N_3^+ \qquad |N_-|^2 = \frac{\hbar}{2p_\Sigma c}\Gamma_- \qquad 36.$$

and the identity

$$\sqrt{2}N_0 + N_+ = N_-. \qquad 37.$$

In other words, if the rule $|\Delta I|=\frac{1}{2}$ is valid the three vectors $\sqrt{2}N_0$, N_+ and N_- must form a triangle as shown in Figure 5. We assume at this point CP or T invariance and neglect the $\pi-N$ phase shifts, so that our "vectors" are real. Table II shows that the experimental rates are all equal (within about 10 per cent) so let us take $|N_-|=|N_0|=|N_+|$. We see that our triangle must have sides in the ratio $\sqrt{2}:1:1$. Such a (45°, 45°, 90°) triangle of course can be constructed, and so the $|\Delta I|=\frac{1}{2}$ rule is consistent with the rates of the three $\Sigma^\pm$ decays.

In Fig. 5 we have drawn N_- at an unknown angle ν_- with the s-axis indicating that we have so far put no restrictions on $P_-/S_-=\tan\nu_-$.

We can now write the asymmetry coefficients α_-, α_0 and α_+ (Eq. 35) in terms of the three known sides of the triangle and of the unknown P_-/S_- ratio. We note that the expression for α_- can be rewritten in terms of the angle ν_-:

$$\alpha_- = \sin 2\nu_-. \qquad 38.$$

We can see by inspection that the angles of N_0 and N_+ in $s-p$ space are $\nu_0=\nu_-\pm 45°$ and $\nu_+=\nu_-\pm 90°$. (The signs arise because the triangle could equally well have been drawn reflected about N_-). Asymmetry expressions like Eq. 38 obviously apply for all three modes of decay, so we have

$$\alpha_0 = \sin 2(\nu_- \pm 45°) = \pm\cos 2\nu_-, \qquad 39.$$

$$\alpha_+ = \sin 2(\nu_- \pm 90°) = -\sin 2\nu_-. \qquad 39a.$$

Eqs. 38, 39, and 39a give us all three asymmetries in terms of the angle ν_-. For example, if ν_- is near 0° or near 90° (pure s- or p-wave in the Σ^- decay),

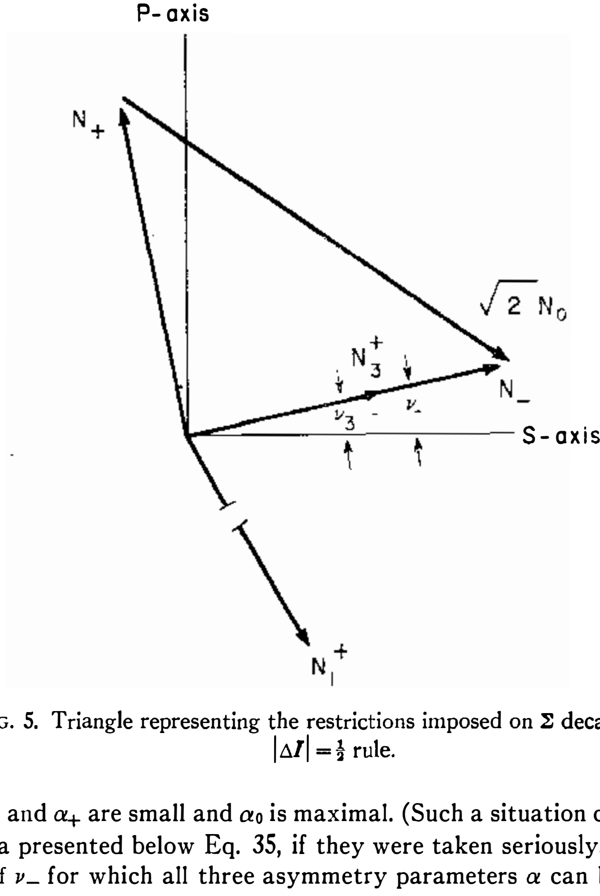

FIG. 5. Triangle representing the restrictions imposed on Σ decay by the $|\Delta I| = \frac{1}{2}$ rule.

then α_- and α_+ are small and α_0 is maximal. (Such a situation could describe the data presented below Eq. 35, if they were taken seriously.) There is no value of ν_- for which all three asymmetry parameters α can be small.

5.8. *The decay of* Ξ.—Very little is known about the Ξ particle. If pionic decay really predominates here, too, then there is only one principal mode of decay for each charge state: the known process $\Xi^- \rightarrow \pi^- + \Lambda$ and the hypothetical process $\Xi^0 \rightarrow \pi^0 + \Lambda$.

If the rule $|\Delta I| = \frac{1}{2}$ applies, then the rate of pionic decay of Ξ^- is twice that of Ξ^0.

As mentioned earlier, it is important to know whether the decay into $\pi + N$ with $|\Delta S| = 2$ is really forbidden.

6. *K* Meson Decay

6.1. *The spin of the K meson.*—There is considerable evidence in favor of spin zero for the K meson. First of all there are two indications that the spin is even: (*a*) The spectrum of the decay $K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$ seems to be inconsistent with odd spin, although quite consistent with spin zero or two (see 6.5). (*b*) A neutral mode of decay of K_1^0 has been discovered that is probably $K_1^0 \rightarrow \pi^0 + \pi^0$ (39). Since the neutral pions are identical bosons, this would rule out odd spin. Then there are three arguments for spin 0 rather than 2, 4, etc.: (*a*) If the spin were 4 or greater, it would seem that the decay of K_1^0 should be delayed by centrifugal barrier effects and should not proceed with a typical hyperonic lifetime of about 10^{-10} sec. Even spin 2 seems unlikely for this reason. (*b*) For any spin greater than zero, Dalitz (40) has estimated that the decay $K^+ \rightarrow \pi^+ + \gamma$ should compete rather favorably with other modes. Experiment, however, indicates that not more than a few per cent of K^+, if any, decay in this way. For a spinless K meson, of course, the decay into $\pi^+ + \gamma$ is strictly forbidden, since it is then a $0 \Longrightarrow 0$ transition. (*c*) Finally, there is the argument based on the absence of anisotropy in the decay of K^+ meson beams.[16]

6.2. *Modes of decay of K mesons.*—In Table VI we list the decay modes and branching ratios for $K^\pm$, K_1^0, and K_2^0, as far as they are known at present. Table VII gives the momenta and energies of the decay products.

The most rapid decay is $K_1^0 \rightarrow 2\pi$ with a rate of around 10^{10}/sec. No competing process has been detected with certainty in the case of K_1^0. In the case of K_2^0, the decay into two pions seems to be rare or lacking. (CP invariance would forbid it altogether, as we saw in 3.9.) The lifetime is correspondingly much longer and the much slower processes $K \rightarrow 3\pi$ and $K \rightarrow$leptons$+\pi$ are observed.

In the decay of $K^\pm$, the 2π mode is somehow inhibited; the rate of 2π decay is smaller than in the case of K_1^0 by a factor of around 500. As in K_2^0 decay, the modes 3π and leptons$+\pi$ have a chance to compete. In the case of the charged K, however, an additional leptonic mode is available, $K^\pm \rightarrow \mu^\pm \pm \nu$ (with no π^0). This is, in fact, the most frequent mode of disintegration of the charged K. As we mentioned in 4.9, the analogous decays $K^\pm \rightarrow e^\pm \pm \nu$ are rare or absent. Experiments on K^+ stopping in emulsion with the emission of e^+ indicate that the ratio $(e^+ + \nu)/(\mu^+ + \nu)$ is probably less than 2 per cent and could be zero.

In 4.8 we mentioned that no decays by neutrino-pair emission are known (except for the ambiguous case of μ^+-decay) but that they might exist, so far as we know. Presumably the best test of neutrino-pair emission would be the hypothetical decay $K^+ \rightarrow \pi^+ + \nu + \bar{\nu}$. Experiments to date rule out a

[16] Barring accidents, we should expect some polarization in a K^+ beam if the spin is greater than zero. If there is polarization then there must be anisotropy in the decay $K^+ \rightarrow \pi^+ + \pi^0$ (and if the K has a spin >2 also for the mode $K \rightarrow \mu + \nu$). In general the decay $K \rightarrow \mu + \nu$ should not only be anisotropic, but should also exhibit an up-down *asymmetry* with respect to the plane of production of K. About one thousand decays of stopped "K_L" mesons have been analyzed to detect asymmetry or anisotropy of the emitted charged particle (K^+ decays which exhibit a single fast track are called "K_L"; nearly $\frac{2}{3}$ of the secondary tracks are from $K \rightarrow \mu + \nu$, $\frac{1}{3}$ from $K \rightarrow \pi + \pi$). Since $K \rightarrow \pi + \pi$ can exhibit anisotropy but not asymmetry it cannot wash out an asymmetry of the $K \rightarrow \mu + \nu$ mode. The 1000 K_L decays show evidence for neither asymmetry nor anisotropy. 600 decays of $K \rightarrow 3\pi$ are also isotropic (41, 42).

TABLE VI

BRANCHING RATIOS AND DECAY RATES IN K DECAY

K^+ decay mode	Branching ratio (%)	Decay rate* (number/sec.)	K^0 decay mode	Branching ratio (%)	Decay rate* (number/sec.)
ified					
$)\rightarrow\pi^++\pi^0$	25.6 ±1.7	20.9 $\pm 10^6$	$\{K_1^0\rightarrow\pi^++\pi^-$	86±6	0.9 $\times 10^{10}$
			$K_1^0\rightarrow\pi^0+\pi^0$ (a)	14±6	0.15 $\times 10^{10}$
				100	1.05 $\times 10^{10}$
$\rightarrow\pi^++\pi^++\pi^-$	5.66±0.30	4.62 $\times 10^6$	$\{K_2^0\rightarrow\pi^++\pi^-+\pi^0$	~30	~3 $\times 10^6$ (c)
$)\rightarrow\pi^++\pi^0+\pi^0$	1.70±0.32	1.38 $\times 10^6$			
$\mu_2)\rightarrow\mu^++\nu$	58.8 ±2.0	48.0 $\times 10^6$	no analogue		
$\beta)\rightarrow e^++\nu+\pi^0$ (b)	4.19±0.42	3.42 $\times 10^6$	$K_2^0\rightarrow e^{\pm}\pm\nu+\pi^{\mp}$	~30	~3 $\times 10^6$ (c)
$\mu_3)\rightarrow\mu^++\nu+\pi^0$	4.0 ±0.77	3.26 $\times 10^6$	$K_2^0\rightarrow\mu^{\pm}\pm\nu+\pi^{\mp}$	~30	~3 $\times 10^6$ (c)
				100	~10 $\times 10^6$ (c)
	100	81.6 $\times 10^6$			
ng (perhaps rigorously orbidden)		*Forbidden because:*			*Forbidden because:*
$\rightarrow\pi^++\gamma$	<2.0	Spinless K	$K_2^0\rightarrow\pi^++\pi^-$	<5	CP invariance
			$K_{1,2}^0\rightarrow\pi^0+\gamma$	<10	spinless K
$\rightarrow\nu+\nu+\pi^+$	<2.0	Neutrino pair emission	$K_{1,2}^0\rightarrow\nu+\nu$	<10	spinless K
$\rightarrow\mu^++e^-+\pi^+$ etc.	<0.01	$\mu-e$ pair emission	$K_{1,2}^0\rightarrow e^{\pm}+\mu^{\mp}$	<5	$\mu-e$ pair emission
ng (rare but perhaps llowed)					
$\rightarrow e^++\nu$	<1.0		$K_{1,2}^0\rightarrow\pi^{\pm}+\pi^0+\mu^{\mp}\nu$, etc.		
$\rightarrow\pi^++\pi^-+\mu^++\nu$, etc.	<0.01		$K_1^0\rightarrow e^{\pm}\pm\nu+\pi^{\mp}$, etc.		

ıalogous identified modes of decay of K^+ and K^0 are shown on the same horizontal line. The symbols (θ), (τ), etc., ler names used to describe the 2π, 3π, etc. decay modes of the K^+.

ie branching ratios of the K^+ are a weighted average of the data of the Berkeley and Dublin emulsion groups lexander, R. H. W. Johnston, C. O'Ceallaigh, private communication (*Nuovo cimento* 6, 478 (1957)); irge, Perkins, Peterson, Stork, and Whitehead, *Nuovo cimento*, 4, 834 (1956)]. For the τ and τ' we have also in- the following branching fractions, τ, (5.1 ± 0.3)%; τ', (1.5 ± 0.2)% supplied by Harris, Orear, and Taylor (private unication).

e branching ratio for the K_1^0 have been determined by the Steinberger propane chamber group studying associ- roduction of strange particles by π^- [Plano, Samios, Schwartz, Steinberger, *Phys. Rev.* (to be published, 1957)]. find that <2% of K_1^0 undergo 3-body decay with charged particles.

e qualitative data on the K_2^0 have been given by the Columbia cloud chamber group [Lande, Lederman, and wsky, *Phys. Rev.*, 105, 1925 (1957)].

'he decay rates are based on the mean lives in Table I.

This mode has not been definitely identified as $K\rightarrow 2\pi^0$; it is known only that some gammas are associated with the decay. Nevertheless the assignment $K\rightarrow 2\pi^0$ is suggested by the energy spectrum of these gammas and is otherwise very plausible (see Sect. 6.5).

The π^0 has not yet been identified, but is suggested by the presence of the analogous charged π's seen in K^0 decay. Dalitz pairs have not yet been seen in association with $K^+\rightarrow e^++\nu+\pi^0$; they have been identified in association with $K^+\rightarrow\mu^++\nu+\pi^0$ (and with $K^+\rightarrow\pi^++\pi^0+\pi^0$).

There is a large uncertainty in this rate (see Table I).

TABLE VII

KINEMATICS OF MESON DECAYS

Decay mode	Momentum, p (Mev/c)*	Total energy $w=T+mc$ (Mev)*	Available kinetic energy, Q (Mev)
$e^{\pm}$ (mass = 0.510976 Mev)	stable	stable	stable
$\mu^{\pm}$ (mass = 105.70 ± .06 Mev) $\mu^{\pm} \to e^{\pm}+\nu+\bar{\nu}$	$p^m = 52.85$	$w_e{}^m = 52.85$	105.19
$\pi^{\pm}$ (mass = 139.63 ± .06 Mev) $\pi^{\pm} \to \mu^{\pm} \pm \nu$	$p = 29.81$	$w_\mu = 109.82$	33.93
π^0 (mass = 135.04 ± .16 Mev) $\pi^0 \to \gamma+\gamma$	$p = 67.52$	$w_\gamma = 67.52$	135.04
K^+ (mass = 494.0 ± 0.14 Mev) $K_{\pi 2} \to \pi^+ + \pi^0$	$p = 205.3$	$w_{\pi^+} = 248.25$	219.33
$K_\tau \to \pi^+ + \pi^+ + \pi^-$	$p_\pi{}^m = 125.5$	$w_\pi{}^m = 187.8$	75.11
$K_{\tau'} \to \pi^+ + \pi^0 + \pi^0$	$p_{\pi^+}{}^m = 133.1$ $p_{\pi^0}{}^m = 132.3$	$w_{\pi^+}{}^m = 192.9$ $w_{\pi^0}{}^m = 189.0$	84.29
$K_{\mu 2} \to \mu^+ + \nu$	$p = 235.7$	$w_\mu = 258.3$	388.3
$K_{\mu 3} \to \mu^+ + \pi^0 + \nu$	$p_\mu{}^m = 215.2$ $p_\pi{}^m = 215.3$	$w_{\mu^+}{}^m = 239.8$ $w_{\pi^0}{}^m = 254.1$	253.26
$K_{e3} \to e^+ + \pi^+ + \nu$	$p_e{}^m = 228.5$ $p_\pi{}^m = 228.5$	$w_{e^+}{}^m = 228.5$ $w_{\pi^0}{}^m = 265.4$	358.45
$K_{e2} \to e^+ + \nu$ not observed	$p = 247.0$	$w_{e^+} = 247.0$	493.49
$K_{\pi\gamma} \to \pi^+ + \gamma$ not observed	$p = 227.2$	$w_{\pi^+} = 266.7$	354.37
K^0 (mass = 493 ± 5 Mev) $K^0 \to \pi^+ + \pi^-$	$p = 203.1$	$w = 246.5$	213.7
$\to \pi^0 + \pi^0$	$p = 206.2$	$w = 246.5$	222.9
$\to \pi^+ + \pi^- + \pi^0$	$p_{\pi^\pm}{}^m = 128.4$	$w_{\pi^\pm}{}^m = 189.7$	78.7
$\to \mu^{\pm} + \pi^{\mp} \pm \nu$	$p_\mu{}^m = 213.2$ $p_\pi{}^m = 213.2$	$w_\mu{}^m = 238.0$ $w_\pi{}^m = 254.9$	247.7
$\to e^{\pm} + \pi^{\mp} \pm \nu$	$p_\pi{}^m = 226.6$ $p_e{}^m = 226.6$	$w_\pi{}^m = 266.2$ $w_e{}^m = 226.7$	352.9

* The momentum and total energy are given in the rest frame of the decaying particle. In three-body decays, the maximum momentum and energy possible for each of the products is given, as indicated by the superscript *m*.

branching ratio of more than a few per cent for this process. If it can be shown with much greater accuracy that this process is absent, our speculation that neutrino pair emission never occurs would be supported.

6.3. *The modes $K \rightarrow 2\pi$.*—We have seen that K_1^0 decays into two pions with a rate comparable to those of hyperon decays. For K_2^0 this mode is absent (or nearly so), but that can be explained either by CP invariance or possibly otherwise. (See Appendix A.) We then must deal with the fact that K^+ decays into two pions about 500 times slower than K_1^0. Let us apply the proposed isotopic spin selection rule $|\Delta I| = \frac{1}{2}$.

The rule requires that the final state differ in isotopic spin from the original one by $\frac{1}{2}$. Since the initial state has $I = \frac{1}{2}$, the final one must have $I = 0$ or 1. After the decay of a spinless K particle, the two pions are in an s-state. Since they obey Bose-Einstein statistics, the complete wave function must be symmetric; thus the isotopic spin function must be symmetric. When we combine two unit isotopic spins in a symmetrical way we get $I = 0$ or $I = 2$. Here only $I = 0$ is permitted. Thus, to the extent that charge independence is rigorous apart from the weak coupling and to the extent that the weak coupling obeys the rule $|\Delta I| = \frac{1}{2}$, we have:

(*a*) K^+ is forbidden to decay into two pions (clearly $\pi^+ + \pi^0$ cannot have $I = 0$, since $I_Z = +1$).

(*b*) In the decay $K_1^0 \rightarrow 2\pi$ we have the branching ratio characteristic of $I = 0$; the fraction f of decays into neutral pions is $\frac{1}{3}$.

Now experimentally the first result is nearly true; the decay $K^+ \rightarrow \pi^+ + \pi^0$ is practically forbidden, and the $|\Delta I| = \frac{1}{2}$ rule has given an explanation of the fact. However, the decay does occur, albeit slowly, and so we must look for corrections to the rule, either from electromagnetic violations of charge independence or from a slight failure of the rule in the weak interaction itself.

The second result is that

$$\equiv \frac{K_1^0 \rightarrow 2\pi^0}{K_1^0 \rightarrow 2\pi}$$

is $\frac{1}{3}$; this is in disagreement with the present experimental value (39) of 0.14 ± 0.06. This discrepancy is discussed in 6.4.

6.4. *Validity of the rule* $|\Delta I| = \frac{1}{2}$.—Let us summarize the evidence on the rule $|\Delta I| = \frac{1}{2}$. We have seen in Section 5 that the rule is in agreement with hyperon results so far and scores a success in predicting the branching ratio of Λ decay. In 6.5 we shall see that it is not far from agreement with results on the decay $K \rightarrow 3\pi$. We have just noted that, apart from requiring a small correction, it explains the slowness of $K^+ \rightarrow \pi^+ + \pi^0$. But the branching ratio in K_1^0 decay appears to require a larger correction.

It is certainly worthwhile, therefore, to discuss the possibility that the $|\Delta I| = \frac{1}{2}$ rule is approximately correct. In order to get a nonzero rate of decay for $K^+ \rightarrow \pi^+ + \pi^0$ we must put in some contribution from $|\Delta I| = \frac{3}{2}$ and/or $|\Delta I| = \frac{5}{2}$. Say we introduce these with complex amplitudes ϵ_3 and ϵ_5 respectively relative to that for $|\Delta I| = \frac{1}{2}$. Then the ratio Γ^+/Γ^0 of the decay rates $K^+ \to 2\pi$ and $K_1^0 \to 2\pi$ and the fraction f of K_1^0 that give $2\pi^0$ are given by the relations:

$$\frac{\Gamma^+}{\Gamma^0} = \frac{1}{3}\,\frac{|3/2\epsilon_3 - \epsilon_5|^2}{1 + |\epsilon_3 + \epsilon_5|^2} \text{ instead of } 0,$$

$$f = \frac{1}{3}\,\frac{|1 - \sqrt{2}(\epsilon_3 + \epsilon_5)|^2}{1 + |\epsilon_3 + \epsilon_5|^2} \text{ instead of } 1/3.$$

If CP invariance holds, then ϵ_3 and ϵ_5 *have the same phase* $\delta_2 - \delta_0$, where δ_I is the *s*-wave pion-pion phase shift in the state with isotopic spin I at the energy of the final two-pion system. This phase shift is, of course, unknown at present.

Let us put $\Gamma^+/\Gamma^0 \approx 1/500$ as indicated by the experiments. Then if we use only ϵ_3 (admixture of $|\Delta I| = \frac{3}{2}$ only (42a) the fraction f must lie between 0.28 and 0.38, which is still in disagreement with 0.14 ± 0.06. If we use both ϵ_3 and ϵ_5 then we can fit $f = 0.14$ with $\epsilon_3 \approx 12$ per cent and $\epsilon_5 \approx 11$ per cent or with $\epsilon_3 \approx 6$ per cent and $\epsilon_5 \approx 17$ per cent. Similarly we can fit $f = 0.20$ with $\epsilon_3 \approx 9$ per cent and $\epsilon_5 \approx 6$ per cent or with $\epsilon_3 \approx 3$ per cent and $\epsilon_5 \approx 12$ per cent. Making the parameters ϵ complex will, in general, require them to be still larger in magnitude.

Can such large corrections come from electromagnetic violations of charge independence? It seems unlikely, since electromagnetic effects are of the order of the fine structure constant and moreover attempts to estimate them theoretically for this problem have not given anomalously large answers.

We are led, then, to say that the $|\Delta I| = \frac{1}{2}$ rule may be approximately valid, but if so it seems to be approximate for the weak interactions themselves, with corrections of the order of 5 or 10 per cent; i.e., it is hard to blame the electromagnetic field for these corrections.

6.5. *The decay* $K^+ \to \pi^+ + \pi^+ + \pi^-$.—The 3π mode of decay of K has been most extensively studied in the case $K^+ \to \pi^+ + \pi^+ + \pi^-$ which accounts for only some 5 per cent of all K^+ decays, but is spectacular in photographic emulsion, cloud chambers, or bubble chambers.

The distribution in energy and angle of the emitted pions has been carefully investigated and is notable for having provided, together with the decay $K^+ \to \pi^+ + \pi^0$, the first evidence that parity is not conserved by the weak interactions. The 3π mode shows a distribution (see below) characteristic of even angular momentum and odd parity for the final state (0^- for a spinless K). In the 2π mode, the final state cannot have odd parity if it has even angular momentum; for a spinless K the state of the two pions is, of course, 0^+.

There was the suggestion (43) that two particles might be involved, one with even parity called the θ and one with odd parity called the τ and that these were degenerate because of a new kind of symmetry. The discovery of nonconservation of parity has made such a complication unnecessary.

The analysis of the "τ" events $K^+ \to 3\pi$ has been carried out by the method of Dalitz (44). The Q-value is 75.11 Mev, shared among the three pions, which we may roughly assume to be nonrelativistic in the rest system of K^+.

Let the kinetic energies of the two π^+ and the π^- be called T_1, T_2, T_3 respectively, with $T_1+T_2+T_3=Q$. A decay event is then described by two independent variables, for which Dalitz chose $x\equiv\sqrt{3}(T_1-T_2)/Q$ and $y\equiv(3T_3-Q)/Q$. In Appendix C we show why these variables were chosen, namely that x^2+y^2 runs between 0 and 1 and the element of volume in phase space is just proportional to $dxdy$. Thus if we plot individual events inside the unit circle in the $x-y$ plane the density of events as a function of x and y is proportional to $|M|^2$, the square of the decay matrix element. Since the two π^+ are identical, M is an even function of x and thus the circle can be folded across the y-axis to form a semicircle. The Dalitz plot of "τ" events to date appears in Figure 6. Each event is a dot inside the semicircle. Relativistic effects slightly distort the semicircle into the dashed curve (45).

Figure 6 shows that the population of events is fairly uniform. Proceeding naively, we may simply conclude that the matrix element M is practically a constant, independent of the pion momenta, so that the three pions are essentially in an overall s-state with a totally symmetric wave function. The spin of K must then be zero. In fact we accept these conclusions, although they are not rigorous consequences of the data in Figure 6. That the K spin is even, however, we may now show with considerable confidence just on the basis of the Dalitz plot.

The spin $\boldsymbol{S}$ of the K^+ meson is equal to the sum of the two independent angular momenta $\boldsymbol{l}$ and $\boldsymbol{L}$, defined as follows: the two π^+ revolve about their mutual center of mass with orbital angular momentum $\boldsymbol{l}$, while the $2\pi^+$ system and the π^- revolve around the three-body center of mass with orbital angular momentum $\boldsymbol{L}$. We have $\boldsymbol{S}=\boldsymbol{l}+\boldsymbol{L}$, where l, of course, is even. Simultaneous eigenstates of l^2 and L^2 may be described by the symbol (L, l). We want to show that amplitudes of the type $(0, l)$ are present, so that $S=l+0$ is even.

We must assume that the K particle has some reasonable "radius" R and consider π^- of such low energy $T_3=\hbar^2k_3^2/2m_\pi$ that $k_3R\lesssim 1$. For $T_3<10$ Mev, we need only suppose that $R\lesssim 5\hbar/m_\pi c$. Then for $L>0$ the existence of a centrifugal barrier would make $|M|^2$ approach zero as $(k_3R)^{2L}$ for small k_3R, which corresponds to the bottom of the semicircle in Figure 6. But the uniform population of dots in this region shows (46) that an appreciable part of the matrix element does not tend to zero for small k_3R and must correspond to $L=0$. We cannot, however, exclude the presence of some $L>0$. In any case S is even. (Furthermore, the parity is -1 for any $L=0$ configuration of the three pseudoscalar pions.)

Inspection of the upper corner of the semicircle, where the relative momentum of the two π^+ is very small, shows in a similar way that there is a large amplitude with $l=0$.

These arguments do not exclude the possibility of spin 2, with a wave function for the pions composed largely of (0, 2) and (2, 0). However, we have seen in 6.1 that there are strong arguments for a K spin of zero and we shall continue the discussion on that basis.

With $S=0$, the pion wave function is composed of (0, 0) (2, 2), etc.

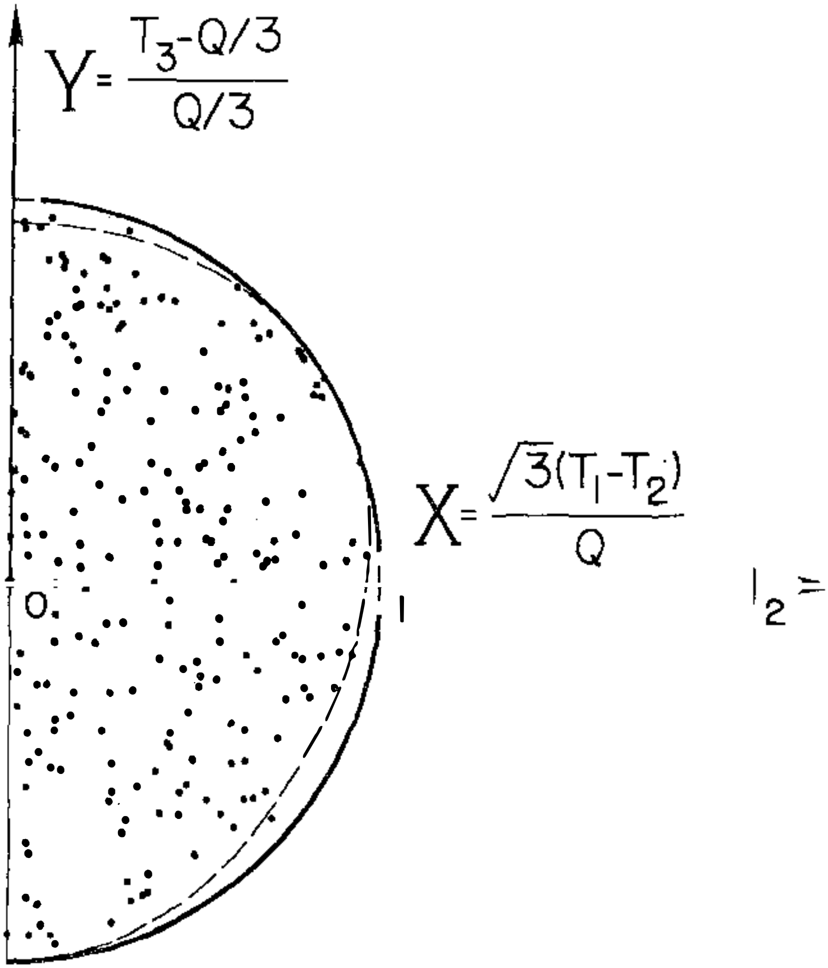

FIG. 6. Dalitz plot of the distribution of pion energies in the "τ" decay $K^+ \to \pi_1^+ + \pi_2^+ + \pi_3^-$. If the problem is treated nonrelativistically the semicircle represents the boundary of the region allowed by conservation of momentum. Relativistic treatment gives the dashed curve. The 219 events (from Berkeley, Columbia, and MIT) were compiled by Orear, Harris, and Taylor, *Phys. Rev.*, **102,** 1676 (1956).

(Note the parity of each of these configurations is -1.) For any reasonable radius R, we may expect centrifugal barrier effects to suppress (2, 2) and higher amplitudes relative to (0, 0). Moreover, the matrix element corresponding to (0, 0) should be a constant plus terms of order $(kR)^2$, where k is the wave number of any of the pions. All of this is perfectly consistent with the appearance of Figure 6. In fact, we can try expanding the matrix element M in a power series in x and y; note odd terms in x must be absent because of the identity of the two π^+. A good fit to the data, obtained from a plot given by Dalitz (R3), is obtained with

$$M \propto 1 + \frac{1}{10} y. \quad 40.$$

The term 1 is of course totally symmetric in the three pions, while the term $1/10y$ is not. We can thus obtain a rough estimate, useful in 6.6, of the fraction of τ decays into nonsymmetric states of the three pions. We square the amplitude M and average the resulting intensity over the semicircle. Since the average of y^2 is $\frac{1}{4}$, the nonsymmetric term $1/10y$ contributes a fraction $\frac{1}{4}\,(1/10)^2 = 1/400$ of the total intensity.[17]

6.6. *Charge dependence of the decay $K \to 3\pi$; the neutral 3π mode.*—There is another charge state of three pions into which K^+ can decay: $\pi^+ + 2\pi^0$. Let us try to predict the ratio of these two modes by means of the $|\Delta I| = \frac{1}{2}$ rule. The isotopic spin of K is $\frac{1}{2}$, and so in the final state we would have $I=1$ or $I=0$. However, in the decay of K^+ the final state has $I_Z = +1$ and thus only $I=1$ is possible. Now there are two ways of combining three unit isotopic spins to make $I=1$, just as there are two ways of constructing a vector trilinear in three vectors $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$. One of these (corresponding to $\boldsymbol{a}\cdot\boldsymbol{b}\boldsymbol{c} + \boldsymbol{a}\cdot\boldsymbol{c}\boldsymbol{b} + \boldsymbol{b}\cdot\boldsymbol{c}\boldsymbol{a}$) gives an isotopic spin function totally symmetric in the three pions. Since the pions are bosons, this must be associated with a matrix element totally symmetric in the pion momenta, like the constant term in Eq. 40. The other kind of isotopic spin function is not totally symmetric (and corresponds to a vector like $\boldsymbol{a} \times [\boldsymbol{b} \times \boldsymbol{c}]$). It is associated with a nonsymmetric matrix element like the term $1/10y$ in Eq. 40. Since the first kind of term seems to predominate in the decay, we can, as a good approximation, use the totally symmetric isotopic spin function. The ratio $(\pi^+ + 2\pi^0)/(\pi^- + 2\pi^+)$ is then uniquely determined to be $\frac{1}{4}$ (47). The experiments (as in Table VI) indicate the value 0.33 ± 0.07. More data are necessary to determine whether there is a serious disagreement with the "predicted" ratio 0.25. In any case, we know from the $K_{\pi 2}$ decay that the $|\Delta I| = \frac{1}{2}$ rule is not exact.

Let us now turn to the "τ" decays of neutral K mesons. If CP invariance holds, we can draw strong conclusions about the decay of K_1^0 and K_2^0 into 3π. We have seen in 6.5 that there is no 0^+ state of 3π; with a spinless K, therefore, the final 3π system has $P=-1$. There are two charge states: $3\pi^0$ and $\pi^+ + \pi^- + \pi^0$. Now π^0 is well-known to be even under charge conjugation (for example, it decays into 2γ) and thus a state of any number of π^0 mesons has $C=+1$. We see, then, that the final $3\pi^0$ system has $CP=-1$ and is a possible decay product of K_2^0 but not of K_1^0 if CP is conserved.

In the case of $\pi^+ + \pi^- + \pi^0$, we cannot conclude that $C=+1$ and $CP=-1$

[17] We have assumed a real coefficient of y in our empirical formula 40 for M. With CP invariance, the phase of the coefficient is specified, but unfortunately in terms of unknown phase shifts of the final three-pion system. Fitting the data with a complex coefficient of y might increase somewhat our estimate of the nonsymmetric fraction.

without some knowledge of the wave function. But to the extent that the matrix element is symmetric in π^+ and π^-, the conclusion does hold. Now in the case of charged τ decay we have seen that the matrix element is practically a constant and that the fraction of decays into totally symmetric pion states is probably >99 per cent. If we make the reasonable assumption that the neutral τ decays behave the same way, then we can say that the *rate* of $K_1^0 \to \pi^+ + \pi^- + \pi^0$ is at least ~ 100 times less than that of $K_2^0 \to \pi^+ + \pi^- + \pi^0$. Moreover, the lifetime of K_1^0 is at least 300 times shorter than that of K_2^0 and therefore the *fraction* of K_1^0 decays into $\pi^+ + \pi^- + \pi^0$ should be at least 3×10^4 smaller than the corresponding *fraction* for K_2^0.

Experimentally, the fraction of K_2^0 decays into 3π is quite appreciable (see Table VI). We can calculate the rates of these decays if we assume the validity of the $|\Delta I| = \frac{1}{2}$ rule and the practically total symmetry of the 3π wave function. The result is that the total rate of $K_2^0 \to 3\pi$ should equal the total rate of $K^+ \to 3\pi$ (6×10^6/sec. in Table VI), while the ratio $K_2^0 \to 3\pi^0 / K_2^0 \to \pi^+ + \pi^- + \pi^0$ should be 3/2.

6.7. *The decay* $K^\pm \to \mu^\pm \pm \nu$.—The fact that a large fraction of $K^\pm$ decay into $\mu^\pm \pm \nu$ makes possible an experiment on asymmetry in the $K \to \mu \to e$ chain analogous to those for the $\pi \to \mu \to e$ chain described in 4.5.

If K is spinless and if the longitudinal theory of the neutrino and conservation of leptons are both correct, then the two chains must behave exactly alike. The forward-backward asymmetry and the polarizations of μ and e must be identical for $K \to \mu \to e$ and $\pi \to \mu \to e$. As mentioned in 4.5, this has been borne out experimentally (57).

The rate of the decay $K^\pm \to \mu^\pm \pm \nu$ and the rarity or absence of $K^\pm \to e^\pm \pm \nu$ have been discussed in 4.9.

Assuming, as in the tetrahedron scheme, that $\nu\bar{\nu}$ and $\mu^\pm e^\mp$ pair emission can never occur, we see that the purely leptonic decay of $K^\pm$ has no neutral counterpart.

6.8. *The modes* $K \to \pi +$ *leptons; spectra.*— The decays of K mesons into $\pi+$leptons have been referred to briefly toward the end of 4.9. We remarked that they are induced by different interactions from those that lead to $K^\pm \to$leptons alone. For example, for a pseudoscalar K, the couplings we have labeled S, V, and T are responsible for decay into $\pi+$leptons, while P and A can induce pure leptonic decay. In Eq. 25 we have given a somewhat special definition of these couplings, employing the tetrahedron scheme and forming S, for example, by coupling $\bar{\mu}(1-\gamma_5)\nu$ to $p\Lambda$. More generally, in order to form S for the muon and neutrino, we might couple $\bar{\mu}(1-\gamma_5)\nu$ to other scalar field operators with the same strangeness properties as $\bar{p}\Lambda$, but let us continue to use Eq. 25 to fix our ideas. For the *electron* and neutrino, of course, we have similar definitions of the couplings with $\bar{\mu}(1-\gamma_5)\nu$ replaced by $\bar{e}(1-\gamma_5)\nu$, etc.

A complete discussion of the decays $K \to \pi +$ leptons involves 18 amplitudes. We must distinguish: (*a*) the electron and muon. (*b*) the three different processes $K^\pm \to \pi^0 + e^\pm \pm \nu$, $K_1^0 \to \pi^\mp + e^\pm \pm \nu$ and $K_2^0 \to \pi^\mp + e^\pm \pm \nu$ (likewise for the muon), and (*c*) the three types of interaction: *S*, *V*, and *T* for a pseudoscalar *K* or *P*, *A*, and *T* for a scalar *K*. The product of $2\times3\times3$ is 18.

(*a*) The relation between and muon couplings is unknown. The rates of decay into $\pi+e^{\pm}\pm\nu$ and $\pi+\mu^{\pm}\pm\nu$ seem to be about equal for both charged and neutral *K* mesons (see Table VI) but we have no idea whether the couplings are identical in form for electron and muon.

(*b*) If the lifetime of K_2^0 is around 10^{-7} sec., then the rates of $K^{\pm}\rightarrow\pi+$leptons and $K_2^0\pi+$leptons are comparable (see Table VI); however, the present limits on the K_2^0 lifetime are rather far apart (see Table I). The process $K_1^0\rightarrow\pi+$leptons has not yet been established. If its rate is comparable with that of $K_2^0\rightarrow\pi+$leptons, that still corresponds to a very small fraction of K_1^0 decays ($\lesssim 0.1$ per cent). Assuming that both K_1^0 and K_2^0 give $\pi+$leptons with similar rates, we must discuss those rare events in which neutral *K* mesons decay into $\pi+$leptons after $\lesssim 10^{-10}$ sec. in terms of interference between K_1^0 and K_2^0. This phenomenon is treated theoretically in 6.9, but has not been observed.

(*c*) Let us consider a particular decay, say $K_2^0\rightarrow\pi^{\mp}+e^{\pm}\pm\nu$, and compare the effects of various couplings. For simplicity we take the *K* pseudoscalar and use *S*, *V*, and *T*. (The same consequences would hold for a scalar *K* if we used *P*, *A*, and *T*.) Also we neglect the electron mass.

Following Pais & Treiman, (48) we may introduce three effective coupling parameters b_S, b_V, and b_T corresponding to the three interactions. Assuming CP invariance and neglecting tiny coulomb effects in the final state, we can take these to be real. In principle the *b*'s may depend on the pion total energy *w*; however, if the decays really proceed through baryon-anti-baryon loops this dependence is probably rather weak. Let $p(w)$ equal the pion momentum and $x(w)$ equal $p(w)/(m_K-w)$; let θ be the angle between *e* and π. Then the distribution of decays per unit energy and unit solid angle is proportional to (48)

$$\frac{(1-x^2)^2(m_K-w)^2}{(1+x\cos\theta)^4}\,p\left\{b_V{}^2x^2\sin^2\theta+\left[b_S(1+x\cos\theta)+\frac{b_Tp}{m_K}(x+\cos\theta)\right]^2\right\}. \qquad 41.$$

(Here and in the rest of the Section, we put $\hbar=c=1$.)

An analysis of a large number of three-body decays using this formula should permit the identification of the couplings involved. The longitudinal neutrino theory predicts that the electrons corresponding to the *V* term are 100 per cent longitudinally polarized in one sense and those corresponding to the *S*, *T* term 100 per cent polarized in the opposite sense. Using conservation of leptons and a right-handed neutrino, we see that the *V* interaction gives right-polarized electrons and left-polarized positrons, as in β-decay, μ-decay, etc.

The analogue of Eq. 41 for the muon case is more complicated; [it is given in ref. (48)]. There is longitudinal polarization of the muons also, again slightly more complicated because of the finite muon mass. This polarization can be detected by means of the asymmetry of $\mu\rightarrow e$ decay.

If CP invariance holds, there should be no appreciable transverse muon polarization, i.e., perpendicular to the plane of the K decay. (See Appendix A.)

6.9. *Interference phenomenon in K decay.*—We have referred in 6.8 to the possibility that both K_1^0 and K_2^0 decay into $\pi^\mp+e^\pm\pm\nu$ and/or $\pi^\mp+\mu^\pm\pm\nu$. In a "fresh" beam of neutral K mesons (see 3.9), out of which the K_1^0, component has not yet largely decayed, a small fraction (perhaps $\sim$0.1 per cent) of the decay products will be π+leptons. These events will arise from interference of K_1^0 and K_2^0 (49). (In a "stale" beam, of course, the decays into π+leptons are to be attributed to K_2^0 alone.)

For simplicity let us treat a definite model of the interference, namely that supplied by the tetrahedron scheme, in which $K^0\to\pi^-+e^++\nu$ and $\overline{K^0}\to\pi^++e^-+\bar{\nu}$ are allowed, but $K^0\to\pi^++e^-+\bar{\nu}$ and $\overline{K^0}\to\pi^-+e^++\nu$ are forbidden, and likewise for the muon (see 4.8).

We must recall the relations, given in 3.9, between K^0, $\overline{K^0}$ and K_1^0, K_2^0:

$$\begin{aligned} |K^0\rangle &= \frac{1}{\sqrt{2}}\,|K_1^0\rangle + \frac{1}{\sqrt{2}}\,|K_2^0\rangle, \\ |\overline{K^0}\rangle &= \frac{1}{\sqrt{2}}\,|K_1^0\rangle - \frac{1}{\sqrt{2}}\,|K_2^0\rangle. \end{aligned} \qquad 42.$$

Let Γ_1 and Γ_2 stand for the reciprocal lifetimes of K_1^0 and K_2^0 respectively, while m_1 and m_2 are their masses, which are presumed to differ by something like 10^{-11} Mev. We may then write the state of a neutral K meson as

$$|\Psi(t)\rangle = a_1|K_1^0\rangle e^{-im_1t}e^{-\Gamma_1t/2} + a_2|K_2^0\rangle e^{-im_2t}e^{-\Gamma_2t/2}. \qquad 43.$$

Suppose we are dealing with a beam that initially is pure K^0, i.e., has pure strangeness +1. Then $a_1=a_2=1/\sqrt{2}$ and we have

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\,|K_1^0\rangle e^{-im_1t}e^{-\Gamma_1t/2} + \frac{1}{\sqrt{2}}\,|K_2^0\rangle e^{-im_2t}e^{-\Gamma_2t/2}. \qquad 44.$$

The quantity $|\langle K^0|\Psi(t)\rangle|^2$ will now give the fraction of the original K^0 that remain in the beam after time t, while $|\langle\overline{K^0}|\Psi(t)|^2$ gives the fraction of the original K^0 that have been turned into $\overline{K^0}$. The sum of these fractions keeps decreasing with increasing t as mesons decay. We have

$$|\langle K^0|\Psi(t)\rangle^2 = |\tfrac{1}{2}e^{-im_1t}e^{-\Gamma_1t/2} + \tfrac{1}{2}e^{-im_2t}e^{-\Gamma_2t/2}|^2 \qquad 45.$$

and

$$|\langle\overline{K^0}|\Psi(t)\rangle|^2 = |\tfrac{1}{2}e^{-im_1t}e^{-\Gamma_1t/2} - \tfrac{1}{2}e^{-im_2t}e^{-\Gamma_2t/2}|^2 \qquad 46.$$

Let us now impose the condition implied by the tetrahedron scheme, that $K^0\to\pi^-$+leptons and $\overline{K^0}\to\pi^+$+leptons are allowed, with a common rate Γ_l (common because of CP invariance), while $\overline{K^0}\to\pi^-$+leptons and $K^0\to\pi^+$+leptons are forbidden. Then the number of leptonic decays per second per original K^0 in the beam is

$$\begin{aligned} L_- &= \Gamma_l|\langle K^0|\Psi(t)\rangle|^2 \quad \text{for } \pi^- + \text{leptons and} \\ L_+ &= \Gamma_l|\langle\overline{K^0}|\Psi(t)\rangle|^2 \quad \text{for } \pi^+ + \text{leptons.} \end{aligned}$$

Using Eqs. 45 and 46, we obtain

$$L_- = \frac{\Gamma_l}{4} \{e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-(\Gamma_1+\Gamma_2)t/2} \cos (m_1 - m_2)t\}, \qquad 47.$$

$$L_+ = \frac{\Gamma_l}{4} \{e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_1+\Gamma_2)t/2} \cos (m_1 - m_2)t\}, \qquad 48.$$

$$L \equiv L_+ + L_- = \frac{\Gamma_l}{2} \{e^{-\Gamma_1 t} + e^{-\Gamma_2 t}\}. \qquad 49.$$

Eq. 49 is perfectly straightforward. Both $K_1{}^0$ and $K_2{}^0$ can decay into leptons at the same rate Γ_l; initially we have 50 per cent of each, but each decays out according to its own lifetime.

The equations for L_- and L_+ separately are rather peculiar, however. L_- starts out at Γ_l and then decreases, while L_+ starts out at 0 and then increases at first. If $|m_1-m_2|$ is $\gtrsim\Gamma_l$, a striking oscillation phenomenon sets in, with a frequency corresponding to the very small mass difference. If $|m_1-m_2|$ is too small, the oscillations are badly damped and would be hard to detect. If $|m_1-m_2|$ is much larger than Γ_1, the oscillations are very rapid compared to the time of traversal of experimental apparatus and may be difficult to detect. If it should turn out, though, that $|m_1-m_2|$ and Γ_1 are comparable, then a measurement could be made of a mass difference $\sim 10^{-11}$ Mev.

Appendix A

Possible Noninvariance under CP and T.—It may still turn out that separate invariance under CP and T fails for weak couplings. We have pointed out in 2.5 that an absolute definition of right and left and of matter and antimatter would then be possible. Moreover, as we discussed in 2.6, the phases of transition matrix elements in weak processes would not be theoretically determined. Despite its unattractiveness, noninvariance under CP and T is a perfectly tenable hypothesis; present experimental evidence does not really distinguish between invariance and noninvariance.

The well-known equality of mass and lifetime for particle-antiparticle pairs like $\pi^\pm$, $\mu^\pm$, etc. is guaranteed by CPT invariance alone (2, 50) and is thus not evidence for CP invariance. The remarkable behavior of $K_1{}^0$ and $K_2{}^0$, which we have explained in the text on the basis of CP invariance, has been discussed by Lee, Oehme, & Yang (50) on the assumption that this principle fails. They show that without the invariance principle one may arrive at a rather similar picture of $K_1{}^0$ and $K_2{}^0$, and that experimental data so far are consistent with either situation.

In a specific field-theoretic model of the weak couplings, like the theory of four-fermion interactions treated in Section 4, CP or T invariance is equivalent to the (relative) reality of the coupling parameters C, D, E, etc. In the text we have written all the formulas for rates, longitudinal polarizations, and angular distributions in weak decays on the assumption that these coefficients are real. To obtain the corresponding formulas in the case of noninvariance under CP, we must replace C_S^2 by $|C_S|^2$, $2C_SC_T$ by $2\ \mathrm{Re}\ C_S^*C_T$, etc., apart from certain corrections (50a).

These corrections are introduced by interactions among the decay products, for example the Coulomb force in β-decay, and they vanish for cases like $\mu^\pm \rightarrow e^\pm + \nu + \bar{\nu}$, where there are no (appreciable) final state interactions. They take the form of terms in $\mathrm{Im}\ C_T^*C_A$, $\mathrm{Im}\ C_S^*C_T$, etc., which are zero if CP invariance holds.

Let us consider, for instance, the formula for the asymmetry parameter a in the decay of Co^{60} (see 2.5 and 4.3) under the assumption that both T and A interactions are present and that the neutrino is right-handed:

$$a = \frac{v}{c}\,\frac{|C_A|^2 - |C_T|^2}{|C_A|^2 + |C_T|^2 + \frac{2m}{E}\,\mathrm{Re}\ C_A^*C_T} - \frac{Zm}{137E}\,\frac{2\ \mathrm{Im}\ C_T^*C_A}{|C_A|^2 + |C_T|^2 + \frac{2m}{E}\,\mathrm{Re}\ C_A^*C_T}. \qquad \text{A-1}$$

The term in $\mathrm{Im}\ C_T^*C_A$ occurs only in the Coulomb correction.

Now there is a class of measurable quantities that follow the reverse pattern: the leading term is proportional to $\mathrm{Im}\ C_S^*C_T$, say, while only the correction due to final state interactions involves $\mathrm{Re}\ C_S^*C_T$, etc. Such quantities, unlike the ones previously considered, are odd under the reversal of time (50b).

We note that momenta $\mathbf{p}$ and angular momenta $\mathbf{J}$ or $\boldsymbol{\sigma}$ change sign under time reversal. The asymmetry parameter a of Eq. A-1 multiplies $\langle \mathbf{J}\rangle\cdot\mathbf{p}$ in an angular distribution; such a term is evidently even under T. Similarly the longitudinal polarization of elections is the coefficient of $\boldsymbol{\sigma}\cdot\mathbf{p}$, again even under T. To obtain a scalar or pseudoscalar that is odd under T, we must use at least three momenta and/or angular momenta, for example $\boldsymbol{\sigma}\cdot\mathbf{p}_1\times\mathbf{p}_2$. In the decay of polarized neutrons, $\mathbf{p}_1$ may stand for the electron momentum and $\mathbf{p}_2$ for the proton momentum.

Experiments are in progress to measure this angular correlation between the spin direction and the plane of decay of polarized neutrons. The coefficient γ of $\boldsymbol{\sigma}_n\cdot\mathbf{p}_e\times\mathbf{p}_p$ is proportional to

$$\mathrm{Im}\ (C_S^*C_T - C_V^*C_A) - \frac{m}{137p_e}\,\mathrm{Re}\ (C_S^*C_A - C_V^*C_T) \qquad \text{A-2}$$

if the neutrino is longitudinal.

The contrast between Eq. A-2 for a time-odd quantity and Eq. A-1 for a time-even quantity is what we have stressed. The measurement of a time-odd quantity can provide a clear-cut test of time reversal invariance. Consider the measurement of γ in neutron decay. Whether the nuclear β-decay coupling is S, T or V, A, there must be a large effect if T invariance is violated appreciably in β-decay. If T invariance holds, there should be no effect, or at most a tiny Coulomb term.

A similar experiment is possible involving the decay $K_2^0 \to \pi^{\mp} + \mu^{\pm} \pm \nu$, namely a search for a correlation $\boldsymbol{\sigma}_\mu \cdot \boldsymbol{p}_\mu \times \boldsymbol{p}_\pi$, where $\boldsymbol{\sigma}_\mu$ is measured by the asymmetry of $\mu \to e$ decay. If T invariance holds, there can be no such transverse muon polarization, except through a Coulomb effect. Longitudinal polarization of the muons is expected, however (see 6.8) and can be detected in the same experiment.

Appendix B: Polarization and Asymmetry; Symmetric Alignment and Anisotropy

We wish to discuss and contrast two different sorts of spin alignments of particles produced in a strong, parity conserving reaction such as

$$\pi + p \Rightarrow \Lambda + K. \qquad \text{B-1}$$

We assume the target protons are unpolarized and call the π and Λ momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ respectively.

B.1. *Polarization and "up-down" asymmetry.*—First we take up polarization, by which we mean a nonzero expectation value of the Λ spin along the direction $\boldsymbol{n} = \boldsymbol{p}_1 \times \boldsymbol{p}_2$ normal to the production plane containing $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$. Our reason for picking the particular direction $\boldsymbol{n}$ and ignoring all others stems from considerations of parity. When we measure the polarization $p(\theta)$ we are actually measuring $\langle \boldsymbol{\sigma} \cdot \boldsymbol{p}_1 \times \boldsymbol{p}_2 \rangle$. This expression is P-invariant, but no other combination of $\boldsymbol{\sigma}$ with $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ can be found which is P-invariant.

Since the target protons were originally unpolarized, the Λ cannot be polarized unless the interaction is capable of distinguishing between spin-up and spin-down by means of some spin-dependent operator like $\boldsymbol{L} \cdot \boldsymbol{\sigma}$. Thus if reaction B1 takes place so close to threshold that it proceeds overwhelmingly through s-wave, there can be no polarization. More precisely, polarization takes place through the interference between two partial waves.

Let us next consider the decay of a polarized particle. If the decay proceeds through two states of opposite parity (thus violating conservation of P) then we may expect the decay products to exhibit an "up-down" asymmetry with respect to the production plane.

B.2. *Symmetric alignment and "polar vs. equatorial" anisotropy.*—For particles with spin $> \frac{1}{2}$ the spin can still be aligned even in the absence of polarization: we may call this "symmetric alignment," meaning "symmetric about the production plane." As an example we discuss Λ production at 0° or 180° by reaction B.1. Let us assume that K is spinless (like the π) and choose the z-axis along the beam direction. The symmetric alignment is introduced by the fact that a particle can have no component of orbital angular momentum along its direction of motion, i.e., $\boldsymbol{L}_z = 0$ both before and after collision. For unpolarized target protons the two spin states $S_z = \pm \frac{1}{2}$ are equally probable, and the same must then apply for Λ. Now if Λ had spin $> \frac{1}{2}$ this restriction to $S_z = \pm \frac{1}{2}$ would mean that $\boldsymbol{S}$ tended to be perpendicular to the z-axis. Thus, as Adair has pointed out (51), we would have a beam of

Λ with their spins aligned symmetrically fore and aft with respect to the z-axis.

The angular distribution of decay products from such *aligned* Λ must exhibit a (fore-aft) symmetry with respect to the z-axis (i.e., must contain no odd terms in $\cos\theta$) but it will in general be *anisotropic* in a "polar vs. equatorial" sense (i.e., will contain terms in $\cos^2\theta$, etc.). For instance suppose that Λ has spin 3/2 and is produced at 0° or 180°. It is then easy to show that the angular distribution of its decay is

$$d\sigma/d\Omega \propto 1/3 + \cos^2\theta. \qquad \text{B-2}$$

Another reaction that will yield the same symmetric anisotropy is the absorption of K^- from an atomic s-state of hydrogen according to

$$K^- + p \Rightarrow \Lambda + \pi^0. \qquad \text{B-3}$$

Since $L_z = L = 0$ in the initial state this example is entirely equivalent to the one already discussed, and the distribution of the angle of decay is given by Eq. B2 (52).

B.3. *General Remarks*.—For spin $\frac{1}{2}$ particles alignment necessarily means polarization; "symmetric alignment" is meaningless. In the decay of spin $\frac{1}{2}$ particles the only possible deviation from isotropy is an "up-down" asymmetry resulting from polarization in production followed by lack of parity conservation in the decay.

For particles with spin $>\frac{1}{2}$ we have so far contrasted polarization with symmetric spin alignment. Of course in general when two or more partial waves interfere in a reaction we expect both polarization and symmetric alignment.

B.4. *Hyperon spins: associated production*.—Adair has recommended the use of reaction B.1 and Eq. B2 as a test of the Λ spin (or of the Σ spin if we use a reaction like $\pi^\pm + p \Rightarrow \Sigma^\pm + K^+$). Of course one cannot in practice restrict oneself to Λ or Σ produced exactly at 0° or 180°; however, as Adair suggests, the effect should not wash out very much if we restrict ourselves to angles within $1/L_{\max}$ radians of the forward and backward directions, where $L_{\max}$ is the largest important orbital angular momentum in the final state. Even far away from the forward and backward directions, we may expect some polar vs. equatorial anisotropy in the distribution of the decay angle if the hyperon has spin $>\frac{1}{2}$. At production angles comparable to or large compared with $1/L_{\max}$, however, the effect could accidentally vanish or be small, whereas near 0° or 180° it must exist in full strength.

The present experimental situation is that less than 1000 hyperon decays have been observed following associated production and only a few tens of these satisfy Adair's condition. Neither the unselected events nor those satisfying Adair's condition permit any useful conclusion about the spin of Λ or Σ; however there is another reaction, Eq. B3, which we now discuss.

B.5. *Hyperon spins: K^- capture from rest*.—Bubble chamber and emulsion groups have reported the decay of nearly 500 hyperons coming from reaction B.3. Table BI shows that the numbers of polar and equatorial decays are almost equal. If we know that the (spinless) K^- cascaded down all the way to an atomic s-state before capture this near-isotropy would surely establish that both Λ and Σ have spin $\frac{1}{2}$. We are not sure, however, that capture from p-states is unimportant, and so it can only be said that the data are consistent with spin $\frac{1}{2}$ but also with spin 3/2.

TABLE BI

POLAR VS EQUATORIAL DECAYS OF Λ AND Σ FROM K^- CAPTURES AT REST

	$n_{\text{polar}}/n_{\text{total}}$
Λ from K^-+P*	$\frac{31}{58}=0.53\pm.06$
$\Sigma^{\pm}$ from K^-+P*	$\frac{87}{159}=0.55\pm.04$
$\Sigma^{\pm}$ from K^-+emulsion†	$\frac{237}{432}=0.55\pm.024$

n_{polar} includes those events with $|\cos\theta|>1/2$. For an isotropic distribution of decays, $n_{\text{polar}}/n_{\text{total}}$ should be 1/2.

* Alvarez *et al.*, UCRL-3774 (1957) and *Nuovo cimento*, **5**, 1026 (1957).

† Compilation of all emulsion data available from all laboratories ($\approx 90\%$ Σ^+, 10% Σ^-) compiled by G. Snow for the Seventh Rochester Conference.

APPENDIX C. CONSTRUCTION OF THE DALITZ PLOT AND DENSITY OF STATES IN τ DECAY

In order to find a convenient representation for the "τ" decay $K\to 3\pi$, Dalitz has taken advantage of the fact that if a point α is plotted inside an equilateral triangle (Fig. 7) and from it perpendiculars of length T_1, T_2, T_3 are dropped to its three sides, then the sum $T_1+T_2+T_3$ of these lengths is equal to the height Q of the triangle. If the pions are treated nonrelativistically, it can be shown that the momentum condition $p_1+p_2+p_3=0$ forces the point α to lie within the inscribed circle. Inspection of the figure gives the Cartesian coordinates x and y in terms of the kinetic energies T; the normalization is chosen so that the circle has unit radius $(0\le x^2+y^2\le 1)$.

We shall now show that unit area $dxdy$ on the Dalitz plot is proportional to unit volume in phase space, even if the τ decay is treated relativistically.

A familiar expression for the decay rate Γ for $K\to 3\pi$ is

$$\Gamma=\frac{2\pi}{\hbar}\frac{\Omega^2}{(2\pi\hbar)^6}\int d^3p_1\int d^3p_2\int d^3p_3\,\delta\left(\sum_i p_i\right)\delta\left(m_Kc^2-\sum_i w_i\right)|R|^2 \qquad \text{C-1.}$$

where R is the transition matrix element, p_i is the momentum of the ith pion, and w_i its total energy. The normalization volume is Ω. Because of the constraints on energy and momentum and the angular symmetries of the

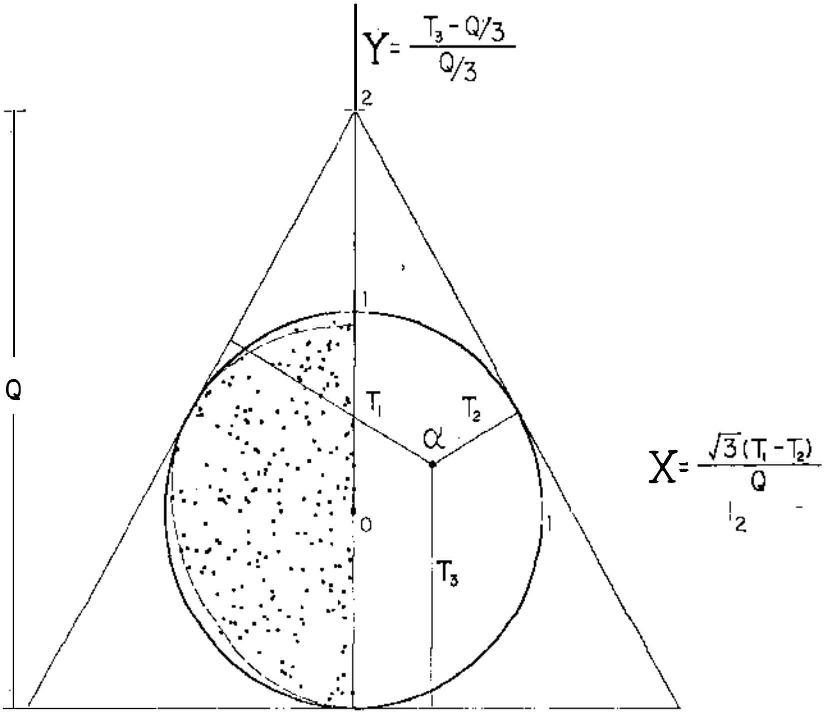

FIG. 7. Dalitz triangular construction for the distribution of pion energies in the "τ" decay $K\to\pi_1^+ + \pi_2^+ + \pi_3^-$.

problem, the matrix element R depends on only two variables, which we may choose to be w_1 and w_2. Thus we may write

$$\Gamma = \frac{2\pi}{\hbar}\int\int dW_1 dW_2 \left| R(W_1, W_2)\right|^2 \frac{\Omega^2}{(2\pi\hbar)^6}\rho(W_1, W_2), \qquad \text{C-2}$$

where

$$\rho(W_1, W_2) = \int\int\int d^3p_1 d^3p_2 d^3p_3 \delta(\sum p_i)\delta(m_K c^2 - \sum w_i)\delta(w_1 - W_1)\delta(w_2 - W_2). \quad \text{C-3}$$

Now we can reduce C-3 to the simple form

$$\rho(W_1, W_2) = \frac{8\pi^2}{c^6} W_1 W_2 W_3; \qquad (W_3 = m_K c^2 - W_1 - W_2) \qquad \text{C-4}$$

by proceeding as follows:

We integrate out the momentum p_3 and all angles except the angle θ between p_1 and p_2, obtaining

$$\rho(W_1, W_2) = \int 4\pi p_1^2 dp_1 \int 2\pi p_2^2 dp_2 \int d(\cos\theta)\delta(m_K c^2 - \sum w_i)\delta(w - W_1)\delta(w_2 - W_2). \quad \text{C-5}$$

Now we can put

$$w_3^2 = (cp_3)^2 + (m_\pi c^2)^2 = (cp_1)^2 + (cp_2)^2 + 2c^2 p_1 p_2 \cos\theta + (m_\pi c^2)^2 \qquad \text{C-6.}$$

and, taking the differential of both sides with p_1 and p_2 fixed, we find

$$2w_3 dw_3 = 2c^2 p_1 p_2 d(\cos\theta). \qquad \text{C-7.}$$

Using C-7 and the relation $p_i dp_i = w_i dw_i/c^2$, we have in place of C-5

$$\rho(W_1, W_2) = \frac{8\pi^2}{c^6}\int\int\int w_1 dw_1 w_2 dw_2 w_3 dw_3 \delta(w_1 - W_1)\delta(w_2 - W_2)\delta(w_3 - W_3), \qquad \text{C-8.}$$

which immediately gives C-4.

It is convenient to factor out of the R matrix element the product

$$\sqrt{N} = \prod_i \frac{1}{\sqrt{2w_i}}$$

that comes from the creation of three pions. We are then left with the Feynman matrix element $M = R/\sqrt{N}$, which is a world-scalar (53). In the nonrelativistic limit, N is a constant and M and R are simply proportional. Relativistically, it is $|M|^2$ that is proportional to the population of dots in the Dalitz plot, as we now show.

We use C-2, C-4, and the definition of M to obtain

$$\Gamma = \frac{2\pi}{\hbar}\int\int dw_1 dw_2 |M(w_1, w_2)|^2 \cdot \frac{\pi^2}{c^6}\frac{\Omega^2}{(2\pi\hbar)^6} \qquad \text{C-9.}$$

We see immediately that if dw_1dw_2 is the element of area in the Dalitz plot, then the density of dots is proportional to $|M|^2$. We must show that $dxdy \propto dw_1dw_2$, where x and y are the Dalitz coordinates of Figures 6 and 7. Clearly $dw_1dw_2 = dT_1dT_2$, since $w_i = T_i + m_\pi c^2$. But x and y are related to T_1 and T_2 by a linear transformation with constant Jacobian, and so $dxdy \propto dT_1dT_2$.

Appendix D

Hypernuclei.—Nuclear matter can bind Λ to form systems stable for a time comparable with the Λ mean life. Such systems are well known and are called hypernuclei or hyperfragments. This topic has been treated thoroughly by Dalitz in his forthcoming review (R3). Experimental data have recently been surveyed by Levi-Setti, Slater, and Telegdi (54).

Mesonic vs. nonmesonic decay.—In the decay of a bound Λ we have two competing processes: the mesonic decay, for example the mode

$$\Lambda \to p + \pi^-_{\text{real}} \text{ at a rate } \Gamma_{\text{real}} \quad (|p_{\text{real}\,\pi}| = 99.8 \text{ Mev}/c), \qquad \text{D-1.}$$

and the nonmesonic mode. These competing rates can be compared most conveniently by assuming a simple model in which the nonmesonic mode arises principally from the "internal conversion" of a virtual π, for example

$$\Lambda \to p_1 + \pi^-_{\text{virt}},\ \pi^-_{\text{virt}} + p_2 \Rightarrow n, \text{ rate } \Gamma_{\text{virtual}} \quad (|p_{\text{virt}\cdot\pi}| \approx |p_n| \approx 380 \text{ Mev}/c). \qquad \text{D-2.}$$

TABLE DI

RATIOS OF NON-π^--MESONIC TO π^--MESONIC DECAYS

I Hyperfragment	II $\dfrac{\text{non-}\pi^-\text{-mesonic}}{\pi^-\text{-mesonic}}$	III $\dfrac{\text{nonmesonic}}{\pi^-\text{-mesonic}}$	IV $Q^{(-)}$ Estimated from data	V Theoretical $Q^{(-)}$
Hydrogen	$\frac{0}{6}$	$\frac{0}{6}$	$\frac{0}{6}=0$	0.5
Helium	$\frac{18}{7}$	$\frac{16}{7}$	$\frac{11}{7}=1.5$	1.1
Lithium	$\frac{31}{2}$	$\frac{31}{2}$	$\frac{20}{2}=10$	—
$Z>3$	$\frac{140}{2}$	$\frac{140}{2}$	$\frac{93}{2}=46$	50.0

Table DI: Internal Conversion Data for Hyperfragments; Col. II [from Schneps, Fry & Swami (56)] is observed "directly" when hyperfragments decay in emulsion. To calculate $Q^{(-)}$ one must go through two steps. First we get from Col. II, which lists "non-π^--mesonic" to Col. III, which lists "nonmesonic," by estimating the number of decays in which an unseen π^0 escaped. To get Col. IV one must multiply the numerators by $\frac{2}{3}$ to correct for the fact that about $\frac{1}{3}$ of the nonmesonic decays result from conversion of virtual neutral pions (see Table II). (We assume that the probability for a π^0 to be absorbed by either protons or neutrons equals the probability for π^- to be absorbed by protons only.) The purpose of the quotation marks around the word "directly" is to emphasize the considerable uncertainties introduced by scanning bias and experimental ambiguities arising from the fact that it is relatively easy to identify mesonic decays, whereas nonmesonic decays can look like π^- captures, nuclear interactions, etc., Col. V gives the calculated $Q_a^{(-)}$ of Ruderman & Karplus (55).

The momentum involved is estimated by dividing the available energy between two nucleons.

Just as in the case of the internal conversion of a nuclear γ ray, the internal conversion coefficient $Q=\Gamma_{\text{virt}}/\Gamma_{\text{real}}$ is a very sensitive function of the orbital angular momentum carried by the (real or virtual) pion. Using the simple model mentioned above, and assuming the pions have angular momentum l, Ruderman & Karplus (55) have shown that Q_l is proportional to the relative probability of penetrating an angular momenta barrier, i.e.,

$$Q_l^{(-)}(Z) = Q_0^{(-)}(Z)(p_{\text{virt}}/p_{\text{real}})^{2l} = Q_0^{(-)}(Z) \times 15^l. \qquad \text{D-3.}$$

$Q_0^{(-)}$ is proportional to the density of protons near the Λ in a nucleus of charge Z. In heavy nuclei $Q_0^{(-)}(Z)$ must also be corrected for many complications such as the Pauli exclusion principle and self-absorption of real π^- before they escape from the nucleus. The subscript $(-)$ means we consider only the decay mode $p+\pi^-$ (real or virtual).

Cols. II of Table D1 gives data obtained with nuclear emulsion and Cols. III and IV convert the data into an experimental estimate of $Q^{(-)}$. Col. V gives the Ruderman-Karplus estimate of $Q_0^{(-)}$. We see that the experimental $Q^{(-)}$ agrees with the theoretical $Q_0^{(-)}$ (i.e., for $l=0$), and disagrees badly with $Q_l^{(-)}$ for $l>0$ (Eq. D-3). Even though Ruderman & Karplus claim that their calculation of $Q_0^{(-)}$ is reliable only within a factor three, the argument for at least some decay amplitude through $l=0$, and therefore the evidence for a Λ spin $S_\Lambda=\frac{1}{2}$, is impressive.

Let us next try to get quantitative (and less reliable) information from the data, namely to estimate $x_- \equiv |S_-/P_-|^2$ where S_- and P_- are the relative amplitudes of s- and p-wave in the decay $\Lambda \rightarrow p+\pi^-$ as defined by Eq. 27. Eq. D-3 generalizes, in the case of mixed s- and p-waves, to

$$Q^{(-)} = 1.1\left(\frac{1+15x_-}{1+x_-}\right) \qquad \text{D-4.}$$

If we took this equation and the data perfectly seriously we would find $x_- < 1/7$. If we assume the value $Q_0=1.1$ is uncertain by no more than a factor 3 and also allow for the experimental uncertainty, we still find $x_- < \frac{1}{2}$. Despite the uncertainties the argument does show that the p-wave channel is not dominant. Of course if x_- is as large as $\frac{1}{2}$, $|P/S|$ is $1/\sqrt{2}$, and the asymmetry parameter α_- of Eq. 32 can be nearly unity.

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